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INTEGRALS OF PLANETARY MOTION SUITABLE FOR AN INDEFINITE LENGTH OF TIME.

By G. W. HILL.

The desirableness of having in our possession a feasible method for procuring expressions for the coordinates of the planets not limited to short intervals of time about an adopted epoch cannot be disputed. The general theory of such expressions is now well known, and the difficulties attending the subject are reduced to those of elaboration. It must be confessed that when all the parameters involved are left indeterminate in the formulas the latter have a degree of complexity that is truly frightful.

Gyldén devoted the latter years of his life to the investigation of this matter. But he left everything in an incomplete state, and it is obvious that he undertook too much in endeavoring to provide for the eight major planets of the solar system at once.

Is it not likely, that, leaving supplementary efforts for the future, a more satisfactory result may be attained by advancing with a lighter load? In accordance with such a view I have undertaken to mark out a practicable route in treating the simplest case suggested by the constitution of the solar system. Let us suppose that *Jupiter* and *Saturn* are alone considered, and that they are made to move in the same plane. Also let the masses of the three bodies concerned, the two constants called protometers by Gyldén, and the two constants attached severally to the integrals of living force and the conservation of areas, be known numerical quantities. It is proposed to treat the problem thus limited. The deviations in passing from the ideal to the actual case can afterwards be estimated by methods similar to Lagrange's variation of arbitrary constants. It is evident that the coefficients of the various inequalities brought out in the treatment of the simplified problem will be functions of two indeterminate constants, which we may designate by e_0 and e_0', and which are the moduli of the deviations of the orbits from circularity. These functions admit of development in powers and products of these constants, the multipliers always turning out as numbers. It is the latter circumstance which renders the treatment at all practicable. If we were to insist on the four linear elements, as well as the masses being kept indeterminate in the coefficients, we should find the latter incapable of expression.

The fundamental conceptions of Gyldén are employed in the following treatment, and I desire to express my high sense of their value, nevertheless, in the interests of brevity, many modifications have been made in the ulterior procedures.

The motions of *Jupiter* and *Saturn* relative to the *Sun* are treated simultaneously, that is, as if we were concerned with the motion of a single point in a space of four dimensions.

Let the following scheme show the notation for masses and rectangular coordinates: —

	Masses	Coordinates	
Sun	M		
Jupiter	m	ξ	η
Saturn	m	ξ'	η'

The motion of the planets relative to the *Sun* is bound up in the expressions of two functions; *first*, T the living force deduced by multiplying half the product of every two masses by the square of the velocity of one relative to the other, adding the three terms thus obtained and dividing by the mass of the system; *second*, Ω the potential function equivalent to the sum of the three terms given by dividing the product of every two masses by their distance. Thus,

$$T = (M+m+m')^{-1}\left[Mm\frac{d\xi^2+d\eta^2}{2dt^2}+Mm'\frac{d\xi'^2+d\eta'^2}{2dt^2}+mm'\frac{(d\xi'-d\xi)^2+(d\eta'-d\eta)^2}{2dt^2}\right]$$

$$\Omega = \frac{Mm}{\sqrt{\xi^2+\eta^2}}+\frac{Mm'}{\sqrt{\xi'^2+\eta'^2}}+\frac{mm'}{\sqrt{(\xi'-\xi)^2+(\eta'-\eta)^2}}.$$

* These equations are identical with those adopted by Lagrange in his *Essai*, except that here the third coordinate is made to vanish. One may consult Laplace, *Mécanique Céleste*, *Première Partie*, Liv. II, Art. 9, especially Equation (7); Tom. I, p. 131, Old Ed.

By putting

$$\mu_1 = \frac{(M+m')\,m}{M+m+m'} \quad , \quad \mu_2 = \frac{(M+m)\,m'}{M+m+m'} \quad , \quad \mu_3 = \frac{m\,m'}{M+m+m'}$$

we may write

$$T = \mu_1\frac{d\xi^2+d\eta^2}{2dt^2} + \mu_2\frac{d\xi'^2+d\eta'^2}{2dt^2} - \mu_3\frac{d\xi d\xi'+d\eta d\eta'}{dt^2}$$

But it is convenient to have a form of this function consisting of two terms, each involving two coordinates. We get this at the expense of complicating the form of Ω. Submitting to this, however, we introduce two hypothetical planets for the actual. Let the coordinates of the former be x, y and x', y' connected with the coordinates of the actual planets (κ being a constant) by the equations

$$\xi = x+\kappa x' \quad , \quad \eta = y+\kappa y' \quad , \quad \xi' = x'+\kappa x \quad , \quad \eta' = y'+\kappa y$$

Now consider the quadratic form

$$\mu_1\xi^2 + \mu_2\xi'^2 - 2\mu_3\xi\xi'$$

By the substitution this becomes

$$[\mu_1+\mu_2\kappa^2-2\mu_3\kappa]\,x^2 + [\mu_2+\mu_1\kappa^2-2\mu_3\kappa]\,x'^2 + 2[(\mu_1+\mu_2)\,\kappa-\mu_3\,(1+\kappa^2)]\,xx'$$

If κ is adopted so as to satisfy the equation

$$\frac{\kappa}{1+\kappa^2} = \frac{\mu_3}{\mu_1+\mu_2}$$

(we can take the smaller root of the quadratic) the term in xx' will disappear. Instead of eliminating κ eliminate μ_3 from the expression, and put

$$\mathrm{m} = \frac{1-\kappa^2}{1+\kappa^2}(\mu_1-\mu_2\kappa^2) \quad , \quad \mathrm{m}' = \frac{1-\kappa^2}{1+\kappa^2}(\mu_2-\mu_1\kappa^2)$$

and the quadratic form becomes

$$\mathrm{m}x^2 + \mathrm{m}'x'^2$$

This linear transformation being applied to the living force T, gives rise to the expression

$$T = \mathrm{m}\frac{dx^2+dy^2}{2dt^2} + \mathrm{m}'\frac{dx'^2+dy'^2}{2dt^2}$$

or, using r for the radius and v for the longitude, in terms of polar coordinates, to

$$T = \mathrm{m}\frac{dr^2+r^2dv^2}{2dt^2} + \mathrm{m}'\frac{dr'^2+r'^2dv'^2}{2dt^2}$$

But since we wish Ω to involve only three variables, we further transform by making

$$v = \tfrac{1}{2}(\psi+\phi) \quad , \quad v' = \tfrac{1}{2}(\psi-\phi)$$

which leads to

$$T = \mathrm{m}\frac{dr^2+\frac{1}{4}r^2(d\psi+d\phi)^2}{2dt^2} + \mathrm{m}'\frac{dr'^2+\frac{1}{4}r'^2(d\psi-d\phi)^2}{2dt^2}$$

Denoting the variables severally conjugate to r, r', ϕ, ψ by the symbols s, s', u, w, we shall have

$$s = \mathrm{m}\frac{dr}{dt}\,,\quad s' = \mathrm{m}'\frac{dr'}{dt}\,,\quad u = \tfrac{1}{4}\mathrm{m}r^2\frac{d\psi+d\phi}{dt} - \tfrac{1}{4}\mathrm{m}'r'^2\frac{d\psi-d\phi}{dt}$$

$$w = \tfrac{1}{4}\mathrm{m}r^2\frac{d\psi+d\phi}{dt} + \tfrac{1}{4}\mathrm{m}'r'^2\frac{d\psi-d\phi}{dt}$$

The derivatives being eliminated from T by means of these equations,

$$T = \frac{1}{2\mathrm{m}}\left[\left(\frac{w+u}{r}\right)^2 + s^2\right] + \frac{1}{2\mathrm{m}'}\left[\left(\frac{w-u}{r'}\right)^2 + s'^2\right]$$

The potential function in terms of the variables last adopted is

$$\Omega = \frac{Mm}{\sqrt{r^2+2\kappa rr'\cos\phi+\kappa^2r'^2}} + \frac{Mm'}{\sqrt{r'^2+2\kappa rr'\cos\phi+\kappa^2r^2}} + \frac{\dfrac{m\,m'}{1-\kappa}}{\sqrt{r'^2-2rr'\cos\phi+r^2}}$$

Putting F for $\Omega - T$, the differential equations of the problem are

$$\frac{ds}{dt} = \frac{\partial F}{\partial r}\,,\quad \frac{ds'}{dt} = \frac{\partial F}{\partial r'}\,,\quad \frac{du}{dt} = \frac{\partial F}{\partial \phi}\,,\quad \frac{dw}{dt} = \frac{\partial F}{\partial \psi}$$

$$\frac{dr}{dt} = -\frac{\partial F}{\partial s}\,,\quad \frac{dr'}{dt} = -\frac{\partial F}{\partial s'}\,,\quad \frac{d\phi}{dt} = -\frac{\partial F}{\partial u}\,,\quad \frac{d\psi}{dt} = -\frac{\partial F}{\partial w}$$

But F does not contain ψ, hence $\frac{dw}{dt} = 0$ and $w = h$ a constant. This value may be substituted in T, and thus

$$T = \frac{1}{2\mathrm{m}}\left[\left(\frac{h+u}{r}\right)^2 + s^2\right] + \frac{1}{2\mathrm{m}'}\left[\left(\frac{h-u}{r'}\right)^2 + s'^2\right]$$

Employing this in F the equations of the problem are the independent system of six:

$$\frac{ds}{dt} = \frac{\partial F}{\partial r}\,,\quad \frac{ds'}{dt} = \frac{\partial F}{\partial r'}\,,\quad \frac{du}{dt} = \frac{\partial F}{\partial \phi}$$

$$\frac{dr}{dt} = -\frac{\partial F}{\partial s}\,,\quad \frac{dr'}{dt} = -\frac{\partial F}{\partial s'}\,,\quad \frac{d\phi}{dt} = -\frac{\partial F}{\partial u}$$

with the equation $w = h$, and the equation (to be treated by a quadrature)

$$\frac{d\psi}{dt} = -\frac{\partial F}{\partial h}$$

We have still one more integral of the problem, viz., $F = C$ a constant, and t is not explicitly involved in the equations. Thus we may dispense with t as the independent variable and employ some other in its place. It is well known that GYLDÉN's aim was to determine the radius of each planet as a function of its longitude, and thus he adopts the latter as the independent variable. But we are almost necessitated to have only one independent variable through the whole treatment of the problem. It will be advantageous to select a variable already contained in the equations. The only one suitable appears to be ϕ or the elongation of the hypothetical planets; this like t can be regarded as passing from $-\infty$ to $+\infty$, and $\frac{d\phi}{dt}$ never vanishes.* It will be seen that Ω involves this variable through the function $\cos\phi$, hence, no elaboration of this factor is needed with the proposed choice.

By division of the differential equations we obtain

$$\frac{ds}{d\phi} = -\frac{\frac{\partial F}{\partial r}}{\frac{\partial F}{\partial u}}\ ,\quad \frac{ds'}{d\phi} = -\frac{\frac{\partial F}{\partial r'}}{\frac{\partial F}{\partial u}}\ ,\quad \frac{du}{d\phi} = -\frac{\frac{\partial F}{\partial \phi}}{\frac{\partial F}{\partial u}}$$

$$\frac{dr}{d\phi} = -\frac{\frac{\partial F}{\partial s}}{\frac{\partial F}{\partial u}}\ ,\quad \frac{dr'}{d\phi} = -\frac{\frac{\partial F}{\partial s'}}{\frac{\partial F}{\partial u}}\ ,\quad \frac{dt}{d\phi} = -\frac{1}{\frac{\partial F}{\partial u}}$$

Also, we may write

$$\frac{d\psi}{d\phi} = \frac{\frac{\partial F}{\partial h}}{\frac{\partial F}{\partial \phi}}$$

A simpler form may be given to these equations; by solving the equation $F = C$, u being regarded as the unknown, we arrive at

$$u = W \text{ a function of } r,\ r',\ s,\ s',\ \phi$$

Then we may write

$$\frac{ds}{d\phi} = \frac{\partial W}{\partial r}\ ,\quad \frac{ds'}{d\phi} = \frac{\partial W}{\partial r'}\ ,\quad \frac{du}{d\phi} = \frac{\partial W}{\partial \phi}$$

$$\frac{dr}{d\phi} = -\frac{\partial W}{\partial s}\ ,\quad \frac{dr'}{d\phi} = -\frac{\partial W}{\partial s'}\ ,\quad \frac{dt}{d\phi} = -\frac{\partial W}{\partial C}$$

to which may be added

* The Julian year being the unit of time, a rude computation has given 81211″ and 50122″ as the greatest and least values of $\frac{d\phi}{dt}$

$$\frac{d\psi}{d\phi} = -\frac{\partial W}{\partial h}$$

The four differential equations bearing on the variables r, r', s, s' constitute an independent system to be integrated by itself. Thus, r, r', s, s' will be got as functions of ϕ the independent variable. The remainder of the work on the problem consists, first, of a quadrature executed on the equation

$$\frac{dt}{d\phi} = -\frac{\partial W}{\partial C}$$

by means of which t will be obtained as a function of ϕ, and, by inversion, ϕ as a function of t: and, second, of a quadrature executed on the equation

$$\frac{d\psi}{d\phi} = -\frac{\partial W}{\partial h}$$

by which ψ will be had as a function of ϕ, and thence of t.

It will be seen that these operations introduce six additional arbitrary constants, which, with h and C, make up the eight demanded by the problem.

If it is thought undesirable to keep h and C evident in the expression of W, we can have recourse to the equations

$$\frac{d\phi}{dt} = -\frac{\partial F}{\partial u}\ ,\quad \frac{dv}{dt} = \frac{h+W}{\mathrm{m}r^2}\ ,\quad \frac{dv'}{dt} = \frac{h-W}{\mathrm{m}'r'^2}$$

We have now to consider the derivation of W. This is obtained from the solution of a quadratic. This quadratic is

$$\frac{(h+W)^2}{\mathrm{m}r^2} + \frac{(h-W)^2}{\mathrm{m}'r'^2} = 2(\Omega - C) - \frac{s^2}{\mathrm{m}} - \frac{s'^2}{\mathrm{m}'}$$

To simplify the solution of this we can put

$$W = hV\ ,\quad \frac{1}{\mathrm{m}r^2} = \rho^2\cos^2 v\ ,\quad \frac{1}{\mathrm{m}'r'^2} = \rho^2\sin^2 v$$

$$\frac{h+W}{\sqrt{\mathrm{m}}\,r} = \sqrt{2(\Omega - C) - \frac{s^2}{\mathrm{m}} - \frac{s'^2}{\mathrm{m}'}}\,\cos\nu$$

$$\frac{h-W}{\sqrt{\mathrm{m}'}\,r'} = \sqrt{2(\Omega - C) - \frac{s^2}{\mathrm{m}} - \frac{s'^2}{\mathrm{m}'}}\,\sin\nu$$

Whence may be derived

$$\cos(\nu - v) = \frac{2h}{\rho\sqrt{2(\Omega - C) - \frac{s^2}{\mathrm{m}} - \frac{s'^2}{\mathrm{m}'}}}\ ,\quad W = h\,\frac{\cos(\nu + v)}{\cos(\nu - v)}$$

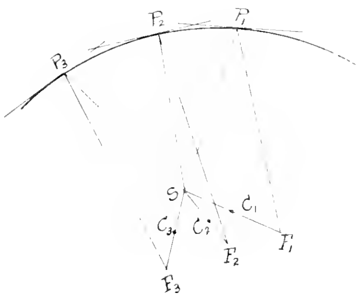
The quadratic in V can be given the form

$$V^2 + 2V\cos 2v + 1 = \frac{1}{h^2\rho^2}\left[2(\Omega - C) - \frac{s^2}{\mathrm{m}} - \frac{s'^2}{\mathrm{m}'}\right]$$

This equation will be more useful to us in deriving the value of V or W than the equation involving the auxiliary angle ν.

Since W involves no less than four square radicals, it is sufficiently plain that, with the four dependent variables r, r', s, s', nothing can be accomplished in the line of integration. We are therefore led to make a transformation of variables such that the radicals in the expression of W may be got rid of. GYLDÉN's notions relative to this step in the treatment of the planetary problem are valuable, but, what is singular, he has never given a philosophical presentation of them. We adopt the essential part of them, reserving the privilege of making extensive modifications in the remainder.

We bear in mind that it is always possible to study the form of orbits without regard to the question as to what particular points the planets occupy at stated times. To show what, in fact, is at the bottom of GYLDÉN's principles, the annexed diagram is drawn. Let the curve $P_1P_2P_3$ be a portion of the relative orbit of a planet about the



sun S. Suppose we have it in our power to draw the tangents to the curve at the points P_1, P_2, P_3. These tangents may be regarded as the traces of mirrors perpendicular to the plane of the orbit, and SP_1, SP_2, SP_3 being rays of light emanating from the *Sun*, let the directions of the reflected rays be P_1F_1, P_2F_2, P_3F_3. Next adopt a linear magnitude a, named the protometer by GYLDÉN, and take the points F so that, generally, $SP+PF$ shall equal double this. The fixed point S being called the occupied focus of the curve, the points F_1, F_2, F_3 may be called the empty foci of the curve severally belonging to the points P_1, P_2, P_3 and correspondent to the protometer a. As the point P moves along the curve from P_1 through P_2 to P_3 the general empty focus F will move on a curve starting from F_1, passing through F_2 to F_3. As we have drawn the radii SP, so we may draw the radii SF. Note the general quotient

$$\frac{SF}{SP+PF}$$

of which the denominator is constant; this is called the eccentricity (GYLDÉN's diastem) of the orbit at the point P correspondent to protometer a. Also the direction of SF is that of the apsides at P correspondent to protometer a; and the difference of the directions SP and FS taken so as to augment with the motion of P is called the true anomaly (GYLDÉN's diastematic argument) at P correspondent to protometer a.

The properties of the orbit may then be studied in the path of the empty focus F. There is nothing which necessitates a determinate value for a, but practical considerations lead us to adopt a value making F move much more slowly than P. It is easy to see that a value may be adopted such that when P makes a movement of the order of the solar mass, F makes a movement of the order of the disturbing planetary masses. If we have no other information as to a proper value for a, we may use the semi-axis of the instantaneous ellipse which prevails at any moment, or half the sum of the radii at a perihelion and an aphelion passage if the latter are consecutive. The protometer is a superabundant constant; if it is left indeterminate in the integrals of the problem, on their substitution in the original differential equations, the latter will fail to be satisfied unless a condition is established enabling us to reduce the number of introduced constants by a unit. It is not necessary, however, that the eliminated constant should be a protometer; we may elect to remove one of the others.

In the theory of *Jupiter* and *Saturn*, by properly adopting a and a', the empty foci of the two orbits may be made to move so slowly that, omitting minor oscillations, they do not accomplish what may be called a relative revolution in their positions in less than 54000 years.

Precisely as we have had a protometer and empty focus for the planet's path, we may have similar things for the path of F. Here the protometer will generally be smaller than the first, and the path of the second order wholly contained within the path of the first F. In stable planetary motion it is to be expected that when the operation of establishing an empty focus is repeated many times, the movement of the last F may be small enough to be neglected, and we thus shall have an empty focus as fixed as the occupied one S.

Limiting our attention to the empty focus of the first order, we see that when we have adopted a value for the protometer and know the position of the corresponding empty focus with the longitude of the planet, we know the position of the latter in space as well as its velocities, it being, of course, assumed that we know the value of W or V at the moment concerned. For convenience in graphic exhibition we have supposed in the diagram that we had the power of drawing the tangent to the planetary path at

any point. It does not, at first sight, seem that the knowledge of the variable

$$s = \mathrm{m}\frac{dr}{dt}$$

could be tantamount to this: but when it is noted that

$$\frac{dr}{dt} = \frac{s}{\mathrm{m}} \quad , \quad \frac{dv}{dt} = h\frac{1+V}{\mathrm{m}r^2}$$

we see that

$$\frac{dr}{dv} = \frac{r^2 s}{h\,(1+V)} = \frac{r^2 s}{h+W}$$

Thus, if we know the values of s, h and W, the tangent can be drawn.

It is apparent from all this that the planet may be conceived to move at each moment in an ellipse with a constant major-axis, but with the eccentricity and the line of apsides in constant variation. The principle of the moving empty focus has been invoked chiefly to find a transformation of the four variables r, r', s, s' suitable for the purpose of enabling us to get rid of the square radicals which appear in W.

Let us call the protometer a, the eccentricity e, and the true anomaly f (Gyldén's symbols for the latter are η and F). Then we propose to replace the four variables r, r', s, s' by the four f, f', e, e'. We immediately have

$$r = a\frac{1-e^2}{1+e\cos f} \quad , \quad r' = a'\frac{1-e'^2}{1+e'\cos f'}$$

and it only remains to consider what functions of f, f', e, e' we shall substitute for the variables s, s'. To this end we appeal to some properties of intermediate orbits. Let us put

$$\begin{aligned}
\Delta^2 &= r^2 + 2\kappa r r'\cos\phi + \kappa^2 r'^2 \\
\Delta'^2 &= r'^2 + 2\kappa r r'\cos\phi + \kappa^2 r^2 \\
\Delta''^2 &= r'^2 - 2 r r'\cos\phi + r^2
\end{aligned}$$

Then the potential function will have the expression

$$\Omega = \frac{Mm}{\Delta} + \frac{Mm'}{\Delta'} + \frac{\frac{mm'}{1-\kappa}}{\Delta''}$$

For the intermediate orbits we may suppose that r takes the place of Δ, and r' the places of Δ' and Δ''. By putting

$$Mm = \mu\mathrm{m} \quad , \quad Mm' + \frac{mm'}{1-\kappa} = \mu'\mathrm{m}'$$

we can write

$$\Omega = \frac{\mu\mathrm{m}}{r} + \frac{\mu'\mathrm{m}'}{r'} + R$$

where

$$R = -Mm\kappa r'\frac{2r\cos\phi + \kappa r'}{r\Delta\,(\Delta+r)} - Mm'\kappa r\frac{2r'\cos\phi + \kappa r}{r'\Delta'\,(\Delta'+r')} + \frac{mm'}{1-\kappa}\,r\,\frac{2r'\cos\phi - r}{r'\Delta''(\Delta''+r')}$$

Let the intermediate orbits be founded upon the potential

$$\Omega = \frac{\mu\mathrm{m}}{r} + \frac{\mu'\mathrm{m}'}{r'}$$

Then it is plain that the variables s and s' in these intermediate orbits will have expressions in terms of f, f', e, e', as follow:

$$s = \mathrm{m}\sqrt{\frac{\mu}{a}}\,\frac{e}{\sqrt{1-e^2}}\sin f \quad , \quad s' = \mathrm{m}'\sqrt{\frac{\mu'}{a'}}\,\frac{e'}{\sqrt{1-e'^2}}\sin f'$$

which are the functions we shall use.

Compute now the two Jacobians

$$\nabla = \frac{\partial s}{\partial e}\frac{\partial r}{\partial f} - \frac{\partial s}{\partial f}\frac{\partial r}{\partial e} \quad , \quad \nabla' = \frac{\partial s'}{\partial e'}\frac{\partial r'}{\partial f'} - \frac{\partial s'}{\partial f'}\frac{\partial r'}{\partial e'}$$

Then the differential equations in terms of the new variables will be

$$\nabla\frac{de}{d\phi} = \frac{\partial W}{\partial f} \quad , \quad \nabla\frac{df}{d\phi} = -\frac{\partial W}{\partial e}$$

$$\nabla'\frac{de'}{d\phi} = \frac{\partial W}{\partial f'} \quad , \quad \nabla'\frac{df'}{d\phi} = -\frac{\partial W}{\partial e'}$$

We have

$$\nabla = \frac{\mathrm{m}\sqrt{\mu a}}{\sqrt{1-e^2}}\,\frac{e\sin^2 f + [2e + (1+e^2)\cos f]\,e\cos f}{(1+e\cos f)^2} = \frac{\mathrm{m}\sqrt{\mu a}\,e}{\sqrt{1-e^2}}$$

consequently

$$\frac{de}{d\phi} = \frac{1}{\mathrm{m}\sqrt{\mu a}}\,\frac{\sqrt{1-e^2}}{e}\,\frac{\partial W}{\partial f} \quad , \quad \frac{de'}{d\phi} = \frac{1}{\mathrm{m}'\sqrt{\mu' a'}}\,\frac{\sqrt{1-e'^2}}{e'}\,\frac{\partial W}{\partial f'}$$

$$\frac{df}{d\phi} = -\frac{1}{\mathrm{m}\sqrt{\mu a}}\,\frac{\sqrt{1-e^2}}{e}\,\frac{\partial W}{\partial e} \quad , \quad \frac{df'}{d\phi} = -\frac{1}{\mathrm{m}'\sqrt{\mu' a'}}\,\frac{\sqrt{1-e'^2}}{e'}\,\frac{\partial W}{\partial e'}$$

These equations have not the canonical form, but it is easy to reduce them to it. For brevity put

$$\frac{1}{\mathrm{m}\sqrt{\mu a}} = k \quad , \quad \frac{1}{\mathrm{m}'\sqrt{\mu' a'}} = k'$$

Then we adopt the variables η and η' of the order of the squares of the eccentricities to replace e and e', and such that

$$\eta = \frac{1}{k}\,(1-\sqrt{1-e^2}) \quad , \quad \eta' = \frac{1}{k'}\,(1-\sqrt{1-e'^2})$$

whence it follows that

$$e = \sqrt{2k\eta - k^2\eta^2} \quad , \quad e' = \sqrt{2k'\eta' - k'^2\eta'^2}$$

With this choice of variables we have to submit to the slight inconvenience of having half powers of η and η' in the expression of W. The differential equations become

$$\frac{d\eta}{d\phi} = \frac{\partial W}{\partial f} \quad , \quad \frac{d\eta'}{d\phi} = \frac{\partial W}{\partial f'}$$

$$\frac{df}{d\phi} = -\frac{\partial W}{\partial \eta} \quad , \quad \frac{df'}{d\phi} = -\frac{\partial W}{\partial \eta'}$$

The two protometers a and a' will be superabundant constants, but we can bring it about that there shall be only one such constant by supposing that a and a' are adopted not in an arbitrary manner, but so as to fulfil the equation

$$\frac{\mu \mathrm{m}}{a} + \frac{\mu' \mathrm{m}'}{a'} = 2C$$

Thus we eliminate C which is replaced by the two constants a and a'. This restriction does not impair the suitableness of these constants for our purpose, and it brings about a marked reduction in the complexity of the quadratic equation which determines W. Here, with profit, we may introduce the variable semi-parameters.

$$p = a(1-e^2) \quad , \quad p' = a'(1-e'^2)$$

By putting the values

$$\frac{\mu \mathrm{m}}{r} = \frac{\mu \mathrm{m}}{p}(1+e\cos f) \quad , \quad \frac{s^2}{\mathrm{m}} = \frac{\mu \mathrm{m}}{p} e^2 \sin^2 f$$

into the expression

$$\frac{\mu \mathrm{m}}{r} - \frac{\mu \mathrm{m}}{2a} - \frac{s^2}{\mathrm{m}}$$

it becomes

$$\frac{\mu \mathrm{m}}{p}\left[1+e\cos f - \tfrac{1}{2}(1-e^2) - e^2\sin^2 f\right] = \frac{\mu \mathrm{m} p}{2r^2}$$

Thus the equation for W takes the form

$$\frac{(h+W)^2}{\mathrm{m} r^2} + \frac{(h-W)^2}{\mathrm{m}' r'^2} = \frac{\mu \mathrm{m} p}{r^2} + \frac{\mu' \mathrm{m}' p'}{r'^2} + 2R$$

where the variables s and s' have been eliminated and replaced by p and p', or, what is the same thing, by e and e'. At first sight, it might appear possible to get rid of the arbitrary constant h by putting it equal to a function of the protometers, and thus escape having any superabundant constants. But h essentially depends on the moduli of the departure of the orbits from circularity, hence there is an incongruity in supposing that h depends on a and a'. After the integration is accomplished, we shall find that the differential equations are not satisfied unless a condition, which may be put in the form

$$h = \text{funct.}\,(a, a', e_0, e'_0)$$

is fulfilled. But we suppose that the numerical values of both h and C have been derived from observation before the investigation is commenced. It is apparent therefore that any incongruity in the values assigned to a and a' are simply thrown on the values of the constants e_0 and e'_0. As the values of the latter are supposed to be determined after the investigation is completed, we need not pay any attention to the matter.

Employing the preceding auxiliary quantities ρ and v, and putting

$$K = \frac{\mu \mathrm{m}^2 a}{h^2}(1-e^2) \;,\quad K' = \frac{\mu' \mathrm{m}'^2 a'}{h^2}(1-e'^2) \;,\; X = \frac{2R}{h^2\rho^2}$$

the quadratic for V takes the form

$$\begin{aligned} V^2 + 2V\cos 2v + 1 &= K\cos^2 v + K'\sin^2 v + X \\ &= \tfrac{1}{2}(K+K') + \tfrac{1}{2}(K-K')\cos 2v + X \end{aligned}$$

Let us make

$$A = \sqrt{\tfrac{1}{2}(K+K') - 1} \quad , \quad M = 2\left[\frac{K-K'}{4} - A\right]$$

Then

$$[V+\cos 2v]^2 = A^2 + (2A+M)\cos 2v + \cos^2 2v + X$$

If we put

$$N = A + \cos 2v \quad , \quad N^2 Q = M\cos 2v + X$$

we have

$$[V+\cos 2v]^2 = N^2(1+Q)$$

whence it follows that

$$V = A + \frac{NQ}{1+\sqrt{1+Q}}$$

The radical in this expression must have the positive sign.

The preceding formulas have been given such a shape that the greatest degree of accuracy may be attained by the use of logarithms of a definite number of decimals. We note that X is of the order of the planetary masses; and, in the case of *Jupiter* and *Saturn*, the numerical value of $\cos 2v$ is always less than $\frac{1}{11}$, and M of the order of the squares of the eccentricities, hence $M\cos 2v$ may be considered as of the same order as X. Thus Q, always within the limits ± 0.004, is of the same order. A is then quite an approximate value of V. The computation of the latter is facilitated by having a table of $\log \frac{2}{1+\sqrt{1+Q}}$ for small values of Q. We have preferred to derive V instead of W, because it is independent of the assumed linear and temporal units; to have W multiply V by the constant h.

It is proposed to develop V in series suitable for use in the further prosecution of the subject by the employment of special values. It will be found convenient to have the development in two forms. First, as a power series of four rectangular coordinates, so to speak, and second as a series of periodic terms depending on arguments whose constituents are ϕ, f and f'. It is comparatively easy to pass from one to the other of these forms. We prefer to attack the development by way of the first form. In the elaboration of this matter it seems a trifle easier to employ parameters somewhat different from those previously suggested. We adopt the four following:

$$e\cos f = x \quad , \quad e'\cos f' = x' \quad , \quad e^2 = y \quad , \quad e'^2 = y'$$

We shall then have

$$V = \Sigma \,.\, A x^i x'^{i'} y^j y'^{j'}$$

where the A are periodic functions of the independent variable ϕ, such that

$$A = C_0 + C_1 \cos\phi + C_2 \cos 2\phi + C_3 \cos 3\phi + \ . \ .$$

the C being constants. The object of the procedure is to discover the values of the C from the special values of V corresponding to chosen values of the five parameters x, x', y, y', ϕ.

The second or polar form for V may be given in terms of η and η' instead of e and e'; it is

$$V = \Sigma\left[A_{00} + A_{10}\,\eta + A_{01}\,\eta' + A_{20}\,\eta^2 + A_{11}\,\eta\eta' + A_{02}\,\eta'^2 + \ . \ .\right] \eta^{\left|\frac{i}{2}\right|}\eta'^{\left|\frac{i'}{2}\right|}\cos\left(j\phi + if + i'f'\right)$$

where j may not receive negative integral values, while i and i' do. The A in this expression are constants whose numerical values result from the proposed method.

With the chosen parameters the radii have the expressions.

$$r = a\,\frac{1-y}{1+x} \quad , \quad r' = a'\,\frac{1-y'}{1+x'}$$

Let us suppose that, in the considered development of V, all terms of an order greater than the eighth with respect to eccentricities may be neglected; and that the quantities A are to be pushed so as to stop with the term $C_{15}\cos 15\phi$; then it is evident that the number of constant coefficients C is

$$16\left[1.5.9 + 2.4.7 + 3.3.5 + 4.2.3 + 5.1.1\right] = 2800$$

We shall thus be obliged to compute 2800 special values of V. But, not to be too greatly appalled at this, we see that very large portions of the computations involved are identical throughout certain groups in the 2800 values. In order to save labor we must arrange our work in such a manner that there are no virtual repetitions even of arithmetical operations. For instance, having to make our computations for the 16 values of ϕ, viz., 0°, 12°, 24°, ... 180°, we notice that the only way ϕ is involved in V is by the factor $\cos\phi$: hence, when ϕ is in the second quadrant, the terms having it as factor are to be got by negativing the corresponding terms when ϕ was in the first quadrant.

It is impossible to give here such a development of V as has just been described, nevertheless I propose to exemplify the process by giving some details for the special value $\phi = 60°$. We must then compute 175 values of V. First, it is necessary to mention the values adopted for the masses, the two protometers and the two constants C and h added severally to the equations of living forces and conservation of areas. Let the Julian year be the unit of time, and the *Earth's* mean distance from the *Sun* the linear unit. Let us assume the data:

	Mass in terms of *Sun's* mass	n	log (n in terms of the radian)
Earth	$\frac{1}{328000}$	1295977″.4238	0.798 1723 029
Jupiter	$\frac{1}{1047.355}$	109256″.61518	9.724 0226 085
Saturn	$\frac{1}{3501.6}$	43996″.08754	9.328 9889 243

Whence follow the values of the logarithms of the masses:

$$\log M = 1.596\,3432\,817 \ , \ \log m = 8.576\,2493\,713$$
$$\log m' = 8.052\,0767\,483$$

Thence are derived the values of the constants employed in the preceding:

$$\log\mu_1 = 8.575\,8350\,290 \ , \ \log\mu_2 = 8.051\,9528\,568$$
$$\log\mu_3 = 5.031\,4441\,859 \ , \ \log\kappa = 6.341\,8974\,798$$

$$\log \mathrm{m} = 8.575\,8349\,808 \quad , \quad \log \mathrm{m}' = 8.051\,9527\,448$$
$$\log\mu \ = 1.596\,7576\,722 \quad , \quad \log\mu' = 1.596\,8818\,368$$

To get the values of a and a', we have the equation already agreed upon,

$$\frac{\mu \mathrm{m}}{a} + \frac{\mu'\mathrm{m}'}{a'} = 2C$$

A discussion of ephemerides, derived from the New Tables of *Jupiter* and *Saturn*, gives

$$2C = 0.33268\,25845$$

It may be arbitrarily assumed that the ratio may be obtained from the equation

$$\alpha^3 = \frac{a^3}{a'^3} = \frac{\mu}{\mu'}\,\frac{n'^2}{n^2}$$

Thence $\log\alpha = 9.736\,6028\,224$; and the two equations combined give

$$\log a = 0.716\,2344\,631 \quad , \quad \log a' = 0.979\,6316\,407$$

From the same discussion of ephemerides we get

$$h = 0.37893\,10781 \quad , \quad \log h = 9.578\,5602\,254$$

Having now the values of the necessary constants, the formulas for the special value $\phi = 60°$ may be set down. The special values of x and x' are selected from the arithmetical progression -0.08, -0.06, . . ., 0.06, 0.08; those of y and y' from the progression -0.0050, -0.0025, 0.0000, 0.0025, 0.0050. Modifying the significations of r and Δ, the following formulas which involve constants are given (the number within brackets are common logarithms):

$$r = \frac{1-y}{1+x} \quad , \quad r' = \frac{1-y'}{1+x'}$$

$$\Delta^2 = 1 + 0.00040\,29904\,\frac{r'}{r} + 0.00000\,01624\,\frac{r'^2}{r^2}$$

$$\Delta'^2 = 1 + 0.00011\,98120\,\frac{r}{r'} + 0.00000\,00144\,\frac{r^2}{r'^2}$$

$$\Delta''^2 = 1 - 0.54525\,89745\,\frac{r}{r'} + 0.29730\,73492\,\frac{r^2}{r'^2}$$

$$\rho^2 = 0.98106\,11839\,\frac{1}{r^2} + 0.97450\,47801\,\frac{1}{r'^2}$$

$$\rho^2\cos 2v = 0.98106\,11839\,\frac{1}{r^2} - 0.97450\,47801\,\frac{1}{r'^2}$$

$$X = \frac{-[7.205\,5623\,922]}{\rho^2}\,\frac{r'}{r^2}\,\frac{1+0.00040\,29904\,\frac{r'}{r}}{\Delta(\Delta+1)}$$
$$\quad \frac{-[5.891\,1982\,364]}{\rho^2}\,\frac{r}{r'^2}\,\frac{1+0.00011\,98120\,\frac{r}{r'}}{\Delta'(\Delta'+1)}$$
$$\quad \frac{+[6.529\,3022\,860]}{\rho^2}\,\frac{r}{r'^2}\,\frac{1-0.54525\,89745\,\frac{r}{r'}}{\Delta''(\Delta''+1)}$$

The following table gives the values of A and $\log M$ for the only combinations of the values of y and y' that are used; d' represents 0.0025.

Arguments y d	y' d'	A	$\log M$
0	0	.42653 77301 599	7.680 9425 816 n
−2	0	.43244 64917 631	8.062 1565 242 n
−1	0	.42950 22721 059	7.913 1787 711 n
1	0	.42355 24392 043	7.134 7777 062 n
2	0	.42054 59573 717	7.324 5524 816
0	−2	.42751 44143 044	7.879 9102 207 n
0	−1	.42702 63514 632	7.791 7616 788 n
0	1	.42604 85484 730	7.531 6407 641 n
0	2	.42555 88044 701	7.302 0514 923 n
−1	−1	.42998 75246 308	7.981 1656 474 n
1	1	.42305 98056 562	5.584 9170 982
2	1	.42004 97978 427	7.546 6291 400
1	2	.42256 65977 901	7.158 9463 351
−1	1	.42901 64707 212	7.832 4670 973 n
1	−1	.42404 45004 359	7.441 7072 155

The 175 values of V and of the derived function G which serve better for the determination of the coefficients follow; all are to 13 places of decimals; the horizontal lines delimit the 16 groups; the first G is omitted as it is identical with the corresponding V.*

No.	Argument i	i	i	i	V	G
1	0	0	0	0	.42607 46965 557	. . . . . .
2	−4	0	0	0	.42658 69065 999	− 12 80525 110½
3	−3	0	0	0	.42643 33658 278	− 11 95564 240⅓
4	−2	0	0	0	.42629 88798 411	− 11 20916 427
5	−1	0	0	0	.42618 01784 162	− 10 54818 605
6	1	0	0	0	.42598 03943 278	− 9 43022 279
7	2	0	0	0	.42589 56291 947	− 8 95336 805
8	3	0	0	0	.42581 90638 313	− 8 52109 081½
9	4	0	0	0	.42574 95984 151	− 8 12745 351½
10	0	−4	0	0	.42569 15912 989	+ 9 57763 142
11	0	−3	0	0	.42577 87633 688	+ 9 86443 956⅓
12	0	−2	0	0	.42587 12863 433	+ 10 17051 062
13	0	−1	0	0	.42596 97198 701	+ 10 49766 856
14	0	1	0	0	.42618 69350 754	+ 11 22385 197
15	0	2	0	0	.42630 72560 656	+ 11 62797 549½
16	0	3	0	0	.42643 66015 374	+ 12 06349 939
17	0	4	0	0	.42657 60587 540	+ 12 53405 495¾
18	0	0	−2	0	.43202 73981 346	−297 63507 894½
19	0	0	−1	0	.42905 14744 700	−297 67779 143
20	0	0	1	0	.42309 71356 838	−297 75608 719
21	0	0	2	0	.42011 88635 244	−297 79165 156½
22	0	0	0	−2	.42699 90167 611	− 46 21601 027
23	0	0	0	−1	.42654 09550 516	− 46 62584 959
24	0	0	0	1	.42560 00405 323	− 47 46560 234
25	0	0	0	2	.42511 67793 591	− 47 89585 983
26	−1	−1	0	0	.42606 56238 373	− 95778 933
27	1	1	0	0	.42608 34132 997	− 92195 478
28	−1	1	0	0	.42630 31492 094	− 1 07322 735
29	1	−1	0	0	.42588 36915 790	− 82739 368
30	−2	−1	0	0	.42617 31408 338	− 1 03811 608½
31	2	1	0	0	.42599 06615 124	− 86031 010
32	−2	1	0	0	.42643 41613 511	− 1 16714 966½
33	2	−1	0	0	.42580 61300 834	− 77387 871½
34	1	−2	0	0	.42595 86133 903	− 90774 067½
35	1	2	0	0	.42619 34273 162	− 97632 607½
36	−1	2	0	0	.42643 55401 477	−1 14011 108
37	1	−2	0	0	.42579 27061 368	− 78610 107
38	−2	−2	0	0	.42605 61728 808	− 98241 869¾
39	2	2	0	0	.42609 17946 109	− 90985 234¼
40	−2	2	0	0	.42658 11299 341	−1 24226 451
41	2	−2	0	0	.42572 16599 647	− 73602 456
42	−3	−1	0	0	.42629 44381 336	−1 13170 028⅔
43	3	1	0	0	.42590 71252 144	− 80590 455⅓
44	−3	1	0	0	.42658 39221 886	−1 27726 137
45	3	−1	0	0	.42573 58809 032	− 72645 858⅓
46	−1	−3	0	0	.42585 83853 549	− 86199 581⅓
47	1	3	0	0	.42631 12095 383	−1 03632 570⅔
48	−1	3	0	0	.42657 85136 233	−1 21434 084⅔
49	1	−3	0	0	.42570 69070 294	− 74819 628⅓
50	3	2	0	0	.42600 05443 053	− 85131 726½
51	2	3	0	0	.42619 96703 738	− 96439 671
52	4	1	0	0	.42583 15338 857	− 75757 622¾
53	1	4	0	0	.42643 76450 170	−1 10278 772¾
54	0	0	−1	−1	.42950 85731 868	− 91597 791
55	0	0	1	1	.42261 31404 539	− 93392 065
56	0	0	−1	1	.42858 61411 708	− 93227 242
57	0	0	1	−1	.42357 25708 152	− 97766 355
58	0	0	2	1	.41962 55137 620	− 93468 695
59	0	0	1	2	.42212 03727 440	− 94228 716
60	−1	0	−1	0	.42924 09891 117	+8 40327 812
61	1	0	1	0	.42307 81172 757	+7 52838 198
62	−1	0	1	0	.42311 83237 274	+8 42938 169
63	1	0	−1	0	.42888 20630 257	+7 51092 164
64	−2	0	−1	0	.42945 42660 584	+8 93041 515
65	2	0	1	0	.42306 09573 591	+7 14445 181½
66	−2	0	1	0	.42314 20669 240	+8 96260 226
67	2	0	−1	0	.42872 98055 735	+7 13007 677½
68	1	0	2	0	.42017 52729 396	+7 53669 918½
69	−1	0	2	0	.42005 55057 317	+8 44197 605½

* The reader is referred to *A.J.*, No. 567, and *Amer. Jour. Math.*, Vol. XXVII, for further explanation.

No.	Argument				V	G
	i	i'	j	j'		
70	3	0	1	0	.42304 54019 971	+6 79663 459
71	−3	0	1	0	.42316 88492 034	+9 56519 175
72	0	−1	−1	0	.42886 99701 766	−7 65276 078
73	0	1	1	0	.42312 68659 512	−8 25082 523
74	0	−1	1	0	.42306 88639 700	−7 67049 718
75	0	1	−1	0	.42924 59662 555	−8 22532 658
76	0	−2	−1	0	.42870 02053 100	−7 39294 738
77	0	2	1	0	.42315 82495 229	−8 57228 354
78	0	−2	1	0	.42304 18786 889	−7 40766 087½
79	0	2	−1	0	.42945 48709 860	−8 54185 030½
80	0	1	2	0	.42006 58396 977	−8 26311 732
81	0	−1	2	0	.42016 74658 880	−7 67895 246
82	0	3	1	0	.42319 15078 648	−8 91776 002⅓
83	0	−3	1	0	.42301 60263 719	−7 16079 583⅓
84	−1	0	0	−1	.42667 87841 281	+3 23472 160
85	1	0	0	1	.42553 54568 952	+2 97185 908
86	−1	0	0	1	.42567 22868 216	+3 32355 712
87	1	0	0	−1	.42641 76926 019	+2 89602 218
88	−2	0	0	−1	.42683 38238 269	+3 43427 449½
89	2	0	0	1	.42547 74072 853	+2 82170 570
90	−2	0	0	1	.42575 35990 131	+3 53142 023
91	2	0	0	−1	.42630 68649 045	+2 75113 930½
92	1	0	0	2	.42508 26988 681	+3 01108 684½
93	−1	0	0	2	.42515 48692 123	+3 36960 036½
94	3	0	0	1	.42542 49737 206	+2 68553 042⅓
95	−3	0	0	1	.42584 57441 246	+3 76552 266
96	0	−1	0	−1	.42640 64709 066	−2 95071 594
97	0	1	0	1	.42567 97523 222	−3 25267 298
98	0	−1	0	1	.42552 53133 670	−3 02795 203
99	0	1	0	−1	.42668 48515 369	−3 16579 656
100	0	−2	0	−1	.42628 04936 972	−2 85255 710
101	0	2	0	1	.42576 50630 014	−3 37685 204
102	0	−2	0	1	.42545 51492 069	−2 92594 435
103	0	2	0	−1	.42683 92013 261	−3 28433 823
104	0	1	0	2	.42516 30638 533	−3 29770 127½
105	0	−1	0	2	.42507 31605 566	−3 06789 415½
106	0	3	0	1	.42585 66437 572	−3 51005 856
107	0	−3	0	1	.42538 90076 135	−2 83000 893⅔
108	1	1	1	0	.42310 58853 156	+ 72573 203
109	−1	1	1	0	.42315 03254 566	+ 84608 147
110	−1	−1	1	0	.42308 80287 364	+ 75546 161
111	1	−1	1	0	.42305 16036 942	+ 65158 073
112	2	1	1	0	.42308 70161 158	+ 67673 446½
113	−2	1	1	0	.42317 67229 171	+ 92086 338
114	−2	−1	1	0	.42310 94233 327	+ 81952 221
115	2	−1	1	0	.42303 59840 003	+ 60896 096⅓
116	1	2	1	0	.42313 50710 980	+ 76832 523½
117	−1	2	1	0	.42318 42689 790	+ 89854 045½
118	−1	−2	1	0	.42305 92346 103	+ 71613 456½
119	1	−2	1	0	.12302 61990 132	+ 61916 445
120	1	1	−1	0	.42906 01583 877	+ 71768 757
121	−1	1	−1	0	.42945 15641 149	+ 83509 742
122	−1	−1	−1	0	.42904 24359 261	+ 74709 989
123	1	−1	−1	0	.42871 53705 895	+ 64544 991
124	2	2	1	0	.42311 42982 526	+ 71552 870¼
125	3	1	1	0	.42306 99606 077	+ 63351 599⅓
126	1	3	1	0	.42316 58726 586	+ 81576 577
127	1	1	2	0	.42012 76456 273	+ 72968 608
128	2	1	−1	0	.42889 36957 902	+ 66976 834
129	1	2	−1	0	.42925 07517 361	+ 75906 420½
130	1	1	0	1	.42560 87729 013	+ 28237 637
131	−1	1	0	1	.42575 94463 515	+ 32845 338
132	−1	−1	0	1	.42559 09515 556	+ 29397 926
133	1	−1	0	1	.42546 64929 123	+ 25407 544
134	2	1	0	1	.42554 51841 848	+ 26356 556½
135	−2	1	0	1	.42584 95142 857	+ 35697 554½
136	−2	−1	0	1	.42566 45089 166	+ 31846 952½
137	2	−1	0	1	.42541 34343 104	+ 23766 919½
138	1	2	0	1	.42568 69241 761	+ 29858 166½
139	−1	2	0	1	.42585 31449 886	+ 34832 618⅓
140	−1	−2	0	1	.42551 48198 654	+ 27895 913½
141	1	−2	0	1	.42540 14541 925	+ 24166 993½
142	1	1	0	−1	.42654 96567 423	+ 27127 971
143	−1	1	0	−1	.42683 65607 230	+ 31478 361
144	−1	−1	0	−1	.42653 18978 509	+ 28242 389
145	1	−1	0	−1	.42629 39283 286	+ 24459 349
146	2	2	0	1	.42561 71692 711	+ 27834 026
147	3	1	0	1	.42548 79163 701	+ 24693 319⅔
148	1	3	0	1	.42577 04606 653	+ 31634 388
149	1	1	0	2	.42512 55279 807	+ 28820 831
150	2	1	0	−1	.42642 84852 575	+ 25349 651½
151	1	2	0	−1	.42669 06819 681	+ 28651 934
152	1	0	1	1	.42262 47874 174	+ 9167 808
153	−1	0	1	1	.42259 99657 374	+ 11271 889
154	−1	0	−1	1	.42874 35363 281	+ 11160 868
155	1	0	−1	1	.42844 55099 179	+ 9383 994
156	1	0	1	−1	.42352 36690 375	+ 9231 478
157	−1	0	1	−1	.42362 72033 437	+ 10972 689
158	2	0	1	1	.42263 51464 200	+ 8750 884
159	−2	0	1	1	.42258 49628 608	+ 12420 143½
160	0	1	1	1	.42260 92414 414	− 11025 501
161	0	−1	1	1	.42261 61124 863	− 9642 259
162	0	−1	−1	1	.42843 39602 100	− 9561 877
163	0	1	−1	1	.42874 91981 917	− 10919 652
164	0	1	1	−1	.42363 50324 212	− 10733 730
165	0	−1	1	−1	.42351 38510 779	− 9405 641
166	0	2	1	1	.42260 43481 118	− 11845 702
167	0	−2	1	1	.42261 82134 318	− 9055 429
168	1	1	1	1	.42262 19060 681	+ 1561 270
169	−1	1	1	1	.42259 48582 373	+ 1954 156
170	−1	−1	1	1	.42260 40168 846	+ 1625 994
171	1	−1	1	1	.42262 68459 135	+ 1309 142
172	1	1	−1	1	.42859 48403 716	+ 1539 074
173	1	1	1	−1	.42358 13051 049	+ 1505 140
174	2	1	1	1	.42263 31294 710	+ 1411 324½
175	1	2	1	1	.42261 81490 030	+ 1711 556

From the preceding data is derived the following development of V for the special case of $\phi = 60°$. It is given in both of the forms.

$$V = \Sigma A \left(\frac{x}{d}\right)^{i} \left(\frac{x'}{d}\right)^{i'} \left(\frac{y}{d'}\right)^{j} \left(\frac{y'}{d'}\right)^{j'} \quad . \qquad V = A x^{a} x'^{b} y^{c} y'^{d}$$

as the first more readily than the second, enables us to see how well the development represents the function in the region played over by the special values of the parameters. The coefficients of the first form are in units of the 13th decimal, and the fraction is appended, so that if substituted in the linear equations they should rigorously reproduce the special values of V. In the second form the coefficients are carried to such a number of decimals as the case seems to warrant.

Fact.		*A*		*A*
			+	0.42607 46965 567
x	—	9 95887 358 $\frac{89}{105}$	—	0.00497 94367 94
x^2	+	55734 500 $\frac{129}{280}$	+	0.01393 36251 1
x^3	—	3024 283 $\frac{43}{45}$	—	0.03780 35494
x^4	+	163 188 $\frac{431}{480}$	+	0.10199 306
x^5	—	8 771 $\frac{289}{720}$	—	0.27410 63
x^6	+	472 $\frac{19}{120}$	+	0.73775
x^7	—	27 $\frac{4009}{5040}$	—	2.1715
x^8	+	1 $\frac{541}{1120}$	+	5.793
x'	+	10 84799 091 $\frac{11}{840}$	+	0.00542 39954 55
x'^2	+	36266 880 $\frac{5233}{10080}$	+	0.00906 67201 3
x'^3	+	1275 486 $\frac{3}{32}$	+	0.01594 3576
x'^4	+	42 241 $\frac{841}{1152}$	+	0.02640 108
x'^5	+	1 147 $\frac{29}{40}$	+	0.04524 14
x'^6	+	48 $\frac{577}{2880}$	+	0.07531
x'^7	+	1 $\frac{449}{672}$	+	0.1303
x'^8	+	$\frac{2035}{40320}$	+	0.197
y	—	297 71813 066 $\frac{1}{6}$	—	1.19087 25226
y^2	—	3914 915 $\frac{1}{2}$	—	0.06263 9128
y^3	+	119 135 $\frac{1}{6}$	+	0.76246 51
y^4	+	157 $\frac{1}{2}$	+	0.40320
y'	—	47 04232 293 $\frac{2}{3}$	—	0.18816 92917
y'^2	—	41984 770 $\frac{1}{3}$	—	0.67175 6325
y'^3	—	340 302 $\frac{5}{6}$	—	2.17793 81
y'^4	—	2 867 $\frac{1}{6}$	—	7.33994
$x\ x'$	—	93780 675 $\frac{11863}{12600}$	—	0.02344 51690
$x\ x'^2$	—	5193 965 $\frac{61}{80}$	—	0.06492 4563
$x\ x'^3$	—	243 845 $\frac{3937}{7200}$	—	0.15253 026
$x\ x'^4$	—	10 525 $\frac{281}{288}$	—	0.32893 7
$x\ x'^5$	—	433 $\frac{209}{288}$	—	0.6777
$x\ x'^6$	—	18 $\frac{121}{288}$	—	1.439
$x\ x'^7$	—	$\frac{3007}{5040}$	—	2.3
$x^2\ x'$	+	6979 301 $\frac{71}{720}$	+	0.08724 1264
$x^2\ x'^2$	+	516 869 $\frac{121}{720}$	+	0.32304 323
$x^2\ x'^3$	+	30 598 $\frac{7}{288}$	+	0.95618 8
$x^2\ x'^4$	+	1 590 $\frac{265}{288}$	+	2.4858
$x^2\ x'^5$	+	80 $\frac{141}{160}$	+	6.319
$x^2\ x'^6$	+	3 $\frac{421}{480}$	+	15.1
$x^3\ x'$	—	178 942 $\frac{53}{72}$	—	0.29933 92
$x^3\ x'^2$	—	44 994 $\frac{23}{144}$	—	1.40606 7
$x^3\ x'^3$	—	3 190 $\frac{77}{240}$	—	4.9849
$x^3\ x'^4$	—	203 $\frac{67}{144}$	—	15.90
$x^3\ x'^5$	—	10 $\frac{71}{720}$	—	39.4
$x^4\ x'$	+	31 293 $\frac{5}{144}$	+	0.97790 7
$x^4\ x'^2$	+	3 418 $\frac{41}{48}$	+	5.3420
$x^4\ x'^3$	+	300 $\frac{59}{72}$	+	23.50
$x^4\ x'^4$	+	21 $\frac{7}{36}$	+	82.8
$x^5\ x'$	—	2 008 $\frac{173}{400}$	—	3.1378
$x^5\ x'^2$	—	270 $\frac{13}{60}$	—	21.111
$x^5\ x'^3$	—	13 $\frac{47}{50}$	—	54.5
$x^6\ x'$	+	131 $\frac{77}{120}$	+	10.284
$x^6\ x'^2$	+	18 $\frac{17}{18}$	+	74.0
$x^7\ x'$		7 $\frac{1327}{2520}$		29.4
$y\ y'$	—	92489 858 $\frac{5}{12}$		1.17983 773
$y\ y'^2$	—	813 790 $\frac{1}{4}$	—	5.20825 76
$y^2\ y'$	—	83 346 $\frac{3}{4}$	—	0.53419 20
$y^2\ y'^2$	+	935 $\frac{1}{4}$	+	2.39424
$y\ y'^3$	—	7 932 $\frac{2}{3}$	—	20.3076
$y^3\ y'$	+	1 927 $\frac{1}{6}$	+	4.9335
$x\ y$	+	7 94380 169 $\frac{8}{15}$	+	1.58876 0339
$x\ y^2$	+	1064 112 $\frac{5}{12}$	+	0.85128 99
$x\ y^3$	—	14 504 $\frac{3}{4}$	—	4 64152
$x^2\ y$	—	44703 833 $\frac{43}{60}$	—	4.47038 337
$x^2\ y^2$	—	213 890 $\frac{17}{24}$	—	8.55563
$x^2\ y^3$	+	740 $\frac{11}{12}$	+	11.85
$x^3\ y$	+	2425 972 $\frac{25}{48}$	+	12.12986 26
$x^3\ y^2$	+	24 985 $\frac{1}{3}$	+	49.9707
$x^4\ y$	—	130 414 $\frac{23}{48}$	—	32.60362
$x^4\ y^2$	—	2 190 $\frac{1}{24}$	—	219.00
$x^5\ y$	+	7 443 $\frac{107}{240}$	+	93.0431
$x^6\ y$	—	397 $\frac{113}{240}$	—	248.42
$x'\ y$	—	7 94043 184 $\frac{7}{12}$	—	1.58808 6369
$x'\ y^2$	—	1064 945 $\frac{7}{12}$	—	0.85195 65
$x'\ y^3$	+	14 502 $\frac{7}{12}$	+	4.64082
$x'^2 y$	—	28791 033 $\frac{31}{120}$	—	2.87910 333
$x'^2 y^2$	—	193 242 $\frac{3}{4}$	—	7.72731
$x'^2 y^3$	+	738 $\frac{7}{12}$	+	11.82
$x'^3 y$	—	955 427 $\frac{1}{8}$	—	4.77713 6
$x'^3 y^2$	—	15 930 $\frac{2}{3}$	—	31.861
$x'^4 y$	—	32 001 $\frac{1}{6}$	—	8.00104
$x'^4 y^2$	—	813 $\frac{1}{2}$	—	81.35
$x'^5 y$	—	1 135 $\frac{1}{6}$	—	14.189
$x'^6 y$	—	47 $\frac{49}{120}$	—	29.63
$x\ y'$	+	3 09677 309 $\frac{3}{5}$	+	0.61935 4619
$x\ y'^2$	+	4091 479 $\frac{7}{12}$	+	3.27318 37
$x\ y'^3$	+	48 913 $\frac{1}{3}$	+	15.65226
$x^2\ y'$	—	17203 697 $\frac{11}{180}$	—	1.72036 97
$x^2\ y'^2$	—	322 540 $\frac{1}{12}$	—	12.90160
$x^2\ y'^3$	—	5 386 $\frac{1}{6}$	—	86.179
$x^3\ y'$	+	925 596 $\frac{11}{48}$	+	4.62798 1
$x^3\ y'^2$	+	25 330 $\frac{11}{12}$	+	50.6618
$x^4\ y'$	—	51 009 $\frac{97}{144}$	—	12.7524
$x^4\ y'^2$	—	2 425 $\frac{5}{12}$	—	24.254
$x^5\ y'$	+	2 180 $\frac{27}{80}$	+	27.254
$x^6\ y'$	+	156 $\frac{289}{720}$	+	97.75
$x'\ y'$	—	3 09527 595 $\frac{9}{20}$	—	0.61905 5191
$x'\ y'^2$	—	4086 908 $\frac{1}{6}$	—	3.26952 65
$x'\ y'^3$	—	48 819 $\frac{5}{12}$	—	15.62221
$x'^2 y'$	—	10977 038 $\frac{17}{96}$	—	1.09770 38
$x'^2 y'^2$	—	242 650 $\frac{1}{3}$	—	9.70601
$x'^2 y'^3$	—	4 183 $\frac{5}{12}$	—	66.935
$x'^3 y'$	—	352 374 $\frac{5}{24}$	—	1.76187 1
$x'^3 y'^2$	—	15 154 $\frac{7}{12}$	—	30.3092
$x'^4 y'$	—	13 057 $\frac{29}{36}$	—	3.26445
$x'^4 y'^2$	+	892 $\frac{1}{12}$	+	8.921
$x'^5 y'$	—	398 $\frac{27}{40}$	—	4.983
$x'^6 y'$	—	9 $\frac{151}{180}$	—	6.30
$x\ x'\ y$	+	73455 843 $\frac{14}{15}$	+	7.34558 439

Fact.		A	A
$x\,x'^2y$	+	$4019\,198\,\frac{1}{16}$	+ 20.09599 0
$x\,x'^3y$	+	$201\,208\,\frac{1}{3}$	+ 50.3021
$x\,x'^4y$	+	$7\,184\,\frac{47}{48}$	+ 89.812
$x\,x'^5y$	−	$237\,\frac{19}{40}$	− 148.4
$x^2x'\,y$	−	$5490\,544\,\frac{5}{6}$	− 27.45272 4
$x^2x'^2y$	−	$400\,177\,\frac{25}{48}$	− 100.0444
$x^2x'^3y$	−	$24\,625\,\frac{7}{24}$	− 307.816
$x^2x'^4y$	−	$38\,\frac{5}{48}$	− 23.8
$x^3x'\,y$	+	$404\,680\,\frac{13}{24}$	+ 101.17014
$x^3x'^2y$	+	$36\,196\,\frac{1}{12}$	+ 452.451
$x^3x'^3y$	−	$5\,775\,\frac{7}{12}$	−3609.7
$x^4x'\,y$	−	$25\,956\,\frac{1}{2}$	− 321.456
$x^4x'^2y$	−	$2\,632\,\frac{1}{2}$	−1645.3
$x^5\,x'y$	−	$1\,071\,\frac{47}{120}$	− 669.6
$x\,x'\,y^2$	+	$397\,308\,\frac{13}{24}$	+ 15.89234
$x\,x'^2y^2$	+	$56\,699\,\frac{3}{8}$	+ 113.399
$x\,x'^3y^2$	+	$8\,653\,\frac{2}{3}$	+ 865.37
$x^2x'\,y^2$	−	$64\,630\,\frac{7}{8}$	− 129.262
$x^2x'^2y^2$	−	$8\,851\,\frac{3}{8}$	− 885.14
$x^3x'\,y^2$	+	$13\,043\,\frac{2}{3}$	+1304.37
$x\,x'\,y^3$	−	$2\,272\,\frac{3}{4}$	− 36.364
$x\,x'\,y'$	+	$28156\,416\,\frac{87}{120}$	+ 2.81564 167
$x\,x'^2y'$	+	$1506\,165\,\frac{1}{24}$	+ 7.53082 5
$x\,x'^3y'$	+	$81\,571$	+ 20.3927
$x\,x'^4y'$	+	$3\,036\,\frac{1}{6}$	+ 37.952
$x\,x'^5y'$	−	$1\,104\,\frac{4}{15}$	− 690.1
$x^2x'\,y'$	−	$2073\,295\,\frac{2}{3}$	− 10.36647 8
$x^2x'^2y'$	−	$146\,532\,\frac{3}{4}$	− 36.6332
$x^2x'^3y'$	−	$8\,774\,\frac{1}{12}$	− 109.676
$x^2x'^4y'$	−	$453\,\frac{11}{12}$	− 283.7
$x^3x'\,y'$	+	$158\,406\,\frac{5}{48}$	+ 39.60153
$x^3x'^2y'$	+	$13\,561\,\frac{1}{6}$	+ 169.515
$x^3x'^3y'$	−	$3\,945\,\frac{3}{4}$	−2466.1
$x^4x'\,y'$	−	$9\,369\,\frac{3}{8}$	− 117.117
$x^4x'^2y'$	−	$1\,096\,\frac{23}{24}$	− 685.6
$x^5x'\,y'$	−	$1\,233\,\frac{5}{48}$	− 770.7
$x\,x'\,y'^2$	+	$558\,638\,\frac{3}{8}$	+ 22.34553
$x\,x'^2y'^2$	+	$46\,613\,\frac{7}{8}$	+ 93.228
$x\,x'^3y'^2$	+	$5\,277$	+ 527.7
$x^2x'\,y'^2$	−	$58\,081\,\frac{5}{8}$	− 116.163
$x^2x'^2y'^2$	−	$6\,246\,\frac{1}{8}$	− 624.61
$x^3x'\,y'^2$	+	$8\,631\,\frac{1}{2}$	+ 863.15
$x\,x'\,y'^3$	+	$9\,453\,\frac{2}{3}$	+ 151.259
$x\quad y\,y'$	+	$10115\,368\,\frac{5}{6}$	+ 8.09229 51
$x^2\quad y\,y'$	−	$874\,429\,\frac{19}{24}$	− 34.97719
$x^3\quad y\,y'$	+	$71\,888\,\frac{5}{12}$	+ 143.777
$x^4\quad y\,y'$	−	$5\,091\,\frac{11}{24}$	− 509.1
$x\quad y\,y'^2$	+	$133\,882\,\frac{1}{2}$	+ 42.8424
$x^2\quad y\,y'^2$	−	$15\,717\,\frac{1}{2}$	− 251.48
$x\quad y^2y'$	+	$48\,708\,\frac{3}{4}$	+ 15.5868
$x^2\quad y^2y'$	−	$6\,801\,\frac{3}{4}$	− 108.828
$x'\quad y\,y'$	−	$10116\,329\,\frac{5}{6}$	− 8.093064
$x'^2\quad y\,y'$	−	$669\,483\,\frac{7}{12}$	− 26.77934
$x'^3\quad y\,y'$	−	$38\,895\,\frac{1}{6}$	− 77.790
$x'^4\quad y\,y'$	−	$1\,982\,\frac{5}{12}$	− 198.2
$x'\quad y\,y'^2$	−	$132\,097\,\frac{1}{4}$	− 42.2711
$x'^2\quad y\,y'^2$	−	$13\,788\,\frac{1}{4}$	− 220.61
$x'\quad y^2y'$	−	$46\,557\,\frac{3}{4}$	− 14.8985
$x'^2\quad y^2y'$	−	$6\,366\,\frac{3}{4}$	− 101.868
$x\;x'y\,y'$	+	$1550\,904\,\frac{2}{3}$	+ 62.0362
$x\;x'^2yy'$	+	$145\,072\,\frac{1}{2}$	+ 290.145
$x\;x'^3yy'$	+	$8\,074$	+ 807.4
$x^2\,x'\,yy'$	−	$177\,434\,\frac{1}{2}$	− 354.869
$x^2\,x'^2yy'$	−	$19\,008\,\frac{1}{2}$	−1900.85
$x^3\,x'\,yy'$	+	$15\,498\,\frac{5}{6}$	+1549.88
$x\;x'\,y^2y'$	+	$11\,098$	+ 177.568
$x\;x'\,yy'^2$	+	$27\,065$	+ 433.04

In order to have V or W as a function of the four variables f, f', η, η' the preceding expression must be transformed by making the substitutions

$$x = \sqrt{2k\eta - k^2\eta^2}\cos f \quad , \quad x' = \sqrt{2k'\eta' - k'^2\eta'^2}\cos f'$$
$$y = 2k\eta - 1k^2\eta^2 \quad , \quad y' = 2k'\eta' - k'^2\eta'^2$$

It is proposed to accomplish the integrations the problem involves by Delaunay transformations. Selecting an argument $j\phi + if + i'f'$, j, i and i' being integers prime to each other, such a transformation of the four variables f, f', η, η' is made as shall make the periodic terms of W depending on this argument disappear. When all the sensible periodic terms have been got rid of by a series of these operations W will be reduced to a function of η and η'. As the differential equations retain their canonical form throughout the whole of this process, in the final stage, η and η' become constant, and if we put

$$-\frac{\partial W}{\partial\eta} = \kappa \quad , \quad -\frac{\partial W}{\partial\eta'} = \kappa'$$

κ and κ' will be constants, and the final expressions for f and f' will be

$$f = \kappa(\phi + c) \quad , \quad f' = \kappa'(\phi + c')$$

c and c' being constants.

In accordance with the principles of the Delaunay method, the mentioned transformations must be made not only in the function W, but also in four functions designed to define the positions of the two planets in the common orbital plane. There is considerable latitude for choice here, but the four functions I propose are these:

$$\frac{a}{r} = (1-k\eta)^{-2}\left[1 + \sqrt{2k\eta - k^2\eta^2}\cos f\right]$$
$$\frac{a'}{r'} = (1-k'\eta')^{-2}\left[1 + \sqrt{2k'\eta' - k'^2\eta'^2}\cos f'\right]$$
$$\frac{dt}{d\phi} = \left[\frac{h+W}{\mathrm{m}r^2} - \frac{h-W}{\mathrm{m}'r'^2}\right]^{-1}$$
$$\frac{d\psi}{d\phi} = \left[\frac{h+W}{\mathrm{m}r^2} + \frac{h-W}{\mathrm{m}'r'^2}\right]\left[\frac{h+W}{\mathrm{m}r^2} - \frac{h-W}{\mathrm{m}'r'^2}\right]^{-1}$$

When computing the special values of V or W, since we employ the values of r and r', it is very little additional labor to derive the special values of the right members of the third and fourth equations of the just-given group.

By applying the same method as for V we have infinite series for $\frac{dt}{d\phi}$ and $\frac{d\psi}{d\phi}$ of the same character as for the former quantity, and the DELAUNAY transformations can be made in them in precisely the same way.

When the latter are concluded we shall have $\frac{a}{r}$ and $\frac{a'}{r'}$ as functions of the independent variable ϕ; but before we can have t and ψ as similar functions, it will be necessary, in each case, to execute an integration with reference to ϕ, which will be easy, as each term is of the form

$$K \cos(\kappa\phi + \beta)$$

With this operation I regard the solution of the problem as completed. The assertion may need justification. We are in the habit of using tables by inversion, the general theory of interpolation sufficing for the purpose. Although tables of anti-logarithms have been published, they are seldom used, and no tables have ever been computed for furnishing the arc to a given sine or tangent. Let this notion be applied to tables for *Jupiter* and *Saturn*; let them be constructed so as to give, in the first instance, the time at which the hypothetical planets have a definite elongation ϕ. The computation being made for a series of values of ϕ as 720°, 721°, 722° . . . , by interpolation we find the value of ϕ corresponding to a definite time; with this as argument we can enter another division of the tables and get the corresponding values of $\frac{a}{r}$ and $\frac{a'}{r'}$ and ψ; thus the positions of the hypothetical planets are known; whence it is possible to get those of the actual. In this way an analytical inversion of series is avoided.

In this connection I must state my conviction that GYLDÉN's device of the reduced time is without sensible advantage.

The application of DELAUNAY transformations will be treated in another memoir.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH AND 26-INCH EQUATORIALS AT THE U.S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N. Superintendent.]

1904–05 Wash'n M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
					(13) *Egeria.*					
1904 h m s			m s	′ ″	h m s	° ′ ″			s	″
Nov. 28 10 34 46	1	29 , 6	−0 14.37	+11 21.1	3 41 35.06	+29 17 12.6	n8.919	0.171	+4.63	+ 6.4
30 8 21 37	2	24 , 5	+1 26.09	− 0 56.7	3 39 13.24	+29 22 21.6	n9.527	0.345	+4.64	+ 7.1
30 8 41 40	3	25 , 5	+1 4.33	− 0 25.3	3 39 12.37	+29 22 23.1	n9.477	0.308	+4.65	+ 7.0
					(103) *Hera.*					
Nov. 30 9 38 53	4	25 , 5	−3 20.91	− 6 39.4	3 21 56.08	+10 1 15.2	n9.113	0.630	+3.98	+ 6.9
Dec. 8 10 39 7	5	25 , 5	+3 49.81	+ 6 0.4	3 15 56.59	+ 9 53 28.2	8.824	0.628	+3.95	+ 7.6
					(116) *Sirona.*					
Dec. 1 10 38 39	6	25 , 5	+1 47.89	+ 7 25.8	4 1 11.40	+20 43 12.4	n8.932	0.440	+4.35	+ 4.1
					(130) *Elektra.*					
Dec. 1 11 27 55	7	25 , 5	+1 21.52	− 5 58.0	4 29 47.82	−14 7 13.2	n8.603	0.845	+3.55	+ 0.1
8 11 50 45	8	5 , 1	+0 6.53	+ 3 53.3	4 24 4.90	−13 46 6.6	8.875	0.842	+3.60	− 1.1
12 10 12 57	9	30 , 6	−0 32.37	+ 2 51.1	4 21 4.89	−13 27 8.0	n8.939	0.840	+3.62	− 1.6
12 10 30 17	10	30 , 6	−1 51.11	+ 0 10.5	4 21 4.29	−13 27 5.2	n8.712	0.841	+3.62	− 1.6
18 10 16 26	11	30 , 6	+0 22.65	− 3 27.4	4 16 55.95	−12 49 3.7	n8.364	0.838	+3.63	− 2.3
18 10 31 21	12	30 , 6	−1 39.25	− 1 54.0	4 16 55.46	−12 48 56.1	8.135	0.838	+3.64	− 2.5
					(44) *Nysa.*					
Dec. 13 10 39 26	13	30 , 6	+2 20.18	+ 3 8.5	4 7 40.24	+15 2 21.4	7.437	0.548	+4.25	+ 2.9
13 11 3 11	14	30 , 6	−2 44.21	− 7 21.1	4 7 39.13	+15 2 20.1	8.715	0.549	+4.27	+ 2.4
16 10 9 46	15	30 , 6	+2 46.67	+ 7 44.6	4 5 6.51	+15 2 9.9	n8.462	0.548	+4.25	+ 3.1
					(415) *Palatia.*					
1905										
Jan. 2 12 51 35	16	29 , 6	+1 8.82	− 3 46.4	6 5 39.63	+15 16 52.3	9.292	0.563	+1.08	−10.1
4 11 8 32	17	25 , 5	+2 52.13	− 3 10.9	6 4 7.25	+15 29 53.7	7.214	0.540	+1.09	−10.1
8 10 42 23	18	30 , 6	−3 38.32	+ 1 59.1	6 1 10.12	+15 57 18.6	n8.144	0.532	+1.11	−10.2
14 10 13 13	19	30 , 6	+0 51.35	− 1 26.3	5 57 28.92	+16 39 18.0	n8.228	0.519	+1.11	−10.1
14 10 53 21	20	14 , 3	+1 14.31	− 0 43.0	5 57 27.93	+16 39 28.0	8.817	0.521	+1.11	−10.1
15 9 25 19	21	30 , 6	+1 41.50	− 2 48.2	5 56 59.03	+16 46 7.3	n9.033	0.524	+1.11	−10.1
15 9 48 17	22	25 , 5	−1 21.79	+ 2 20.3	5 56 58.69	+16 46 15.8	n8.783	0.519	+1.11	−10.2
16 10 5 23	23	20 , 10	+0 18.77	− 2 21.5	5 56 29.10	+16 53 22.7	n8.199	0.514	+1.11	−10.2
20 10 13 10	24	30 , 6	−1 27.11	− 3 42.0	5 51 51.62	+17 21 36.0	8.566	0.506	+1.11	−10.1*

1905 Wash'n M.T.	*	Comp	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
					(78) *Diana*.					
			m s	′ ″	h m s	° ′ ″			″	″
Jan. 4 11 55 58 (h m s)	25	25 , 5	+1 50.04	− 0 33.0	6 36 52.17	+36 6 22.5	8.585	9.614	+1.27	− 9.8
4 12 10 58	26	20 , 4	−0 59.23	− 5 41.3	6 36 51.34	+36 6 21.1	8.878	9.642	+1.27	− 9.9
15 10 47 34	27	30 , 6	−1 8.64	− 2 54.0	6 25 11.66	+35 18 42.0	7.703	9.720	+1.33	− 8.4
15 11 8 16	28	25 , 5	−0 12.69	− 6 43.0	6 25 10.80	+35 18 36.3	8.746	9.736	+1.33	− 8.4
16 10 50 28	29	30 , 6	+1 12.00	− 2 9.6	6 24 16.18	+35 13 10.9	8.380	9.734	+1.33	− 8.3
					(51) *Nemausa*.					
Jan. 27 9 58 11	30	30 , 6	+0 22.32	+ 3 41.2	8 18 47.73	+ 5 53 36.5	n9.340	0.686	+1.25	−12.1
28 9 59 36	31	25 , 5	+2 28.94	+ 3 18.3	8 17 50.16	+ 6 0 41.5	n9.317	0.684	+1.26	−12.2
30 10 2 40	32	25 , 5	+2 34.59	− 0 22.6	8 15 55.67	+ 6 15 25.7	n9.266	0.680	+1.27	−12.5
30 10 18 55	33	25 , 5	−1 18.41	− 0 15.8	8 15 54.88	+ 6 15 31.4	n9.188	0.678	+1.27	−12.4
Feb. 3 10 57 32	34	30 , 6	−0 43.11	+ 1 8.3	8 12 10.82	+ 6 46 41.6	n8.604	0.668	+1.28	−12.8
					(15) *Eunomia*.					
Jan. 28 10 57 23	35	25 , 5	+3 17.66	+ 8 27.5	8 41 17.16	+13 9 34.5	n9.167	0.588	+1.23	−11.9
Feb. 3 11 43 20	36	25 , 5	+0 52.70	− 8 6.2	8 34 55.88	+13 9 34.0	7.809	0.579	+1.27	−12.3
4 9 46 3	37	30 , 6	+0 49.77	− 6 11.1	8 33 59.18	+13 9 35.6	n9.331	0.598	+1.28	−12.3
7 10 8 53	38	25 , 5	+4 0.28	+ 2 39.2	8 30 57.69	+13 9 53.0	n9.161	0.588	+1.28	−12.3
10 10 26 5	39	30 , 6	+1 48.02	+ 6 11.4	8 28 4.45	+13 10 18.4	n8.910	0.582	+1.29	−12.4
14 10 42 46	38	24 , 5	−2 28.82	+ 3 36.2	8 24 28.59	+13 10 49.8	n7.886	0.579	+1.28	−12.5
					(362) *Havnia*.					
Jan. 28 11 56 2	40	29 , 6	−0 9.14	+ 8 21.6	9 22 27.86	+28 44 16.1	n9.096	0.210	+1.17	−12.3
30 11 23 9	41	22 , 5	+4 16.08	− 1 39.9	9 20 23.63	+28 52 19.4	n9.243	0.230	+1.21	−12.2
Feb. 4 10 42 8	42	30 , 6	−1 9.08	− 7 46.4	9 15 4.80	+29 10 9.3	n9.321	0.240	+1.27	−11.9
10 11 58 42	43	23 , 5	+3 43.77	− 1 23.0	9 8 37.35	+29 25 53.1	8.455	0.154	+1.33	−11.2
					1905 *P. S.*					
Feb. 10 9 51 41	44	22 , 5	−3 13.13	− 1 56.6	7 59 26.98	+19 27 28.4	n8.983	0.469	+1.27	−11.5
14 9 52 42	45	25 , 5	+4 27.69	+ 3 35.5	7 56 34.55	+19 30 21.8	n8.741	0.464	+1.24	−11.4
15 10 10 23	46	25 , 5	+3 43.56	+ 7 17.7	7 55 55.51	+19 30 54.4	n7.898	0.461	+1.24	−11.4
17 10 28 29	47	20 , 4	+5 20.97	+ 2 7.4	7 54 44.27	+19 31 45.2	8.696	0.463	+1.22	−11.4*
18 10 18 15	48	25 , 5	+3 51.30	− 4 44.0	7 54 12.20	+19 32 5.1	8.573	0.462	+1.22	−11.3*
24 11 18 4	49	30 , 6	+3 38.73	+ 5 32.0	7 51 39.86	+19 32 32.3	9.327	0.494	+1.16	−11.0
26 10 34 48	50	23 , 5	+4 40.28	− 1 47.1	7 51 6.69	+19 32 10.1	9.157	0.476	+1.14	−11.0
					(192) *Nausikaa*.					
Mar. 2 11 5 14	51	19 , 5	−3 27.34	+ 0 48.5	10 31 45.49	+10 19 44.7	n8.961	0.624	+1.38	−11.7
5 10 22 34	52	24 , 5	+2 24.90	− 1 40.4	10 28 47.43	+10 30 26.7	n9.166	0.625	+1.39	−11.8
5 10 40 49	53	27 , 6	+0 23.64	+ 0 32.8	10 28 46.61	+10 30 28.2	n9.044	0.622	+1.39	−11.8
10 11 39 54	54	24 , 5	+3 38.82	− 2 17.1	10 23 56.50	+10 47 12.3	8.764	0.616	+1.38	−11.7
13 9 26 32	55	30 , 6	−0 7.52	− 3 9.1	10 21 18.25	+10 55 54.7	n9.254	0.623	+1.38	−11.7
13 9 40 14	56	30 , 6	−0 47.73	+ 0 20.2	10 21 17.97	+10 55 56.6	n9.185	0.620	+1.38	−11.7
					(117) *Lomia*.					
Mar. 13 10 32 0	57	27 , 6	−0 14.80	− 5 26.9	11 20 33.10	+ 3 43 51.5	n9.216	0.705	+1.46	−10.9
13 10 44 42	58	9 , 2	+2 11.00	+ 3 11.0	11 20 32.81	+ 3 43 53.3	n9.148	0.704	+1.46	−11.0
15 10 5 32	58	28 , 6	+0 22.47	+ 5 35.7	11 18 44.29	+ 3 46 17.9	n9.291	0.706	+1.47	−11.1
25 12 10 4	59	25 , 5	+0 55.84	+ 0 33.2	11 9 54.17	+ 3 56 51.6	9.152	0.702	+1.46	−11.4*
27 9 46 27	60	30 , 6	+1 52.49	− 3 12.0	11 8 21.62	+ 3 58 20.2	n9.091	0.701	+1.45	−11.5*
					(198) *Ampella*.					
Mar. 16 10 45 43	61	24 , 5	+0 38.27	− 0 48.4	11 0 17.54	− 8 30 46.0	n8.886	0.809	+1.58	−12.3
16 10 58 27	62	25 , 5	+1 40.37	− 1 14.5	11 0 17.06	− 8 30 43.1	n8.711	0.809	+1.57	−12.4
16 11 14 47	63	25 , 5	+1 4.60	+ 3 43.0	11 0 16.47	− 8 30 38.6	n8.268	0.810	+1.58	−12.3
25 10 54 23	64	20 , 4	+4 2.81	− 0 57.6	10 52 48.67	− 7 35 8.2	8.427	0.803	+1.52	−13.3
25 11 14 6	65	25 , 5	+2 16.91	+ 3 46.8	10 52 48.05	− 7 35 2.8	8.822	0.802	+1.53	−13.3
28 9 48 35	66	25 , 5	−2 42.84	+ 4 28.4	10 50 35.86	− 7 16 14.7	n8.888	0.800	+1.52	−13.4
					(42) *Isis*.					
Mar. 28 10 37 24	67	25 , 5	−2 9.74	− 1 1.8	12 5 4.30	+13 47 8.9	n9.114	0.576	+1.46	− 9.2
30 11 25 20	68	25 , 5	+2 50.49	− 3 32.8	12 3 10.12	+13 56 25.2	n8.112	0.566	+1.46	− 9.2
30 11 38 54	69	18 , 6	+0 23.30	− 6 7.3	12 3 9.69	+13 56 28.1	8.177	0.566	+1.46	− 9.1
Apr. 7 10 30 31	70	25 , 5	+2 53.50	− 7 37.8	11 56 4.00	+14 26 5.0	n8.668	0.559	+1.44	− 8.4

1905 Wash'n M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(487) *Venetia.*					
h m s			m s	′ ″	h m s	° ′ ″			″	″
Mar. 28 11 21 29	71	25 , 5	+1 30.22	+ 0 55.2	11 53 45.46	+15 43 32.5	n8.253	0.536	+1.44	− 9.2
28 11 33 23	72	18 , 6	−0 19.38	− 3 57.6	11 53 44.98	+15 43 35.3	7.835	0.536	+1.45	− 9.2
31 10 6 20	73	25 , 5	+3 33.62	− 0 22.4	11 51 27.55	+15 57 28.5	n9.154	0.543	+1.44	− 9.0
Apr. 7 11 14 9	74	25 , 5	−1 15.57	+ 0 56.8	11 46 22.35	+16 23 28.2	8.805	0.526	+1.43	− 8.2
					(122) *Gerda.*					
Mar. 29 11 14 39	75	25 , 5	−1 1.67	+ 1 35.0	12 59 30.47	− 5 41 22.9	n9.182	0.786	+1.63	− 7.8
29 11 30 21	76	25 , 5	+1 11.38	− 4 27.6	12 59 30.20	− 5 41 20.2	n9.086	0.787	+1.63	− 7.9
31 10 53 34	77	20 , 4	+3 58.88	+ 3 15.7	12 58 5.71	− 5 31 32.0	n9.241	0.784	+1.64	− 8.2
31 11 6 53	78	25 , 5	−0 13.34	+ 5 24.2	12 58 5.49	− 5 31 28.5	n9.173	0.785	+1.64	− 8.0
Apr. 9 9 43 17	79	25 , 5	−1 19.26	+ 1 14.8	12 51 38.98	− 4 46 41.2	n9.353	0.776	+1.68	− 8.3
16 9 31 21	80	18 , 6	+0 24.00	− 2 59.3	12 46 48.17	− 4 13 0.8	n9.276	0.774	+1.68	− 8.7
19 9 43 49	81	25 , 5	+2 8.23	+10 31.1	12 44 50.36	− 3 59 21.0	n9.145	0.774	+1.67	− 8.8
					(190) *Ismene.*					
Apr. 9 10 30 46	82	25 , 5	−1 10.96	+ 4 56.1	12 45 15.80	− 2 18 11.8	n9.099	0.760	+1.65	− 8.5
16 10 24 35	83	25 , 5	+3 10.94	− 1 49.7	12 41 17.44	− 1 45 54.1	n8.881	0.756	+1.64	− 8.8
22 11 22 25	84	30 , 6	−0 55.74	+ 6 38.8	12 38 8.93	− 1 20 35.7	8.964	0.752	+1.63	− 8.6
					(163) *Erigone.*					
Apr. 9 11 25 23	85	30 , 6	+1 38.98	+ 1 11.7	13 12 1.72	− 2 7 56.3	n9.851	0.759	+1.66	− 7.5
16 10 59 55	86	5 , 1	−2 5.98	− 4 5.6	13 5 48.68	− 1 19 27.1	n8.737	0.752	+1.68	− 7.6
23 11 35 17	87	30 , 6	−0 16.39	+ 1 56.3	13 0 3.83	− 0 37 32.3	8.914	0.746	+1.67	− 7.6
May 1 10 10 16	88	25 , 5	−1 3.97	+ 0 21.4	12 54 37.95	− 0 0 45.6	n8.137	0.740	+1.64	− 7.4
					(53) *Kalypso.*					
Apr. 24 10 55 16	89	25 , 5	+3 16.87	− 1 29.8	14 47 39.57	− 8 8 5.0	n9.300	0.801	+1.84	− 3.3
May 1 11 0 1	90	18 , 6	+0 16.26	−11 23.7	14 41 28.62	− 7 35 16.0	n9.103	0.801	+1.90	− 3.4
7 10 52 31	91	25 , 5	+1 50.46	− 1 35.2	14 36 11.60	− 7 9 47.3	n8.928	0.799	+1.93	− 3.5
7 11 7 38	92	25 , 5	+0 57.41	− 3 5.4	14 36 11.24	− 7 9 48.0	n8.737	0.800	+1.93	− 3.4
					(345) *Tercidina.*					
May 21 11 5 25	93	25 , 5	+2 9.77	+ 2 12.5	16 27 35.02	−12 52 12.5	n9.234	0.831	+2.16	+ 2.6
23 11 21 21	94	25 , 5	−2 17.90	− 3 32.7	16 25 38.10	−12 39 19.9	n9.083	0.834	+2.18	+ 2.7
27 10 14 48	95	25 , 5	+0 29.48	+ 8 28.9	16 21 45.83	−12 14 47.8	n9.323	0.824	+2.22	+ 2.5
28 9 53 52	96	20 , 4	+3 30.21	+ 3 17.3	16 20 47.89	−12 8 52.3	n9.379	0.820	+2.22	+ 2.3
28 10 23 39	97	30 , 6	−0 59.74	− 5 20.8	16 20 46.54	−12 8 46.8	n9.267	0.826	+2.22	+ 2.6
					(386) *Siegena.*					
May 27 11 40 5	98	25 , 5	+2 12.20	+ 5 32.0	15 59 46.24	+ 7 34 58.3	7.045	0.658	+1.99	+ 2.2
27 11 52 39	99	30 , 6	+0 4.96	− 3 13.2	15 59 45.75	+ 7 34 59.9	8.423	0.658	+2.00	+ 2.2
June 8 9 37 27	100	10 , 10	−0 0.66	+ 4 9.9	15 50 56.69	+ 7 51 27.8	n9.119	0.658	+2.04	+ 3.7
11 9 35 30	101	25 , 5	+2 6.51	+ 1 52.3	15 46 59.99	+ 7 48 29.8	n8.910	0.656	+2.03	+ 4.5
18 10 48 46	102	25 , 5	+2 22.47	− 0 39.9	15 44 36.17	+ 7 42 30.5	9.008	0.658	+2.02	+ 4.9
18 11 8 42	103	25 , 5	+2 13.91	+ 3 19.7	15 44 36.13	+ 7 42 27.9	9.148	0.660	+2.02	+ 4.9
					(276) *Adelheid.*					
June 8 10 36 26	104	20 , 4	+3 46.12	− 1 37.7	16 32 36.65	− 1 45 51.2	n8.985	0.756	+2.19	+ 4.1
8 10 51 23	105	20 , 4	−2 57.91	− 6 32.4	16 32 36.07	− 1 45 49.0	n8.787	0.756	+2.20	+ 4.4
					(470) *Kilia.*					
June 11 10 43 18	106	25 , 5	−1 2.98	+ 5 58.1	17 0 32.51	− 9 32 38.0	n8.963	0.815	+2.35	+ 5.7
18 11 11 25	107	25 , 5	+0 41.54	+ 4 21.1	16 56 57.50	− 9 31 55.1	8.841	0.816	+2.38	+ 5.7
18 11 55 14	108	18 , 6	+0 22.70	+ 5 55.4	16 56 57.15	− 9 31 55.3	8.959	0.815	+2.38	+ 5.7
25 10 36 51	109	25 , 5	−2 0.39	− 2 14.6	16 51 22.52	− 9 36 58.2	n6.130	0.817	+2.40	+ 5.9
					(58) *Concordia.*					
June 25 11 37 2	110	20 , 4	+3 37.42	− 4 17.2	16 34 56.74	−14 6 39.3	9.190	0.840	+2.13	+ 4.0

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	3 41 44.80	+29 5 44.8	Camb. Eng., A.G. 1847	56	10 22 4.32	+10 55 48.1	Leipzig I, A.G. 4025
2	3 37 42.51	+29 23 11.2	Camb. Eng., A.G. 1809	57	11 20 46.41	+ 3 49 29.3	Albany, A.G. 4268
3	3 38 3.39	+29 22 41.4	Camb. Eng., A.G. 1815	58	11 18 20.35	+ 3 40 53.3	Albany, A.G. 4259
4	3 25 13.01	+10 7 47.7	Leipzig II, A.G. 1287	59	11 8 56.87	+ 3 56 29.8	Albany, A.G. 4224
5	3 12 2.83	+ 9 47 20.2	Leipzig II, A.G. 1217	60	11 6 27.68	+ 4 1 43.7	Albany, A.G. 4214
6	3 59 19.16	+20 35 42.5	Berlin B, A.G. 1317	61	10 59 37.69	− 8 29 45.3	Wien, A.G. 4184
7	4 28 22.75	−14 1 15.3	Washington, A.G.Zones	62	10 58 35.12	− 8 29 16.2	Wien, A.G. 4179
8	4 23 54.77	−13 49 58.8	Washington, A.G.Zones	63	10 59 10.29	− 8 34 9.3	Wien, A.G. 4182
9	4 21 33.64	−13 29 57.5	Camb.(U.S.), A.G.Zones	64	10 48 44.34	− 7 33 57.3	Wien, A.G. 4129
10	4 22 52.11	−13 27 14.1	Camb.(U.S.), A.G.Zones	65	10 50 29.61	− 7 38 36.3	Wien, A.G. 4135
11	4 16 29.67	−12 45 34.0	Camb.(U.S.), A.G.Zones	66	10 53 17.18	− 7 20 29.7	Wien, A.G. 4154
12	4 18 31.07	−12 46 59.6	Camb.(U.S.), A.G Zones	67	12 7 12.58	+13 48 19.9	Leipzig I, A.G. 4508
13	4 5 15.51	+14 59 10.0	½(Leip.I,1223+Ber.A.,1101)	68	12 0 18.17	+14 0 7.2	Leipzig I, A.G. 4481
14	4 10 19.10	+15 9 38.8	½(Leip.I,1247+Ber.A.,1118)	69	12 2 44.93	+14 2 44.5	Leipzig I, A.G. 4492
15	4 2 15.59	+14 54 22.2	Leipzig I, A.G. 1207	70	11 53 9.06	+14 33 51.2	½(Leip.I,4452+Ber.A.,4573)
16	6 1 29.73	+15 20 48.8	Berlin A, A.G. 1948	71	11 52 13.80	+15 42 46.5	Berlin A, A.G. 4568
17	6 1 14.03	+15 33 14.7	Berlin A, A.G. 1906	72	11 54 2.91	+15 47 42.1	Berlin A, A.G. 4575
18	6 4 47.33	+15 55 29.4	Berlin A, A.G. 1954	73	11 47 52.49	+15 57 59.9	Berlin A, A.G. 4546
19	5 56 36.46	+16 40 54.4	Berlin A, A.G. 1856	74	11 47 36.49	+16 22 39.6	Berlin A, A.G. 4544
20	5 55 42.48	+16 40 21.1	Berlin A, A.G. 1846	75	13 0 30.51	− 5 42 50.1	Strassburg, A.G. Zones
21	5 55 16.42	+16 49 5.6	Berlin A, A.G. 1838	76	12 58 17.19	− 5 36 41.7	Strassburg, A.G. Zones
22	5 58 19.37	+16 44 5.7	Berlin A, A.G. 1876	77	12 54 5.19	− 5 34 39.5	Strassburg, A.G. Zones
23	5 56 9.22	+16 55 54.4	Berlin A, A.G. 1850	78	12 58 17.19	− 5 36 44.7	Strassburg, A.G. Zones
24	5 56 17.62	+17 25 28.1	Berlin A, A.G. 1853	79	12 52 56.56	− 4 47 47.7	Strassburg, A.G. Zones
25	6 35 0.86	+36 7 5.3	Lund, A.G. 3438	80	12 46 22.49	− 4 9 52.8	Strassburg, A.G. Zones
26	6 37 19.30	+36 12 12.3	Lund, A.G. 3479	81	12 42 40.46	− 4 9 43.3	Strassburg, A.G. Zones
27	6 26 18.97	+35 21 44.4	Lund, A.G. 3358	82	12 46 25.11	− 2 22 59.4	Nicolajew, A.G. 3472
28	6 25 22.16	+35 25 27.7	Lund, A.G. 3346	83	12 38 4.86	− 1 43 55.6	Nicolajew, A.G. 3450
29	6 23 2.85	+35 15 28.8	Lund, A.G. 3321	84	12 39 3.04	− 1 27 5.9	Nicolajew, A.G. 3457
30	8 18 24.16	+ 5 50 7.4	Leipzig II, A.G. 4552	85	13 10 24.08	− 2 9 0.5	Nicolajew, A.G. 3555
31	8 15 19.96	+ 5 57 38.4	Leipzig II, A.G. 4509	86	13 7 52.98	− 1 15 13.9	Nicolajew, A.G. 3544
32	8 13 19.81	+ 6 16 0.8	Leipzig II, A.G. 1488	87	13 0 18.55	− 0 39 21.0	Nicolajew, A.G. 3525
33	8 17 12.02	+ 6 15 59.6	Leipzig II, A.G. 1533	88	12 55 40.28	− 0 0 59.6	Nicolajew, A.G. 3509
34	8 12 52.65	+ 6 45 16.1	Leipzig II, A.G. 1482	89	14 44 20.86	− 8 6 31.9	Wien, A.G. 5198
35	8 37 58.27	+13 1 18.9	Leipzig I, A.G. 3515	90	14 41 10.46	− 7 23 48.9	Wien, A.G. 5182
36	8 34 1.91	+13 17 52.5	Leipzig I, A.G. 3492	91	14 34 19.21	− 7 8 8.6	Wien, A.G. 5149
37	8 33 8.13	+13 15 59.0	Leipzig I, A.G. 3486	92	14 35 11.90	− 7 6 39.2	Wien, A.G. 5153
38	8 26 56.13	+13 7 26.1	Leipzig I, A.G. 3136	93	16 25 23.09	−12 54 27.6	Camb.(U.S.), A.G.Zones
39	8 26 15.14	+13 4 19.4	Leipzig I, A.G. 3434	94	16 27 53.82	−12 35 49.9	Camb.(U.S.), A.G.Zones
40	9 22 35.83	+28 36 6.8	Camb. Eng., A.G. 4964	95	16 21 14.13	−12 23 19.2	Camb.(U.S.), A.G.Zones
41	9 16 6.34	+28 54 11.5	Camb. Eng., A.G. 4924	96	16 17 15.46	−12 12 11.9	Camb.(U.S.), A.G.Zones
42	9 16 12.61	+29 18 7.6	Camb. Eng., A.G. 4927	97	16 21 44.06	−12 3 28.6	Camb.(U.S.), A.G.Zones
43	9 4 52.25	+29 27 27.3	Camb. Eng., A.G. 4849	98	15 57 32.05	+ 7 29 24.1	Leipzig II, A.G. 7165
44	8 2 38.81	+19 29 36.5	Berlin A, A.G. 3202	99	15 59 38.79	+ 7 38 10.9	Leipzig II, A.G. 7179
45	7 52 5.62	+19 26 57.7	Berlin A, A.G. 3120	100	15 50 55.31	+ 7 47 14.2	Leipzig II, A.G. 7128
46	7 52 10.71	+19 23 18.1	Berlin A, A.G. 3124	101	15 44 51.42	+ 7 46 33.0	Leipzig II, A.G. 7093
47	7 49 22.08	+19 29 49.2	Battermann, 455	102	15 42 11.68	+ 7 43 5.5	Leipzig II, A.G. 7085
48	7 50 19.68	+19 37 0.4	Berlin A, A.G. 3100	*103	15 41 50.20	+ 7 39 3.3	Leipzig II, A.G. 7083
49	7 47 59.97	+19 27 11.3	Berlin A, A.G. 3078	104	16 28 48.34	− 1 44 17.6	Nicolajew, A.G. 4166
50	7 46 25.27	+19 34 8.2	Berlin A, A.G. 3067	105	16 35 31.81	− 1 39 21.0	Nicolajew, A.G. 4185
51	10 35 11.45	+10 19 7.9	Leipzig I, A.G. 4081	106	17 1 33.17	− 9 38 41.8	Wien, A.G. 5846
52	10 26 21.14	+10 32 18.9	Leipzig I, A.G. 4045	107	16 56 13.58	− 9 36 24.9	Wien, A.G. 5830
53	10 28 21.58	+10 30 7.2	Leipzig I, A.G. 4054	108	16 56 32.07	− 9 37 56.1	Wien, A.G. 5832
54	10 20 16.30	+10 49 41.1	Leipzig I, A.G. 4017	109	16 53 20.51	− 9 31 49.5	Wien, A.G. 5821
55	10 21 24.39	+10 59 15.5	Leipzig I, A.G. 4021	110	16 31 17.19	−14 2 26.1	Washington, A.G.Zones

Nos. 13, 103, 116, 130, 44, 415, 78, 51, 15, 362, 1905 P. S., 192 and 117 were observed with the 12-inch equatorial with the exception of those observations marked with a * which were observed with the 26-inch equatorial; all the other planets were observed with the latter instrument.

* Star No. 103 shows indication of proper motion.

Nos. 13, 103, 116, 130, 44, 415, 51, 362, 1905 P. S., 192 and 117 were found photographically by Mr. G. H. PETERS.

The star places from the Strassburg and Cambridge (U. S.) A. G. Zones were furnished through the courtesy of the Directors of the Observatories at those places.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN PENN., WITH A $4\frac{1}{2}$-INCH REFRACTOR,

By A. W. QUIMBY.

1905		Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Jan.	1	9	..	2	15	3	good
	2	8	..	1	8	..	poor
	4	8	..	1	3	..	poor
	5	8	..	1	1	1	poor
	6	8	..	..	..	..	poor
	7	8	..	1	3	..	poor
	8	8	3	4	16	4	fair
	9	8	1	5	23	1	fair
	10	8	1	6	16	3	poor
	11	9	..	6	24	..	poor
	12	3	..	5	8	..	poor
	13	8	..	5	17	2	poor
	14	9	1	6	32	1	fair
	15	8	1	8	37	3	fair
	16	8	..	5	23	3	fair
	17	8	..	4	42	4	good
	18	8	1	6	54	4	good
	19	11	..	4	63	1	fair
	20	9	1	4	27	2	poor
	21	8	..	4	22	3	fair
	22	12	..	4	16	3	fair
	23	9	..	3	12	3	fair
	26	9	1	2	5	3	poor
	27	9	..	2	11	2	poor
	28	9	1	3	18	3	fair
	29	8	2	5	12	2	fair
	30	2	..	3	50	2	fair
	31	10	..	2	45	1	poor
Feb.	1	8	..	2	45	1	poor
	2	10	1	2	50	1	poor
	3	9	1	3	92	1	fair
	4	9	2	5	125	3	fair
	5	10	..	5	109	3	fair
	6	1	1	6	78	3	fair
	7	8	1	7	118	3	good
	8	11	1	5	35	2	poor
	10	8	1	7	25	5	fair
	11	9	..	6	23	2	fair
	13	5	..	5	23	2	poor
	14	10	..	6	27	2	fair
	15	4	..	3	19	2	fair
	16	10	..	3	13	2	poor
	17	4	3	5	14	3	poor
	18	2	..	3	16	3	fair
	19	8	..	3	20	3	fair
	21	3	1	2	11	1	fair
	23	9	..	3	12	..	poor
	24	10	..	3	23	1	fair
	25	9	1	3	36	2	fair
	26	3	2	5	42	2	fair
	27	9	..	1	36	2	fair
	28	1	1	1	11	3	fair
Mar.	1	9	1	3	84	2	fair
	2	9	..	3	101	2	fair
	3	9	1	1	107	3	fair
	4	3	..	1	138	2	fair
	5	8	..	4	150	2	fair
Mar.	6	8	..	4	150	2	fair
	8	10	..	2	95	1	poor
	10	8	..	1	48	..	poor
	11	7	..	1	40	3	fair
	12	8	..	1	20	1	poor
	13	8	..	1	5	1	fair
	14	8	1	1	5	4	good
	*15	7	..	1	3	1	poor
	*16	8	3	4	9	..	fair
	*17	8	..	4	14	..	fair
	18	3	..	3	18	..	fair
	19	4	1	3	10	1	fair
	22	2	1	3	5	..	poor
	23	2	1	4	14	1	fair
	24	8	..	1	5	..	fair
	25	3	1	4	16	2	fair
	26	8	..	3	19	2	good
	27	7	..	3	22	1	fair
	28	7	..	3	36	3	fair
	29	4	..	3	22	2	fair
	30	7	..	2	20	1	fair
	31	7	..	2	32	1	fair
Apr.	1	3	..	1	14	..	poor
	2	5	1	3	18	2	fair
	3	7	..	2	13	2	fair
	4	7	..	2	8	2	poor
	5	10	..	1	5	1	poor
	6	9	..	1	5	2	fair
	7	3	..	1	5	2	fair
	8	5	1	2	9	2	fair
	9	7	..	2	5	3	fair
	10	8	1	3	10	4	fair
	11	8	..	3	18	2	fair
	12	9	1	4	25	2	fair
	13	5	1	5	24	4	fair
	14	7	..	5	27	4	fair
	15	8	..	5	21	2	fair
	16	8	1	5	24	..	fair
	17	7	..	5	20	..	fair
	18	7	..	3	1	1	fair
	19	7	..	2	3	1	fair
	20	7	1	3	4	2	fair
	21	7	..	2	4	3	fair
	22	7	..	1	9	2	fair
	23	4	2	3	24	2	fair
	24	7	..	3	31	2	fair
	25	7	..	3	26	2	fair
	26	11	..	3	12	..	poor
	27	4	1	4	27	3	fair
	28	7	1	5	33	4	fair
	29	4	..	1	6	..	poor
	30	6	..	3	7	2	fair
May	1	7	1	4	5	2	fair
	2	6	..	2	13	3	fair
	3	6	..	2	14	2	fair
	4	10	1	2	22	2	fair
	5	10	2	1	9	..	poor
May	6	4	..	1	5	..	poor
	7	5	2	3	24	..	poor
	8	6	1	4	31	1	fair
	9	7	..	4	32	1	fair
	10	5	2	6	43	3	fair
	11	5	1	7	36	4	fair
	12	6	..	6	34	4	fair
	13	3	..	5	22	2	poor
	14	1	..	4	17	..	poor
	15	3	..	5	34	2	fair
	16	5	1	5	36	2	fair
	17	10	..	3	17	..	poor
	18	7	..	4	32	3	fair
	19	6	..	3	31	4	fair
	20	6	..	3	20	4	fair
	21	5	3	6	21	4	v. good
	22	5	..	4	8	5	v. good
	23	5	..	1	2	4	good
	24	5	..	..	..	3	good
	25	5	1	1	4	..	fair
	26	6	1	2	10	..	fair
	27	5	..	2	15	3	fair
	28	5	..	1	14	3	fair
	29	5	..	1	8	3	fair
	30	6	1	2	6	3	fair
	31	10	..	1	4	..	v. poor
June	1	5	..	1	7	2	poor
	2	4	2	3	17	3	fair
	3	6	..	1	10	1	fair
	4	6	1	2	20	1	fair
	5	6	1	3	38	2	fair
	6	6	..	3	26	2	fair
	7	9	..	2	8	..	poor
	8	1	1	4	8	2	poor
	9	5	2	7	15	4	fair
	10	6	..	6	20	3	fair
	11	9	..	3	8	..	poor
	12	5	..	3	6	..	fair
	13	6	..	3	10	1	fair
	14	6	1	4	9	2	fair
	15	5	1	5	10	2	fair
	16	6	..	5	8	2	poor
	17	5	..	5	8	3	fair
	18	5	1	5	5	3	fair
	19	5	..	2	4	3	good
	20	5	..	1	3	2	good
	21	9	..	1	3	..	poor
	22	8	1	2	14	..	fair
	23	6	1	3	30	2	good
	24	5	1	4	26	..	fair
	25	5	..	4	43	4	good
	26	6	..	4	47	3	poor
	27	5	..	3	61	5	good
	28	6	..	2	48	4	fair
	29	6	..	2	31	3	fair
	30	6	2	3	27	3	fair

* 2 inch refractor.

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AN ANALYTICAL DETERMINATION OF THE LAW OF LINEARLY COMBINING A SERIES OF INDIRECT OBSERVATION-EQUATIONS SO THAT THE PROBABLE ERRORS OF THE UNKNOWN QUANTITIES BECOME MINIMA.

By J. MIDZUHARA.

AIRY always considered that to find, analytically, the law of linearly combining a series of indirect observation-equations so that the probable errors of the unknown quantities become minima, would be a troublesome matter.[1] This, however, is not the case; for, since then it has been easily solved by TODHUNTER,[2] and afterward by GLAISHER,[3] so that we can not now plan any improvement of that solution. In this paper, I shall describe another solution, of the same problem, which is the result that a difficult course of the analysis, to be regarded as if it was the cause of the erroneous consideration of AIRY, has been simplified by an application of some remarkable theorems of determinants.

Let

$$(1)\qquad \left\{\begin{array}{l} a_1x + b_1y + c_1z + \;.\;.\;.\;.\;. = f_1 \\ a_2x + b_2y + c_2z + \;.\;.\;.\;.\;. = f_2 \\ a_3x + b_3y + c_3z + \;.\;.\;.\;.\;. = f_3 \\ .\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \end{array}\right.$$

be the series of the given observation-equations in which all the observations are subject to the same probable error: and let us confine our attention, for instance, to the discussion of the value of x. Suppose that, if we multiply the observation-equations successively by n_1, n_2, &c., and add together, we get

$$(2)\qquad [na]x = [nf]$$

$$(3)\qquad [nb] = [nc] = [nd] = \&c. = 0;$$

then our present problem is to find the values of n_1, n_2, &c., which render the probable error of the x a minimum, that is, whose variations δn_1, δn_2, &c., satisfy the following conditions:

$$0 = \frac{[n\,.\,\delta n]}{[n^2]} - \frac{[n\,.\,\delta n]}{[na]} \qquad (4)$$

$$0 = [b\,.\,\delta n] = [c\,.\,\delta n] = \&c. \qquad (5)$$

Now let us put

$$\begin{vmatrix} b_1, b_2, \ldots\ldots b_{\mu-1} \\ c_1, c_2, \ldots\ldots c_{\mu-1} \\ \ldots\ldots\ldots\ldots \end{vmatrix} = \Delta$$

$$b_\mu\,\delta n_\mu + b_{\mu+1}\,\delta n_{\mu+1} + \;.\;.\;.\;.\; + b_m\,\delta n_m = B$$
$$c_\mu\,\delta n_\mu + c_{\mu+1}\,\delta n_{\mu+1} + \;.\;.\;.\;.\; + c_m\,\delta n_m = C$$
$$.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.$$

where

m = the number of the observation-equations

μ = the number of the unknown quantities:

then solving (5) for δn_1, δn_2, ... $\delta n_{\mu-1}$ in terms of B, C, ... and known quantities we get

$$\delta n_1 = \frac{-\begin{vmatrix} B, b_2, b_3, \ldots b_{\mu-1} \\ C, c_2, c_3, \ldots c_{\mu-1} \\ \ldots\ldots\ldots \end{vmatrix}}{\Delta} = M'_\mu\delta n_\mu + M'_{\mu+1}\delta n_{\mu+1} + \ldots \text{ (say)}$$

$$\delta n_2 = \frac{-\begin{vmatrix} b_1, B, b_3, \ldots b_{\mu-1} \\ c_1, C, c_3, \ldots c_{\mu-1} \\ \ldots\ldots\ldots \end{vmatrix}}{\Delta} = M''_\mu\delta n_\mu + M''_{\mu+1}\delta n_{\mu+1} + \ldots \text{ (say)}$$

$$.\;.$$

and these expressions being substituted in the equation (4) each of the coefficients of δn_μ, $\delta n_{\mu+1}$, ... δn_m in that equation must vanish; that is, we must have

[1] "Theory of Errors of Observations" (London; 1861, 1st edition; 1875, 2d edition).

[2] "History of the Theory of Probability" (Cambridge and London, 1865)

[3] "On the Law of Facility of Errors of Observations, and of the method of Least-Squares" (*Royal Astron. Soc.* Vol. XXXIX, 1872).

$$
(6)\quad \left\{
\begin{aligned}
&n_1 M'_\mu + n_2 M''_\mu + \ldots + n_{\mu-1} M_\mu^{(\mu-1)} + n_\mu = \frac{[n^2]\, F_\mu}{[na]}\\
&n_1 M'_{\mu+1} + n_2 M''_{\mu+1} + \ldots + n_{\mu-1} M_{\mu+1}^{(\mu-1)} + n_{\mu+1} = \frac{[n^2]\, F_{\mu+1}}{[na]}\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
\right.
$$

where

$$
\begin{aligned}
F_\mu &= a_1 M'_\mu + a_2 M''_\mu + \ldots + a_{\mu-1} M_\mu^{(\mu-1)} + a_\mu\\
F_{\mu+1} &= a_1 M'_{\mu+1} + a_2 M''_{\mu+1} + \ldots + a_{\mu-1} M_{\mu+1}^{(\mu-1)} + a_{\mu+1}\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

Therefore, from (3) and (6) we have immediately (for instance),

$$
(7)\qquad n_1 \times (\text{constant}) = \begin{vmatrix}
0 & b_2 & b_3 & \ldots & b_{\mu-1} & b_\mu & b_{\mu+1} & b_{\mu+2} & \ldots & b_m\\
0 & c_2 & c_3 & \ldots & c_{\mu-1} & c_\mu & c_{\mu+1} & c_{\mu+2} & \ldots & c_m\\
. & . & . & & . & . & . & . & & .\\
F_\mu & M''_\mu & M'''_\mu & \ldots & M_\mu^{(\mu-1)} & 1 & 0 & 0 & \ldots & 0\\
F_{\mu+1} & M''_{\mu+1} & M'''_{\mu+1} & \ldots & M_{\mu+1}^{(\mu-1)} & 0 & 1 & 0 & \ldots & 0\\
. & . & . & & . & 0 & 0 & 1 & \ldots & 0\\
. & . & . & & . & 0 & 0 & 0 & \ldots & 0\\
F_m & M''_m & M'''_m & \ldots & M_m^{(\mu-1)} & 0 & 0 & 0 & \ldots & 1
\end{vmatrix}
$$

which, from the first column, subtracting the remaining columns, multiplied respectively by a_2, a_3, ... a_m, and dividing by a_1, becomes

$$
(8)\qquad \frac{n_1 \times (\text{constant})}{a} = \begin{vmatrix}
b_1 - \dfrac{[ab]}{a_1}, & b_2, & b_3 & \ldots & b_{\mu-1} & b_\mu & b_{\mu+1} & \ldots & b_m\\
c_1 - \dfrac{[ac]}{a_1}, & c_2, & c_3 & \ldots & c_{\mu-1} & c_\mu & c_{\mu+1} & \ldots & c_m\\
. & . & . & & . & . & . & & .\\
M'_\mu & M''_\mu & M'''_\mu & \ldots & M_\mu^{(\mu-1)} & 1 & 0 & \ldots & 0\\
M'_{\mu+1} & M''_{\mu+1} & M'''_{\mu+1} & \ldots & M_{\mu+1}^{(\mu-1)} & 0 & 1 & \ldots & 0\\
. & . & . & & . & 0 & 0 & \ldots & 0\\
. & . & . & & . & . & . & \ldots & 0\\
M'_m & M''_m & M'''_m & \ldots & M_m^{(\mu-1)} & 0 & 0 & \ldots & 1
\end{vmatrix}
$$

Now let us suppose that, if we put

$$[ab] = [ac] = [ad] = \&c. = 0,$$

the second member of (8) becomes equal to N; then developing it we have

(9)

$$
\begin{aligned}
N = (b_1 c_2 d_3 e_4 \ldots) &- \Sigma(b_i c_2 d_3 e_4 \ldots) M'_i - \Sigma(b_1 c_i d_3 e_4 \ldots) M''_i - \ldots\\
&+ \Sigma(b_i c_j d_3 e_4 \ldots)(M'_i M''_j) + \Sigma(b_1 c_i d_j e_4 \ldots)(M''_i M'''_j) + \ldots\\
&- \Sigma(b_i c_j d_k e_4 \ldots)(M'_i M''_j M'''_k) - \ldots
\end{aligned}
$$

where i, j, k, &c., denote $m-\mu+1$ variable numbers between μ and m, both inclusive, satisfying the following conditions:

$$i < j < k < \&c.$$

Again, to simplify the development of the equation (9), we now propose to demonstrate the following theorem:

$$
(10)\qquad \text{“}\begin{vmatrix}
(b_i c_2 d_3 e_4 f_5), & (b_1 c_i d_3 e_4 f_5), & (b_1 c_2 d_i e_4 f_5)\\
(b_j c_2 d_3 e_4 f_5), & (b_1 c_j d_3 e_4 f_5), & (b_1 c_2 d_j e_4 f_5)\\
(b_k c_2 d_3 e_4 f_5), & (b_1 c_k d_3 e_4 f_5), & (b_1 c_2 d_k e_4 f_5)
\end{vmatrix}
= (b_i c_j d_k e_4 f_5)(b_1 c_2 d_3 e_4 f_5)^{p-1}\text{”}
$$

where, for simplicity, we have written a particular case such that:

No. of the columns or the rows of the compound determinant $= p = 3$,

No. of the order of the determinant of every element of the compound determinant $= \mu - 1 = 5$,

though the theorem is generally true.

(*Demonstration of the Theorem*) — Let us suppose that the following equation is true:

$$
\begin{vmatrix}
(b_i c_2 d_3 e_4 f_5), & (b_1 c_i d_3 e_4 f_5)\\
(b_j c_2 d_3 e_4 f_5), & (b_1 c_j d_3 e_4 f_5)
\end{vmatrix}
= (b_i c_j d_3 e_4 f_5)(b_1 c_2 d_3 e_4 f_5) \qquad (11)
$$

which is a particular case of (10) when $p = 2$; then by this and similar hypotheses we have

$$
\frac{R}{(b_1 c_2 d_3 e_4 f_5)} = (b_i c_j d_3 e_4 f_5)(b_1 c_2 d_k e_4 f_5) + (b_j c_k d_3 e_4 f_5)(b_1 c_2 d_i e_4 f_5) + (b_k c_i d_3 e_4 f_5)(b_1 c_2 d_j e_4 f_5)
$$

where R denotes the value of the first member of (10).

Now let us denote the minors corresponding to the constituents b_k, c_k, d_k, &c., in the determinant $(b_1 c_2 d_k e_4 f_5)$ respectively by B, C, D, &c.; then we have evidently

$$
\begin{aligned}
(b_1 c_2 d_k e_4 f_5) &= b_k B + c_k C + d_k D + e_k E + f_k F\\
(b_1 c_2 d_i e_4 f_5) &= b_i B + c_i C + d_i D + e_i E + f_i F\\
(b_1 c_2 d_j e_4 f_5) &= b_j B + c_j C + d_j D + e_j E + f_j F
\end{aligned}
$$

Therefore, if we denote the multiples of B, C, D, &c., in

$$\frac{R}{(b_1 c_2 d_3 e_4 f_5)}$$

respectively by (B), (C), (D), &c., we have, first,

$$(B) = \{(b_i c_j d_3 e_4 f_5) b_k + (b_j c_k d_3 e_4 f_5) b_i + (b_k c_i d_3 e_4 f_5) b_j\} B$$

$$= \begin{vmatrix} b_i & b_j & b_k & 0 & 0 & 0 \\ b_i & b_j & b_k & b_3 & b_4 & b_5 \\ c_i & c_j & c_k & c_3 & c_4 & c_5 \\ d_i & d_j & d_k & d_3 & d_4 & d_5 \\ e_i & e_j & e_k & e_3 & e_4 & e_5 \\ f_i & f_j & f_k & f_3 & f_4 & f_5 \end{vmatrix} \times B = - \begin{vmatrix} 0 & 0 & 0 & b_3 & b_4 & b_5 \\ b_i & b_j & b_k & b_3 & b_4 & b_5 \\ c_i & c_j & c_k & c_3 & c_4 & c_5 \\ d_i & d_j & d_k & d_3 & d_4 & d_5 \\ e_i & e_j & e_k & e_3 & e_4 & e_5 \\ f_i & f_j & f_k & f_3 & f_4 & f_5 \end{vmatrix} \times B$$

$$= (b_i c_j d_k e_4 f_5) b_3 B - (b_i c_j d_k e_3 f_5) b_4 B + (b_i c_j d_k e_3 f_4) b_5 B$$

and similarly

$$\begin{aligned}
(C) &= (b_i c_j d_k e_4 f_5)\, c_3 C - (b_i c_j d_k e_3 f_5)\, c_4 C + (b_i c_j d_k e_3 f_4)\, c_5 C \\
(D) &= (\quad \text{"} \quad)\, d_3 D - (\quad \text{"} \quad)\, d_4 D + (\quad \text{"} \quad)\, d_5 D \\
(E) &= (\quad \text{"} \quad)\, e_3 E - (\quad \text{"} \quad)\, e_4 E + (\quad \text{"} \quad)\, e_5 E \\
(F) &= (\quad \text{"} \quad)\, f_3 F - (\quad \text{"} \quad)\, f_4 F + (\quad \text{"} \quad)\, f_5 F
\end{aligned}$$

Therefore addition of these equations, remembering that

$$\begin{aligned}
b_3 B + c_3 C + d_3 D + e_3 E + f_3 F &= (b_1 c_2 d_3 e_4 f_5) \\
b_4 B + c_4 C + d_4 D + e_4 E + f_4 F &= 0 \\
b_5 B + c_5 C + d_5 D + e_5 E + f_5 F &= 0
\end{aligned}$$

immediately gives

$$\frac{R}{(b_1 c_2 d_3 e_4 f_5)} = (b_i c_j d_k e_4 f_5)(b_1 c_2 d_3 e_4 f_5)$$

which is identical with (10), since in this case we have $p = 3$. Thus we have demonstrated that if the theorem be true when $p = 2$, it is also true when $p = 3$; and since it is evident that this reasoning may be equally applied for an arbitrary number of p (and also of $\mu - 1$) in the hypothetical theorem (11), and when $p = 1$ the equation (10) becomes the identical equation

$$(b_1 c_2 d_3 e_4 f_5) = (b_1 c_2 d_3 e_4 f_5)\,(b_1 c_2 d_3 e_4 f_5)^0$$

we may say that the theorem (10) is generally true for arbitrary values of p and $\mu - 1$ where $\mu - 1 > p$.

Now the equation (9) being developed by the application of the above theorem we have immediately

(12)

$$\begin{aligned}
A.N. = {} & (b_1 c_2 d_3 e_4 \ldots)^2 + \Sigma (b_i c_2 d_3 e_4 \ldots)^2 + \Sigma (b_1 c_i d_3 e_4 \ldots)^2 + \ldots \\
& + \Sigma (b_i c_j d_3 e_4 \ldots)^2 + \Sigma (b_1 c_i d_j e_4 \ldots)^2 + \ldots \\
& + \Sigma (b_i c_j d_k e_4 \ldots)^2 + \ldots
\end{aligned}$$

which, by the well-known theorem of determinants, may be transformed into the following form:

$$(13) \qquad = \begin{vmatrix} [bb] \,, & [bc] \,, & [bd] & \ldots \\ [cb] \,, & [cc] \,, & [cd] & \ldots \\ [db] \,, & [dc] \,, & [dd] & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix} = D_a \text{ (say)}$$

Now if we substitute

$$b_1 - \frac{[ab]}{a_1} \,, \quad c_1 - \frac{[ac]}{a_1} \,, \quad \text{\&c.}$$

for

$$b_1 \,, \quad c_1 \,, \quad \text{\&c.}$$

in (9), the values of M'_i, M'', ... M, M'', ... remaining unchanged, we get the same value as (8): that is to say, if we substitute $(b_1^2 + E_b)$, $(c_1^2 + E_c)$, ... $(b_1 c_1 + E_{bc})$ ... where

$$\left.\begin{aligned}
E_b &= - \frac{b_1 [ab]}{a_1} \\
E_c &= - \frac{c_1 [ac]}{a_1} \\
& \ldots \ldots \ldots \ldots \\
E_{bc} &= - \tfrac{1}{2} \left\{ \frac{[ab]\, c_1 + [ac]\, b_1}{a_1} \right\} \\
& \ldots \ldots \ldots \ldots
\end{aligned}\right\} \qquad (14)$$

for b_1^2, c_1^2, ... $b_1 c_1$, ... respectively in (12) we get the same value as $A \times (8)$ which, since the equation (12) involves only the quantities of the zero and the second orders of b_1, c_1, &c., must have the following form:

$A \times (8) = A.N.$ + terms of the first order of E_b, E_c, ... E_{bc}, ...: or, on account of (13), we must have

$$A \times (8) = \begin{vmatrix} [bb] + E_b \,, & [bc] + E_{bc} \,, & [bd] + E_{bd} & \ldots \\ [bc] + E_{bc} \,, & [cc] + E_c \,, & [cd] + E_{cd} \,, & \ldots \\ [bd] + E_{bd} \,, & [cd] + E_{cd} \,, & [dd] + E_d \,, & \ldots \\ [be] + E_{be} \,, & [ce] + E_{ce} \,, & [de] + E_{de} & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix}$$

$$= D_a + \begin{vmatrix} E_b \,, & [bc] \,, & [bd] \,, & \ldots \\ E_{bc} \,, & [cc] \,, & [cd] \,, & \ldots \\ E_{bd} \,, & [cd] \,, & [dd] \,, & \ldots \\ E_{be} \,, & [ce] \,, & [de] \,, & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix} + \begin{vmatrix} [bb] \,, & E_{bc} \,, & [bd] \,, & \ldots \\ [bc] \,, & E_c \,, & [cd] \,, & \ldots \\ [bd] \,, & E_{cd} \,, & [dd] \,, & \ldots \\ [be] \,, & E_{ce} \,, & [de] \,, & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix}$$

$$+ \text{ (terms of the first order of } E_{bd} \,, E_{cd} \,, \text{\&c.)};$$

and this being transformed by (14) we can easily arrive at the result,

$$A \times (8) = D_a + \frac{b_1 D_{ab}}{a_1} + \frac{c_1 D_{ac}}{a_1} + \ldots$$

that is, by (8),

$$n_1 \times (\text{constant}) = a_1 D_a + b_1 D_{ab} + c_1 D_{ac} + \ldots$$

where

D_{ab}, D_{ac}, &c. = the minors corresponding to the constituents $[ab]$, $[ac]$, &c., of the determinant formed from all of the coefficients of the unknown quantities in the normal equations.

SUNSPOT OBSERVATIONS,

MADE AT THE AMHERST COLLEGE OBSERVATORY,

By ROBERT H. BAKER.

1905			New		Disapp.		Reapp.		Total		Def.
	d	h	Gr.	Spots	Gr.	Spots	Gr.	Spots	Gr.	Spots	
Apr.	1	0	–	–	–	–	–	–	2	18	5
	1	21	–	1	–	–	–	–	2	11	3
	2	5	1	1	–	–	1	1	3	12	3
	3	22	–	2	–	–	–	–	3	13	4
	12	0	3	20	–	–	–	–	4	22	3
	13	21	–	–	1	2	–	–	2	9	4
	14	21	–	5	–	–	–	–	2	14	5
	15	21	–	–	–	–	–	–	2	9	3
	16	21	–	1	–	–	–	–	2	10	2
	17	5	–	1	–	–	–	–	2	9	2
	17	21	–	–	–	–	–	–	2	5	3
	18	22	–	–	–	–	–	–	2	3	4
	19	5	–	–	–	–	–	–	2	5	5
	19	19	–	1	–	–	–	–	2	4	5
	22	0	1	13	–	–	1	13	1	13	4
	23	0	1	4	–	–	1	1	2	17	5
	23	22	–	–	–	–	–	–	2	13	2
	24	19	–	1	–	–	–	–	2	18	3
	25	6	1	15	–	–	–	–	3	33	4
	25	22	–	–	–	–	–	–	3	16	4
	27	21	–	–	–	–	–	–	1	17	4
	28	5	–	–	–	–	–	–	1	21	5
	29	21	1	1	–	–	–	–	2	8	4
	30	21	1	2	1	7	1	2	1	2	5
May	1	21	–	5	–	–	–	–	1	7	4
	2	19	–	7	–	–	–	–	1	14	5
	3	5	–	16	–	–	–	–	1	30	5
	7	20	3	30	–	–	–	–	4	38	3
	8	20	–	8	–	–	–	–	4	37	3
	9	20	–	–	–	–	–	–	4	33	4
	10	20	2	7	–	–	2	7	6	32	5
	12	0	–	6	1	4	–	–	5	30	3
	18	0	1	25	–	–	–	–	4	47	4
	19	3	–	4	1	16	–	–	3	32	3
	20	5	–	–	–	–	–	–	2	20	3
	21	5	–	–	–	–	–	–	2	14	3
	21	20	–	–	1	4	–	–	1	3	2
	22	20	–	–	1	3	–	–	–	–	4
	23	19	–	–	–	–	–	–	–	–	5
	24	6		–	–	–	–	–	–	–	5
	24	20	–	–	–	–	–	–	–	–	4
	25	5	–	–	–	–	–	–	–	–	2
	26	0	1	11	–	–	–	–	–	–	3
	27	0	–	6		–	–	–	1	17	3
	28	0	–			–	–	–	1	13	3
	28	22	–	–			–	–	1	12	4
	30	0	1	2		–	1	2	2	8	3
	30	21	–	7	–	–		–	2	12	4
	31	6		–				–	2	9	2
June	1	19	–	1		–	–	–	1	10	3
	2	19		6		–			1	16	5
	3	5				–	–	–	1	15	5
	3	21	–			–			1	12	3
	5	3				–		–	1	10	4
June	8	4	4	14	–	–	2	3	5	29	5
	8	21	–	–	–	–	–	–	5	10	3
	9	6	–	–	–	–	–	–	5	10	5
	9	21	–	5	–	–	–	–	5	16	4
	12	21	–	5	1	4	–	–	3	10	4
	13	21	1	2	–	–	1	1	4	12	4
	15	0	–	4	–	–	–	–	4	12	3
	15	20	–	–	–	–	–	–	4	8	3
	18	4	–	–	–	–	–	–	4	6	2
	24	7	1	16	–	–	–	–	2	17	3
	25	21	1	32	–	–	–	–	3	48	3
	26	21	–	3	1	1	–	–	2	44	3
	27	21	–	–	–	–	–	–	2	29	3
	28	6	–	–	–	–	–	–	2	33	5
	28	21	–	–	–	–	–	–	2	28	4
	29	5	–	–	1	1	–	–	1	17	3
	29	21	–	–	–	–	–	–	1	9	2
	30	5	1	6	–	–	1	1	3	19	3
July	1	3	1	3	–	–	1	1	4	15	2
	2	21	–	4	1	7	–	–	3	9	4
	3	6	–	–	–	–	–	–	3	8	5
	3	21	–	–	–	–	–	–	2	4	3
	4	22	–	–	–	–	–	–	2	5	4
	5	21	1	3	–	–	1	3	3	6	4
	6	6	–	6	–	–	–	–	3	12	5
	7	2	–	–	–	–	–	–	3	7	3
	8	0	–	11	–	–	–	–	3	22	5
	9	0	–	9	–	–	–	–	3	30	4
	9	20	1	2	–	–	1	2	4	26	5
	10	20	–	10	–	–	–	–	3	34	5
	11	21	–	15	–	–	–	–	3	49	5
	12	20	1	9	–	–	1	1	4	56	5
	13	20	1	18	1	1	1	1	4	59	5
	14	19	–	20	–	–	–	–	4	79	5
	15	20	–	–	–	–	–	–	4	46	5
	17	0	1	9	1	15	1	9	4	30	4
	17	20	–	–	–	–	–	–	4	26	3
	18	21	1	5	–	–	1	1	5	45	5
	19	20	–	6	–	–	–	–	6	48	5
	20	20	–	6	–	–	–	–	6	51	4
	21	20	–	2	–	–	–	–	5	32	5
	22	22	–	4	1	7	–	–	4	14	2
	24	20	–	–	2	6	–	–	–	–	3
	27	0	–	–	–	–	–	–	–	–	5
	27	22	–	–	–	–	–	–	–	–	3
Aug.	1	19	2	5	–	–	–	–	2	5	3
	2	21	1	7	–	–	1	1	3	12	4
	3	21	–	1	–	–	–	–	3	12	3
	4	22	–	1	–	–	–	–	3	11	2
	6	21	1	11	–	–	–	–	4	21	3
	10	0	1	6	1	1	1	1	3	19	3
	11	4	–	4	–	–	–	–	3	19	4
	12	4	–	–	–	–	–	–	3	14	4
	12	21	–	5	1	1	–	–	2	17	4

1905		New Gr.	New Spots	Disapp. Gr.	Disapp. Spots	Reapp. Gr.	Reapp. Spots	Total Gr.	Total Spots	Def.
	d	h								
Aug. 13	20	–	3	–	–	–	–	2	21	4
17	4	3	4	1	15	1	1	4	10	4
20	1	4	1	–	–	1	1	5	11	3
21	0	–	8	–	–	–	–	5	19	5
22	3	1	3	–	–	–	–	6	22	4
23	2	–	1	–	–	–	–	5	13	3
24	4	4	3	1	6	1	3	4	7	3
25	4	–	–	1	2	–	–	2	4	2
26	2	–	5	–	–	–	–	2	9	3
27	5	–	–	–	–	–	–	2	7	4
28	4	1	1	–	–	–	–	3	7	4
29	2	2	7	1	1	1	1	4	13	4
31	4	–	28	–	–	–	–	4	36	5
Sept. 1	4	–	–	–	–	–	–	4	21	4
4	2	1	4	–	–	1	3	5	14	5
Sept. 5	0	–	–	1	1	–	–	4	8	5
8	0	1	1	–	–	1	1	5	5	5
9	0	–	8	1	1	–	4	4	12	3
10	0	1	9	1	1	1	3	4	19	3
11	3	1	20	1	2	–	–	4	38	5
15	3	1	1	–	–	–	–	5	20	2
21	3	–	–	1	1	–	–	1	1	3
22	0	–	–	–	–	–	–	–	–	4
24	3	1	11	–	–	–	–	1	11	3
25	4	–	–	–	–	–	–	1	9	3
26	5	–	–	–	–	–	–	1	7	4
27	4	2	4	–	–	1	2	3	7	4
28	4	–	12	–	–	–	–	4	18	5
29	5	–	1	1	4	–	–	3	15	4
30	0	–	2	–	–	–	–	3	11	3

Observed with 6-inch Reflector.

THE SECULAR PERTURBATIONS OF *MARS* FROM THE ACTION OF *MERCURY*.

By ERIC DOOLITTLE.

The elements adopted in the following computation are from Dr. G. W. Hill's "*New Theory of Jupiter and Saturn*," pages 192 and 554.

Mars.	*Mercury.*
$\pi = 333^\circ\ 17'\ 51''.74$	$\pi' = 75^\circ\ 7'\ 13''.62$
$i = 1\ 51\ 2.24$	$i' = 7\ 0\ 7.71$
$\Omega = 48\ 23\ 54.59$	$\Omega' = 46\ 33\ 8.63$
$e = 0.093\,26803$	$e' = 0.205\,60476$
$n = 689050''.784$	$n' = 5381016''.260$
$\log a = 0.182\,8971$	$\log a' = 9.587\,8217$
$m = 1 \div 3{,}093{,}500$	$m' = 1 \div 7{,}500{,}000$

Epoch 1850.0, G.M.T.

The values of the preliminary constants were as follows:

$I = 5^\circ\ 9'\ 10''.165$	$\log k = p9.999\,5819$
$\Pi = 287\ 24\ 19.31$	$\log k' = p9.998\,6621$
$\Pi' = 29\ 13\ 54.31$	$\log C = p7.801\,7097$
$K = 258\ 16\ 20.56$	$C = +0.006\,334\,4618$
$K' = 258\ 4\ 28.68$	

Dr. G. W. Hill's first modification of Gauss's method was employed, the orbit of *Mars* being divided into twelve parts with regard to the eccentric anomaly. After completing the work, it was duplicated from the beginning, the form of the equations being changed whenever this was possible, and all known tests were applied. The equation arising from the constancy of the major axis,

$$\sin q \,.\, \tfrac{1}{2} B_1^{(0)} + \cos q \,.\, B_0^{(1)} = 0,$$

was found to give the residual,

$$+0.000\,000\,008.$$

The results of the computation were as follows:

$$\left[\frac{de}{dt}\right]_{00} = +0''.000\,335\,67000$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +0.006\,184\,1007$$

$$\left[\frac{d\pi}{dt}\right]_{00} = +0.006\,191\,8174$$

$$\left[\frac{di}{dt}\right]_{00} = +0.000\,074\,481672$$

$$\left[\frac{d\Omega}{dt}\right]_{00} = +0.014\,794\,833$$

$$\left[\frac{dL}{dt}\right]_{00} = +0.194\,017\,85$$

In obtaining the above, the value $m' = 1 \div 7{,}500{,}000$ was employed. If the mass of *Mercury* is left indefinite, the following values result:

		log coeff.
$\left[\frac{de}{dt}\right]_{00} =$	$+\ 2517''.5250\ m'$	$p3.400\,9738$
$\left[\frac{d\chi}{dt}\right]_{00} =$	$+\ 46380.761\ m'$	$p4.666\,33785$
$\left[\frac{d\pi}{dt}\right]_{00} =$	$+\ 46438.628\ m'$	$p4.666\,8794$
$\left[\frac{di}{dt}\right]_{00} =$	$+\ 558.61256\ m'$	$p2.747\,1107$
$\left[\frac{d\Omega}{dt}\right]_{00} =$	$+\ 110961.28\ m'$	$p5.045\,1714$
$\left[\frac{dL}{dt}\right]_{00} =$	$+1455134.1\ m'$	$p6.162\,9030$

The values obtained by LeVerrier are in the "*Annales de l'Observatoire de Paris*," Vol. II, page 59, and Vol. VI, page 189; those of Newcomb are in the "*Secular Variations of the Orbits of the Four Inner Planets*," pages 336 and 378. If these results are reduced to the above value of m', they will compare with those here obtained as follows:

	LeVerrier	Newcomb	Method of Gauss
	"	"	"
$\left[\frac{de}{dt}\right]_{00}$	+0.00036	+0.00033	+0.0003357
$e\left[\frac{d\pi}{dt}\right]_{00}$	+0.00058	+0.00057	+0.0005775
$\left[\frac{di}{dt}\right]_{00}$	+0.00008	+0.00007	+0.0000745
$\sin i\left[\frac{d\Omega}{dt}\right]_{00}$	+0.00047	+0.00048	+0.0004778

The Flower Observatory, 1905 *Sept.* 10.

RELATION OF THE TRUE ANOMALIES IN A PARABOLA AND A VERY ECCENTRIC ELLIPSE HAVING THE SAME PERIHELION DISTANCE,

By A. HALL.

When it was found that some of the comets move in very eccentric ellipses, and it was difficult to compute the true anomaly with accuracy, the plan was adopted of passing from the anomaly in a parabola of the same perihelion distance to the anomaly in the ellipse. Thomas Simpson, the English mathematician, investigated this question in 1757, and published tables giving the first term of the reduction. About a century ago Bessel gave the coefficients of this reduction to the third power of the quantity $1-e$, e being the eccentricity of the ellipse, and computed tables for the first two powers of $1-e$. These serve very well so long as $1-e$ is less than 0.03, but leave some uncertainty. This question was completely solved by Gauss, *Theoria Motus*, §§ 36–46, and this is the only exact solution. However, the solution by series is shorter, and in his *Astronomical Notices* Brünnow has given formulas and tables for this reduction in powers of $\frac{1-e}{1+e}$. This makes his series more convergent than Bessel's, and by the introduction of an auxiliary angle Brünnow's coefficients are less complicated than Bessel's. But it is curious that Simpson's direct method has not been carried out. Perhaps the complication of the coefficients has prevented this. Let us see how this solution will proceed.

If q be the perihelion distance the semi-parameter in the parabola is $2q$. If p be the semi-parameter in the ellipse,

$$q = \frac{p}{1+e}$$

Let v and w be the true anomalies in the ellipse and parabola, the radii vectores are

$$r = \frac{p}{1+e\cos v}$$

$$r = \frac{p}{1+\cos w} = \frac{2q}{1+\cos w}$$

By the law of areas.

$$\int r^2 dv \;:\; \int r^2 dw \;::\; \sqrt{p} \;:\; \sqrt{2q}$$
$$::\; 1 \;:\; \sqrt{\frac{2}{1+e}}$$

$$\int \frac{p^2 \,.\, dv}{(1+e\cos v)^2} : \int \frac{4q^2 \,.\, dw}{(1+\cos w)^2} :: 1 : \sqrt{\frac{2}{1+e}}$$

and we have

$$\int \frac{(1+e)^{\frac{3}{2}} \,.\, dv}{(1+e\cos v)^2 \,.\, \sqrt{2}} = \int \frac{2dw}{(1+\cos w)^2}$$

On the right side we can write

$$\frac{2dw}{(1+\cos w)^2} = \frac{dw}{2\cos\frac{1}{2}w^4} = \frac{d\,.\tan\frac{1}{2}w}{\cos\frac{1}{2}w^2}$$
$$= (1+\tan\tfrac{1}{2}w^2)\,.\,d\tan\tfrac{1}{2}w$$

and the integral is

$$\tan\tfrac{1}{2}w + \tfrac{1}{3}\tan\tfrac{1}{2}w^3, \quad \text{from } w = 0, \text{ to } w = w$$

If we notice that $\cos v = \cos\frac{1}{2}v^2 - \sin\frac{1}{2}v^2$, we have

$$1 + e\cos v = (1+e)\cos\tfrac{1}{2}v^2 + (1-e)\sin\tfrac{1}{2}v^2$$

Put $\frac{1-e}{1+e} = \alpha$, so that $1+\alpha = \frac{2}{1+e}$, and the integral on the left side is

$$\int \frac{(1+e)^{\frac{3}{2}} \,.\, dv}{(1+\alpha\tan\frac{1}{2}v^2)^2 \,.\, (1+e)^2 \,.\, \sqrt{2} \,.\, \cos\frac{1}{2}v^4}$$

or
$$\sqrt{1+\alpha}\,.\int \frac{(1+\tan\frac{1}{2}v^2)\,.\,d\tan\frac{1}{2}v}{(1+\alpha\tan\frac{1}{2}v^2)^2}$$

Expand the denominator by the binomial theorem, multiply the series by $(1+\tan\frac{1}{2}v^2)\,.\,d\tan\frac{1}{2}v$, and integrate from 0 to v. If we put $\theta = \tan\frac{1}{2}v$, the result is

$$\sqrt{1+\alpha}\,.\,\left\{\theta + \tfrac{1}{3}\theta^3(1-2\alpha) - \tfrac{1}{5}\theta^5(2\alpha-3\alpha^2)\right.$$
$$\left. + \tfrac{1}{7}\theta^7(3\alpha^2-4\alpha^3) - \tfrac{1}{9}\theta^9(4\alpha^3-5\alpha^4) + \ldots\ldots\right\}$$

This is the symmetrical series given by Gauss, and it can be continued at pleasure.

Multiply this series by

$$(1+\alpha)^{\frac{1}{2}} = 1 + \tfrac{1}{2}\alpha - \tfrac{1}{8}\alpha^2 + \tfrac{1}{16}\alpha^3 - \ldots.$$

arrange in powers of α, and put $u = \tan\frac{1}{2}w$. We have

$$\left.\begin{array}{l}\theta+\frac{1}{3}\theta^3+\left(\frac{\theta}{2}-\frac{\theta^3}{2}-\frac{2}{5}\theta^5\right).\alpha+\left(-\frac{\theta}{8}-\frac{3}{8}\theta^3+\frac{2}{5}\theta^5+\frac{3}{7}\theta^7\right).\alpha^2\\ \quad+\left(\frac{\theta}{16}+\frac{5}{18}\theta^3+\frac{7}{20}\theta^5-\frac{5}{14}\theta^7-\frac{4}{9}\theta^9\right).\alpha^3\end{array}\right\}=u+\frac{1}{3}u^3$$

Write this equation in the form:

$$A+B\alpha+C\alpha^2+D\alpha^3=A_0$$

where A_0 is the value of A when $\alpha=0$, or $\theta=u$. Consider these coefficients as functions of w plus a small increment h, or of the form $f(w+h)$, and expand by Taylor's theorem. Then

$$\left.\begin{array}{l}h.\frac{dA}{dw}+\frac{h^2}{1.2}.\frac{d^2A}{dw^2}+\frac{h^3}{1.2.3}.\frac{d^3A}{dw^3}+\ldots\\ +\left(B+h.\frac{dB}{dw}+\frac{h^2}{1.2}.\frac{d^2B}{dw^2}+\ldots\right).\alpha\\ +\left(C+h.\frac{dC}{dw}+\frac{h^2}{1.2}.\frac{d^2C}{dw^2}+\ldots\right).\alpha^2\\ +\left(D+h.\frac{dD}{dw}+\ldots\right).\alpha^3\end{array}\right\}=0$$

Assume

$$h=a\alpha+b\alpha^2+c\alpha^3+\ldots$$

and substitute in the preceding equation, omitting powers of α higher than α^3. In this way we have

$$\left.\begin{array}{l}+\left(a.\frac{dA}{dw}+B\right).\alpha+\left(b.\frac{dA}{dw}+\frac{a^2}{2}.\frac{d^2A}{dw^2}+a.\frac{dB}{dw}+C\right).\alpha^2\\ +\left(c.\frac{dA}{dw}+ab.\frac{d^2A}{dw^2}+\frac{a^3}{6}.\frac{d^3A}{dw^3}+b.\frac{dB}{dw}+\frac{a^2}{2}.\frac{d^2B}{dw^2}+a.\frac{dC}{dw}+D\right).\alpha^3\end{array}\right\}=0$$

As this is an identical equation the coefficients of α are each zero, and hence

(1)

$$-a.\frac{dA}{dw}=B$$

$$-b.\frac{dA}{dw}=\frac{a^2}{2}.\frac{d^2A}{dw^2}+a.\frac{dB}{dw}+C$$

$$-c.\frac{dA}{dw}=ab.\frac{d^2A}{dw^2}+\frac{a^3}{6}\frac{d^3A}{dw^3}+b.\frac{dB}{dw}+\frac{a^2}{2}.\frac{d^2B}{dw^2}+a.\frac{dC}{dw}+D$$

These equations solve the problem, since we can express a, b and c in terms of u. Here I think the analytical work should stop, and the numerical work should begin. We have six differential coefficients to compute. Putting

$$\theta=u=\tan\tfrac{1}{2}w$$

we have

$$\frac{dA}{dw}=\frac{(1+u^2)^2}{2}$$

$$\frac{d^2A}{dw^2}=u(1+u^2)^2$$

$$\frac{d^3A}{dw^3}=\tfrac{1}{2}(1+u^2).(1+6u^2+5u^4)$$

$$\frac{dB}{dw}=\frac{(1+u^2)}{4}.(1-3u^2-4u^4)$$

$$\frac{d^2B}{dw^2}=-\tfrac{1}{2}u(1+u^2).(1+7u^2+6u^4)$$

$$\frac{dC}{dw}=(1+u^2).(-\tfrac{1}{16}-\tfrac{3}{16}u^2+u^4+\tfrac{3}{2}u^6)$$

As an example I take Halley's comet, due five years hence. We have

$$e=0.96764567$$
$$\log q=9.7656500$$
$$\log\alpha=8.2159855$$

Take $t=200.4046$ days, Barker's tables give

$$w=130°\ 9'\ 41''.78$$
$$\log u=0.3329284$$

From these data I find

$$a\alpha=+4786''.53$$
$$b\alpha^2=+\quad 77.51$$
$$c\alpha^3=+\quad 1.37$$

and for v.

$w=$		130°	9′	41″.78
	+	1	19	46.53
	+		1	17.51
	+			1.37
$v=$		131	30	47.19

Simpson's tables give	131°	28′	48″.
Bessel's " "	131	30	41.94
Brünnow's " "	131	30	47.16

By Gauss' method the exact value is

$$v=131°\ 30'\ 47''.18$$

For computing tables we could use equations (1), and first compute a for the whole range from 0° to 160°. For this work five-figure logarithms would answer to 35°; then six figures to 85°, and for the rest seven figures. For b five-figure logarithms will be enough to 100°, and for the rest six figures. For c four figures will suffice to 135°, and five for the rest.

1905 *November* 1.

OBSERVATIONS OF SUNSPOTS,

MADE AT BOSTON UNIVERSITY WITH A 5-INCH REFRACTOR,

BY A. H. AVERY, C. S. RICKEY AND W. T. WILLIAMS.

Definition, poor, fair, good, excellent.

1904		Hour	Total Gps.	Total Spots	Def.	Obs.
Oct.	5	1	2	23	F	W
	6	3	4	36	G	A
	7	12	4	21	F	W
	10	4	4	18	P	A
	14	2	4	18	P	A
	15	3	5	10	F	A
	17	9	4	12	G	W
	18	2	4	9	F	W
	20	1	5	22	P	W
	22	3	2	15	F	A
	24	10	4	12	F	R
	24	4	3	17	P	A
	25	11	4	34	G	W
	27	11	4	54	F	W
	28	12	4	33	P	W
	29	11	3	22	P	A
	31	10	3	15	F	R
Nov.	1	2	5	11	F	R
	2	10	2	6	P	A
	7	11	3	4	F	W
	8	11	2	8	G	W
	10	11	2	3	F	W
	12	11	3	7	G	A
	15	11	2	13	G	W
	17	11	4	15	F	W
	18	2	5	11	F	A
	19	11	6	16	G	A
	22	11	4	23	G	W
	23	10	4	23	F	W
	25	10	5	36	G	W
	26	10	4	30	G	R
	28	11	3	26	P	W
Dec.	1	11	3	9	F	W
	1	2	3	13	F	A
	2	12	2	10	P	W
	5	10	2	5	P	R
	6	11	3	19	G	W
	7	12	2	30	F	R
	9	2	2	46	G	A
	12	8	3	9	F	R
	14	10	5	42	P	A
	15	11	4	28	P	W
	16	1	5	64	F	W
	17	12	3	21	P	R
	20	11	3	19	G	W
	20	3	4	15	F	A
	21	11	3	12	F	A
	22	11	3	7	F	W
	23	12	0	0	P	R
	28	2	1	2	P	R

1904-5		Hour	Total Gps	Total Spots	Def.	Obs.
Dec.	29	2	2	10	P	A
	30	12	2	21	P	A
1905 Jan.	4	2	1	2	P	R
	5	1	2	4	P	W
	9	10	5	25	F	W
	10	12	4	17	F	W
	11	12	5	31	F	A
	13	12	5	39	F	W
	14	11	6	26	F	A
	14	2	4	15	G	R
	16	2	5	20	F	A
	17	1	5	31	G	A
	18	2	4	23	G	R
	20	2	4	21	F	R
	23	1	2	16	F	A
	24	12	3	9	P	A
	26	2	2	4	F	R
	27	2	2	8	P	R
	31	12	1	–	P	R
Feb.	1	10	1	20	F	R
	2	12	2	69	G	W
	3	12	3	67	G	W
	8	12	6	64	G	W
	10	1	7	62	G	W
	11	12	6	47	F	R
	14	3	5	35	F	A
	15	12	2	19	P	W
	16	10	4	14	F	R
	16	3	5	27	F	A
	18	2	5	16	F	R
	21	11	4	21	F	W
	23	8	4	9	F	R
	24	10	2	8	P	R
	25	12	3	12	P	R
	28	3	2	31	G	W
Mar.	1	12	3	44	G	W
	2	3	3	50	G	W
	3	10	3	48	P	W
	4	10	4	49	P	R
	6	10	3	38	F	R
	7	12	2	35	F	W
	9	10	3	35	F	R
	11	12	1	7	F	R
	13	8	1	4	F	R
	13	2	1	3	G	W
	14	8	1	1	F	R
	15	12	2	5	G	W
	16	8	1	8	F	R
	17	12	2	9	F	W

1905		Hour	Total Gps.	Total Spots	Def.	Obs.
Mar.	18	11	2	8	F	A
	22	3	3	23	G	W
	23	12	5	16	F	A
	27	10	3	11	F	R
	28	10	2	27	F	W
	29	11	4	36	F	A
	30	2	2	14	P	A
	31	9	2	34	G	W
	31	3	2	28	G	A
Apr.	1	10	2	23	P	A
	3	3	3	21	F	W
	7	9	2	11	G	W
	8	11	2	9	F	A
	10	4	3	9	G	R
	12	4	5	22	F	A
	14	9	3	27	F	W
	15	11	3	30	G	A
	17	10	4	7	F	R
	17	3	3	6	G	W
	18	10	5	10	F	R
	20	10	1	1	P	A
	26	12	3	25	P	A
	27	2	3	24	P	R
	29	11	1	10	P	A
May	1	3	2	6	G	W
	2	10	1	7	F	R
	2	3	2	14	F	A
	3	9	2	27	G	W
	3	2	2	15	F	R
	4	1	2	18	F	A
	5	12	1	13	F	A
	6	11	1	13	F	A
	8	9	4	36	G	W
	8	3	4	33	F	A
	9	11	4	18	G	W
	9	1	4	37	G	A
	10	9	4	31	F	W
	10	1	5	33	G	A
	11	10	6	22	F	R
	11	3	6	29	F	A
	12	3	5	26	F	A
	18	1	4	44	F	A
	19	9	3	29	E	W
	19	3	4	23	G	A
	22	3	1	3	G	W
	23	8	0	0	P	R
	24	2	0	0	F	R
	25	10	0	0	P	R
	26	10	2	7	P	R

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OBSERVATIONS OF THE FIFTH SATELLITE OF *JUPITER*,

MADE WITH THE 40-INCH REFRACTOR IN 1903 AND 1904,

By E. E. BARNARD.

The following measures of the Fifth Satellite of *Jupiter* have been made with the 40-inch refractor in the past two oppositions of the planet.

Though *Jupiter* has been getting into better position for observation, the measures in nearly every case have been made with great difficulty. This has been due to poor seeing. The satellite has been seen well only on one occasion — 1903 August 31 — when it was extremely easy to observe with seeing 4 on a scale of 5.

In the past opposition close attention has been paid to measures from the polar limbs of *Jupiter*. The inclination of the orbit of the satellite seems to be the least accurately determined of the elements, and these measures were made to remedy this defect.

Miss E. E. DOBBIN, whose work on the orbit of this satellite appeared in *A.J.* 562, intends to take up the matter of the inclination as soon as sufficient observations have been secured.

The latitude measures were obtained by placing the wires perfectly parallel to the belts of *Jupiter*, and the measures were made from both limbs of the planet. There is a discordance sometimes of a degree or two in these settings which would seriously affect the measures. The position-angles of the wires at each of these measures have been collected in tabular form, and with these data the measures can accurately be reduced to the known position of the equator of *Jupiter*.

For a new determination of the position of the *Jovian* equator a series of measures of the position-angle of the belts of the planet was made both in 1903 and 1904. These measures, combined with those previously made by me, ought to give a good value for this quantity.

When the opportunity has offered in the past opposition for a comparison of the Fifth Satellite with some of the older satellites, careful measures have been made.

The apparent semi-diameter of *Jupiter* used in the reduction of the measures to the center of the planet, have been computed from my measures of the planet's equatorial and polar diameters published in *A.J.* 325, and are collected in tabular form for easy reference.

A few determinations of the elongation times of the satellite have been made by plotting the measures, and drawing a curve through them. The observations have not been such as would give the best values for the elongation times.

During the observations of the satellite the great Red Spot was seen a number of times. Want of time prevented as careful observations of it as I should have wished, but several transits were observed, and I think they are fairly good, though more time and care would have been desirable in the observations.

In the measures of the satellite I have referred it to the near and distant limbs of the planet as much as possible. It is, however, more difficult to measure from the distant limb unless the seeing is very good. The shorter distances will be more accurate.

In the tables of measures that follow, the first column contains the date; the second the Central Standard Time ($6^h\ 0^m$ slow of Greenwich Mean Time) of the observation. The third column contains the distance from the limb, corrected when necessary for phase. The fourth column is the resulting distance from the center of *Jupiter*; and the fifth column gives the number of comparisons.

In all cases the satellite has been following the planet, with the exception of the observations of 1903 Oct. 13, and the second set of measures on Oct. 26 of the same year, at which times it was preceding.

In all the measures of this satellite that have been made with the 40-inch, a power of 460 diameters has been used.

MEASURES IN 1903.

1903	Central Stand. Time	From limb	From center	Comp.	
	h m s	″	″		
July 21	13 1 43	27.19	50.33	3	From fol. limb
	13 7 3	28.15	51.30	3	
	13 16 31	30.04	53.18	3	
	13 25 10	32.47	55.61	3	
	13 33 10	33.23	56.37	3	
	13 36 38	33.69	56.84	3	
	13 40 22	33.98	57.13	3	
	13 44 39	33.92	57.06	3	
	13 49 30	33.93	57.07	3	
	13 52 20	34.27	57.42	3	
	13 55 21	34.64	57.78	3	
	13 58 25	34.79	57.93	3	
	14 1 40	34.97	58.11	3	
	14 4 25	34.74	57.88	3	
	14 6 50	34.21	57.35	3	
	14 9 35	34.45	57.59	3	
	14 12 36	34.26	57.41	3	
	14 16 13	33.65	56.79	3	
	14 19 10	33.78	56.92	3	
	14 21 54	32.96	56.10	3	
	14 24 32	33.05	56.20	3	
	14 26 47	33.02	56.16	3	
	14 30 39	32.63	55.77	3	
	14 33 43	31.84	54.98	3	
	14 37 8	31.08	54.23	3	
	14 40 28	30.71	53.85	3	
	14 43 31	30.44	53.58	3	
	14 46 43	29.36	52.51	2	
Aug. 11	11 43 44	34.98	59.41	3	From fol. limb
	11 47 31	34.79	59.21	3	
	11 50 41	36.03	60.45	3	
	11 53 46	36.09	60.51	3	
	11 56 56	35.76	60.18	3	
	12 0 11	36.41	60.83	3	
	12 4 3	36.02	60.44	3	
	12 8 6	36.30	60.72	2	
17	10 51 31	33.38	58.08	3	
	10 55 23	33.24	58.04	3	
	10 58 23	33.81	58.51	3	
	11 0 56	34.27	58.97	3	
	11 3 46	34.57	59.27	3	
	11 5 43	35.50	60.20	3	
	11 8 25	35.04	59.74	3	
	11 11 32	35.58	60.28	3	
	11 13 48	36.09	60.79	3	
	11 16 3	35.99	60.69	3	
	11 18 12	36.18	60.88	3	
	11 20 26	36.55	61.25	3	
	11 22 35	36.48	61.18	3	
	11 24 34	36.19	60.89	3	
	11 26 43	36.87	61.57	3	
	11 28 57	37.27	61.97	3	
	11 31 45	37.34	62.04	3	
	11 34 25	36.71	61.41	3	
	11 37 6	36.45	61.15	3	
	11 39 54	36.69	61.39	3	
	11 42 40	36.90	61.60	3	
	11 45 5	36.04	60.74	3	
	11 47 44	36.43	61.13	3	
	11 50 21	36.13	60.83	3	
Aug. 17	11 52 20	35.70	60.41	3	
	11 54 39	35.89	60.59	3	
	11 57 26	35.10	59.81	3	
	12 0 28	34.69	59.39	3	
	12 4 23	34.28	58.98	3	
	12 6 40	33.82	58.52	3	
	12 9 36	33.51	58.21	3	
	12 13 0	33.20	55.90	3	
	12 16 16	32.46	57.16	3	
	12 21 55	31.41	56.11	4	
24	10 58 36	36.11	61.05	1	
	11 8 5	36.46	61.36	2	
	11 20 23	35.19	60.13	3	
	11 25 8	34.67	59.61	3	
	11 32 19	33.97	58.91	3	
	11 42 23	31.27	56.21	5	
31	9 34 36	34.14	59.30	3	
	9 36 57	34.83	59.99	3	
	9 39 15	34.88	60.04	3	
	9 41 38	35.23	60.39	3	
	9 44 1	35.73	60.89	3	
	9 46 17	36.23	61.40	3	
	9 48 49	36.41	61.57	3	
	9 50 17	36.29	61.45	2	
	9 52 26	86.62	61.46	4	From pr. limb
	9 54 52	87.53	62.37	3	
	9 57 12	37.29	62.45	3	From fol. limb
	9 59 46	37.36	62.52	3	
	10 1 31	37.30	62.46	3	
	10 3 20	37.30	62.46	3	
	10 4 57	37.64	62.81	3	
	10 6 49	37.65	62.82	3	
	10 9 4	37.69	62.85	3	
	10 11 31	37.83	62.98	3	
	10 13 47	37.79	62.95	3	
	10 15 29	37.78	62.94	4	
	10 17 52	88.20	63.04	3	From pr. limb
	10 20 32	87.65	62.49	3	
	10 21 59	87.83	62.67	2	
	10 24 11	37.60	62.76	3	From fol. limb
	10 26 36	37.65	62.81	3	
	10 28 36	37.39	62.56	3	
	10 30 47	37.12	62.28	3	
	10 33 12	36.80	61.96	3	
	10 34 51	36.38	61.54	3	
	10 36 26	36.42	61.59	3	
	10 38 22	36.30	61.46	3	
	10 41 14	35.66	60.82	3	
	10 43 21	86.26	61.10	3	From pr. limb
	10 46 1	85.78	60.62	4	
	10 48 28	34.30	59.46	3	From fol. limb
	10 49 59	34.28	59.14	2	
	10 51 34	34.18	59.34	3	
	10 53 17	33.77	58.93	2	
		From south limb.			
Aug. 31	10 56 47	22.29	−1.30	3	
	10 58 32	22.96	−0.62	2	
		From north limb.			
Aug. 31	11 0 40	25.48	−1.89	3	
	11 2 39	25.46	−1.87	2	

1903	Central Stand. Time h m s	From limb ″	From center ″	Comp.	
Sept. 1	9 46 29	36.51	61.67	4	From fol. limb
	9 52 43	36.82	61.99	3	
	9 58 12	37.81	62.98	3	
	10 0 58	37.84	63.01	3	
	10 4 19	37.64	62.80	4	
	10 8 20	37.26	62.43	2	
	10 10 20	37.48	62.65	3	
	10 15 28	37.51	62.68	3	
	10 17 35	37.61	62.77	3	
	10 19 52	37.25	62.42	3	
	10 22 20	37.45	62.62	3	
	10 25 3	37.17	62.34	3	
	10 27 23	37.10	62.27	3	
	10 29 45	36.23	61.39	3	
21	7 58 46	37.25	62.41	3	From fol. limb
	8 0 48	36.98	62.14	3	
	8 3 16	37.32	62.47	3	
	8 5 14	37.45	62.61	3	
	8 7 28	37.72	62.88	3	
	8 9 53	88.27	63.16	3	From pr. limb
	8 13 33	88.35	63.20	3	
	8 16 42	38.26	63.41	3	From fol. limb
	8 19 19	38.08	63.24	3	
	8 21 47	37.41	62.57	3	
	8 24 23	37.59	62.74	3	
	8 26 55	37.73	62.89	3	
	8 29 18	37.42	62.57	3	
	8 32 1	37.53	62.69	3	
	8 34 33	37.22	62.38	3	
	8 37 29	36.86	62.02	3	
	8 40 53	36.10	61.56	3	
	8 42 58	36.52	61.67	3	
	8 44 58	86.84	61.69	3	From pr. limb
	8 47 36	86.23	61.07	3	
	8 50 3	85.59	60.42	2	
	8 52 33	35.03	60.18	3	From fol. limb
	8 55 51	34.08	59.24	3	
	8 58 23	34.02	59.18	3	
	9 0 36	33.57	58.72	–	

Measures from south limb.

Sept. 21	9 5 46	22.72	−0.86	3
	9 8 5	22.93	−0.66	4

Measures from north limb.

Sept. 21	9 11 36	21.85	−1.27	3
	9 14 1	21.74	−1.16	3
	9 17 26	21.63	−1.04	4

Measures from south limb.

Sept. 21	9 20 28	22.49	−1.09	3
	9 23 41	21.71	−1.87	3

Satellite preceding.

Sept. 21	14 8 23	36.23	61.39	3	With pr. limb
	14 14 50	37.80	62.96	5	
	14 17 32	38.20	63.35	4	

1903	Central Stand. Time h m s	From limb ″	From center ″	Comp.	
Sept. 22	7 58 37	37.86	63.01	4	From fol. limb
	8 6 55	37.88	63.03	4	
	8 10 39	37.72	62.87	3	
	8 13 58	37.36	62.51	4	
	8 18 9	37.36	62.50	4	
	8 22 3	37.55	62.70	4	
	8 26 3	36.87	62.02	5	
	8 30 59	36.91	62.06	3	
	8 40 22	35.30	60.45	3	
28	8 43 10	27.59	52.58	4	
	8 47 19	27.50	52.47	4	
	8 51 1	26.00	51.00	3	
	8 54 47	24.72	49.71	4	
	9 1 32	22.21	47.11	4	
Oct. 13	11 58 58	35.30	59.62	4	From pr. limb Satellite precedes planet
	12 3 5	35.46	59.79	3	
	12 5 36	35.78	60.10	3	
	12 7 41	36.29	60.62	3	
	12 10 55	35.73	60.06	3	
	12 14 39	35.56	59.88	3	
	12 17 54	36.65	60.97	3	
	12 21 1	35.88	60.20	4	
	12 28 31	35.71	60.04	4	
Oct. 20	6 32 8	29.41	53.34	3	
	6 35 1	29.03	52.96	3	
	6 37 43	28.46	52.39	3	
	6 39 33	27.89	51.82	2	
	6 41 21	27.58	51.51	3	

Measures from south limb.

Oct. 20	6 46 45	22.41	−0.02	4

Measures from north limb.

Oct. 20	6 51 35	23.38	−0.95	7

Oct. 26	6 45 14	15.45	39.00	3	From fol. limb Satellite fol. the planet
	6 48 12	15.00	38.55	3	
	6 50 17	14.01	37.55	3	
	6 52 9	12.92	36.47	2	

Measures from north limb.

Oct. 26	6 58 6	23.38	−1.60	3

Oct. 26	10 41 36	33.47	57.01	3	From pr. limb Satellite precedes planet
	10 44 36	33.81	57.35	3	
	10 46 49	34.50	58.05	3	
	10 48 52	33.94	57.48	3	
	10 50 24	34.13	57.67	2	
	10 52 5	33.86	57.40	3	
	10 54 2	34.68	58.22	3	
	10 56 4	34.71	58.25	3	
	10 58 39	34.88	58.42	3	
	11 0 29	34.94	58.48	3	
	11 5 32	34.84	58.39	3	
	11 8 30	34.69	58.23	4	
	11 10 44	34.79	58.33	3	
	11 13 6	35.01	58.58	3	
	11 17 29	34.57	58.12	3	
	11 21 21	34.00	57.63	3	
	11 25 16	34.56	58.11	3	
	11 29 39	33.88	57.42	3	

Measures in 1904.

1904	Central Stand. Time	From limb	From center	Comp.	
	h m s	″	″		
Aug. 22	15 7 11	24.12	47.03	3	From fol. limb
	15 15 12	25.80	48.60	4	
	15 22 38	73.55	50.65	–	From pr. limb
The satellite was excessively difficult in all the measures.					
Aug. 27	14 37 6	22.78	46.02	3	From fol. limb
	14 39 24	23.65	46.89	3	
	14 41 32	23.92	47.16	3	
	14 43 9	24.70	47.93	2	
	14 45 19	71.90	48.66	2	From pr. limb
	14 47 0	72.41	49.18	3	
	14 48 19	72.69	49.46	2	
Measures from north limb.					
Aug. 27	14 52 35	20.17	+1.62	3	
	14 54 29	20.13	+1.59	3	
Measures from south limb.					
Aug. 27	14 56 47	23.41	+1.63	3	
	14 58 54	23.27	+1.48	3	
	15 0 50	23.43	+1.64	3	
Measures from north limb.					
Aug. 27	15 3 44	20.30	+1.48	3	
	15 5 34	20.19	+1.59	2	
	15 6 54	20.21	+1.57	2	
Measures from south limb.					
Aug. 27	15 8 49	23.54	+1.75	3	
	15 10 33	23.56	+1.78	2	
Aug. 27	15 15 20	31.72	54.96	3	From fol. limb
	15 17 23	32.19	55.42	4	
	15 20 12	78.98	55.74	3	From pr. limb
	15 21 57	79.42	56.18	3	
	15 23 56	79.83	56.59	4	
	15 26 24	79.52	56.28	3	
	15 28 25	33.34	56.58	3	From fol. limb
	15 29 57	33.94	57.18	3	
	15 31 26	33.92	57.15	4	
Observations interrupted.					
Aug. 27	15 53 54	34.51	57.74	3	
	15 56 11	34.76	57.99	3	
	15 57 40	34.06	57.30	3	
	15 59 47	82.38	59.14	3	From pr. limb
	16 1 21	81.22	57.98	3	
	16 3 14	81.00	57.76	2	
	16 4 59	33.72	56.96	3	From fol. limb
	16 6 57	34.71	57.95	3	
	16 8 44	34.03	57.27	3	
	16 10 39	33.59	56.82	3	
	16 13 7	33.26	56.50	3	
	16 16 18	33.23	56.47	4	
	16 19 9	32.92	56.16	2	
Satellite following.					
Sept. 3	13 57 44	23.09	46.78	3	From fol. limb
	14 2 1	24.50	48.19	4	
	14 6 21	73.72	50.03	3	From pr. limb
	14 9 5	74.56	50.87	4	

1904	Central Stand. Time	From limb	From center	Comp.	
Measures from south limb.					
	h m s	″	″		
Sept. 3	14 11 40	25.97	+3.76	3	
	14 17 50	25.15	+2.94	4	
Measures from north limb.					
Sept. 3	14 21 19	19.43	+2.78	3	
	14 24 29	19.11	+3.10	3	
	14 28 13	19.60	+2.61	2	
Sept. 3	14 34 19	31.86	55.55	3	From fol. limb
	14 37 55	32.43	56.12	4	
	14 42 39	81.28	57.59	3	From pr. limb
	14 45 35	81.58	57.90	4	
	14 49 2	34.07	57.76	4	From fol. limb
	14 51 58	34.25	57.94	3	
	14 54 14	34.36	58.05	3	
	14 55 57	34.59	58.28	2	
	14 58 16	82.41	58.72	3	From pr. limb
	15 0 44	82.95	59.26	3	
	15 3 24	82.53	58.84	3	
	15 5 58	83.22	59.53	3	
	15 8 33	35.06	58.75	3	From fol. limb
	15 11 18	34.92	58.11	3	
	15 13 46	34.94	58.63	3	
	15 16 6	35.52	59.21	3	
	15 19 9	34.84	58.53	3	
	15 21 59	82.73	59.04	3	From pr. limb
	15 24 22	82.65	59.06	3	
	15 26 29	82.15	58.46	3	
	15 28 33	34.28	57.97	3	From fol. limb.
	15 31 11	34.21	57.90	3	
	15 33 21	34.17	57.86	3	
	15 35 36	33.84	57.53	3	
	15 37 49	33.50	57.19	3	
	15 39 58	33.40	57.09	3	
	15 42 14	33.06	56.74	3	
Measures from north limb.					
Sept. 3	15 48 40	22.34	−0.13	4	
	15 52 1	22.36	−0.15	2	
Measures from south limb.					
Sept. 3	15 55 36	21.77	−0.44	3	
	15 58 2	21.73	−0.48	4	
Measures from north limb.					
Sept. 3	16 4 33	22.69	−0.49	5	
Measures from south limb.					
Sept. 3	16 8 12	21.63	−0.58	5	
Sept. 5	14 41 29	34.29	58.11	3	From fol. limb
	14 46 5	34.70	58.51	3	
	14 52 11	34.46	58.27	3	
	14 56 36	34.81	58.63	3	
	14 59 32	83.11	59.30	3	From pr. limb
	15 1 40	83.37	58.56	2	
	15 4 17	83.28	59.46	3	
	15 7 6	34.80	58.62	3	From fol. limb
	15 12 3	34.96	58.77	3	
	15 16 58	34.58	58.10	3	
	15 23 49	33.90	57.71	3	

1904	Central Stand. Time	From limb	From center	Comp.	
		Measures from north limb.			
	h m s	″	″		
Sept. 5	15 34 9	24.65	−2.32	4	
		Measures from south limb.			
Sept. 5	14 42 28	20.25	−2.08	6	
		Measures from south limb.			
Sept. 12	13 24 34	25.54	+2.86	5	
		Measures from north limb.			
Sept. 12	13 44 26	20.85	+1.83	5	
Sept. 12	14 3 37	35.18	59.37	5	From fol. limb
Sept. 24	13 31 24	36.47	61.27	3	From fol. limb
	13 33 36	36.41	61.22	3	
	13 35 22	36.05	60.85	3	
	13 37 29	85.31	60.50	3	From pr. limb
	13 39 21	84.81	59.99	3	
	13 41 24	84.84	60.04	2	
	13 44 0	80.27	55.46	4	
	13 46 47	34.51	59.31	3	From fol. limb
Oct. 1	12 49 0	36.34	61.36	3	From fol. limb
	12 51 22	36.55	61.57	3	
	12 54 9	36.99	62.01	2	
	12 57 2	86.74	61.72	3	From pr. limb
	12 59 24	86.54	61.52	3	
	13 0 54	86.71	61.69	2	
	13 3 30	35.26	60.38	3	From fol. limb
	13 5 19	35.17	60.19	3	
	13 6 31	34.92	59.94	2	
		Satellite V. and Satellite I			
Oct. 1	13 11 40	232.91	. .	5	
	13 16 4	. .	28.17	5	
	13 20 41	. .	28.90	5	
	13 25 21	234.91	. .	3	
		Measures from south limb.			
Oct. 1	13 40 31	20.85″	−2.61	5	
		Measures from north limb.			
Oct. 1	13 46 26	26.77	−3.32	7	
		Measures from south limb.			
Oct. 1	13 54 28	20.17	−3.28	4	
	14 6 30	20.31	−3.15	4	
		Measures from north limb.			
Oct. 1	14 10 37	26.87	−3.42	4	
Oct. 3	12 36 52	37.05	62.16	4	From fol. limb
	12 41 4	36.45	61.56	4	
		Clouds.			
	13 11 44	82.53	57.41	4	From pr. limb
	13 14 46	82.08	56.97	4	
	13 16 55	30.93	56.01	2	From fol. limb
		Satellite excessively difficult — between clouds.			
		Measures from south limb.			
Oct. 15	11 23 48	23.47	−0.23	3	
	11 26 38	22.55	−1.15	2	
		Measures from north limb.			
Oct. 15	11 29 41	24.52	−0.82	3	
	11 31 55	24.43	−0.73	2	

1904	Central Stand. Time	From limb	From center	Comp.	
		Measures from south limb.			
	h m s	″	″		
Oct. 15	12 0 53	22.95	−0.75	3	
	12 2 18	22.16	−1.54	2	
		Measures from north limb.			
Oct. 15	12 5 16	25.28	−1.58	3	
	12 7 23	25.96	−2.26	2	
		Satellite V and Satellite I.			
Oct 15	11 36 53	251.26	. .	5	
	11 43 11	. .	58.93	5	
	11 49 14	. .	58.11	5	
	11 55 2	251.64	. .	5	
Oct. 15	12 14 11	29.28″	54.56	3	
	12 16 33	29.98	55.26	2	
Oct. 17	10 31 21	33.86	59.14	3	From fol. limb
	10 33 18	34.37	59.65	3	
	10 35 47	34.15	59.43	4	
	10 38 33	85.63	60.35	3	From pr. limb
	10 40 0	86.10	60.82	2	
	10 42 35	86.56	61.28	3	
	10 44 43	86.39	61.11	3	
	10 46 31	35.93	61.21	3	From fol. limb
	10 48 13	36.38	61.66	3	
	10 50 18	36.51	61.79	3	
	10 52 5	36.82	62.11	3	
	10 53 58	36.74	62.02	3	
	10 56 33	37.04	62.32	4	
	10 59 32	88.07	62.79	4	From pr. limb
	11 2 48	36.96	62.24	3	
	11 6 15	37.06	62.34	3	
	11 8 42	37.12	62.40	3	
	11 10 57	37.32	62.60	3	
	11 12 42	37.42	62.71	3	
	11 15 21	37.30	62.58	3	
	11 17 17	37.35	62.63	3	
	11 19 20	37.52	62.80	4	
	11 22 20	36.86	62.15	3	
	11 25 17	36.50	61.78	3	
	11 28 4	36.69	61.97	3	
	11 30 52	36.65	61.93	3	
	11 34 37	35.68	60.96	3	
	11 37 12	35.66	60.95	2	
		Satellite V and Satellite II.			
Oct. 17	11 40 28	229°.15	. .	5	
	11 45 10	. .	38.42	5	
	11 51 0	. .	39.38	5	
		Satellite V and Satellite II fol.			
Oct. 17	9 56 13	146.80°	. .	1	
	9 57 43	147.60	. .	1	
	9 58 43	150.90	. .	1	
	9 59 33	152.95	. .	1	
	10 0 13	153.60	. .	1	
	10 3 2	. .	16.56	5	Pos.-Angles of
	10 6 9	. .	16.79	5	wires for these
	10 9 18	171.00	. .	1	2 sets of meas-
	10 10 18	173.50	. .	1	ures = 68°.1

1904	Central Stand. Time	From limb	From center	Comp.	
		Satellite V and Satellite II fol.			
	h m s	°	″		
Oct. 17	10 11 8	173.75	. .	1	
	10 12 8	175.55	. .	1	
	10 12 48	176.50	. .	1	

The very rapid change in angle may make a correction to the distances necessary.

1904	Central Stand. Time	From limb	From center	Comp.	
		Measures from north limb.			
		″			
Oct. 17	10 16 56	21.05	+2.65	3	
	10 18 33	20.94	+2.75	2	
		Measures from south limb.			
Oct. 17	10 20 36	26.18	+2.48	3	
	10 22 15	25.88	+2.18	2	
		Measures from north limb.			
Oct. 17	10 24 38	21.42	+2.27	3	
	10 26 36	21.16	+2.54	2	
		Measures from north limb.			
Oct. 17	11 59 14	24.91	−1.24	3	
	12 2 49	24.45	−0.75	3	
		Measures from south limb.			
Oct. 17	12 5 47	22.38	−1.33	3	
	12 9 13	22.20	−1.50	4	
		Measures from north limb.			
Oct. 29	9 10 38	22.15	+1.41	3	
	9 12 48	22.01	+1.55	3	
		Measures from south limb.			
Oct. 29	9 16 18	24.86	+1.30	3	
	9 18 58	25.40	+1.84	3	
Oct. 29	9 25 8	33.54	58.67	3	From fol. limb
	9 26 59	33.77	58.90	3	
	9 29 26	34.15	59.28	2	
	9 31 13	85.31	60.17	3	From pr. limb
	9 33 29	85.70	60.57	3	
	9 35 24	85.86	60.72	3	
	9 38 19	35.38	60.52	3	From fol. limb
	9 40 48	36.15	61.28	3	
	9 43 11	36.44	61.57	3	
	9 45 16	35.94	61.08	3	
	9 48 48	37.06	62.19	3	
	9 52 20	36.97	62.10	4	
	9 55 24	37.46	62.59	3	
	9 57 33	37.06	62.19	3	
	9 59 29	36.98	62.11	3	
	10 1 24	37.41	62.55	3	
	10 35 4	37.70	62.84	3	
	10 7 8	37.27	62.40	4	
	10 9 56	37.13	62.57	3	
	10 13 5	37.11	62.55	3	
	10 15 49	37.19	62.33	3	
	10 17 53	37.07	62.20	3	
	10 20 1	36.86	62.00	3	
	10 23 50	36.82	61.96	4	
	10 27 25	36.25	61.39	3	
	10 29 29	35.91	61.04	3	
	10 32 5	35.37	60.50	3	
	10 35 21	35.95	61.09	3	
	10 38 47	34.75	59.88	1	

1904	Central Stand. Time	From limb	From center	Comp.	
		Satellite V and Satellite I fol.			
	h m s	°	″		
Oct. 29	10 45 19	65.79	. .	6	
	10 50 31	. .	87.70	3	
	10 52 23	. .	88.37	2	
	10 55 7	. .	89.61	3	
	10 56 57	. .	90.18	2	
	11 2 1	65.47	. .	5	
		Measures from north limb.			
		″			
Oct. 29	11 6 49	25.51	−1.95	3	
	11 9 58	25.91	−2.35	4	
		Measures from south limb.			
Oct. 29	11 13 57	21.02	−2.54	3	
	11 16 38	20.75	−2.81	4	
		Measures from south limb.			
Oct. 31	9 8 36	24.73	+0.78	3	
	9 11 31	25.19	+1.68	3	
	9 13 8	24.74	+1.23	3	
	9 14 18	24.64	+1.13	2	
		Measures from north limb.			
Oct. 31	9 16 42	22.63	+0.88	3	
	9 19 21	22.49	+1.02	3	
	9 21 18	22.33	+1.18	2	
	9 22 38	22.33	+1.18	2	
Oct. 31	9 29 18	35.90	60.98	2	From fol. limb
	9 33 1	35.79	60.87	3	
	9 36 16	35.93	61.01	3	
	9 42 21	36.17	61.25	3	
	9 45 11	36.02	61.10	3	
	9 48 19	37.01	62.09	3	
	9 50 26	36.95	62.03	3	
	9 53 2	36.92	62.00	4	
	9 56 15	36.85	61.93	3	
	9 59 36	36.83	61.91	3	
	10 2 10	36.71	61.79	3	
	10 5 18	36.81	61.88	3	
	10 9 0	36.82	61.90	3	
	10 12 41	36.65	61.73	4	
	10 16 50	35.71	60.79	3	
	10 20 15	35.37	60.45	3	
		Measures from south limb.			
Oct. 31	10 25 25	22.14	−1.37	3	
	10 28 5	22.87	−0.64	3	
	10 30 13	22.26	−1.25	2	
	10 31 57	22.55	−0.96	2	
		Measures from north limb.			
Oct. 31	10 35 46	24.75	−1.24	3	
	10 39 10	24.54	−1.03	3	
	10 40 55	25.17	−1.66	2	
	10 42 18	25.21	−1.70	2	
		Satellite V and Satellite I (n.p. of 2).			
		°			
Oct. 31	10 46 52	81.65	. .	5	
	10 51 55	. .	47.90	3	
	10 56 21	. .	47.17	3	
	10 59 18	. .	47.82	2	

1904	Central Stand. Time	From limb	From center	Comp.	
	h m s	"	"		
Nov. 5	9 15 47	37.18	62.10	3	From fol. limb
	9 27 39	37.30	62.21	3	
	9 30 3	36.45	61.37	3	
	9 32 34	36.81	61.73	3	
	9 35 4	36.92	61.84	3	
	9 40 19	36.74	61.65	3	
	9 43 0	36.18	61.10	3	
	9 45 28	36.18	61.09	4	
	Measures from north limb.				
Nov. 12	9 35 46	23.43	−0.34	3	
	9 39 14	23.63	−0.54	3	
	Measures from south limb.				
Nov. 12	9 45 24	23.43	+0.34	2	Both very uncertain. Reject
	Measures from south limb.				
Nov. 14	9 2 48	21.52	−1.48	3	
	9 8 18	21.04	−1.96	3	
	Measures from north limb.				
Nov. 14	9 11 53	24.74	−1.74	3	
	9 17 6	25.62	−2.62	3	
	9 20 38	25.03	−2.03	3	
	9 24 13	24.67	−1.66	2	
	Measures from south limb.				
Nov. 14	9 43 53	20.22	−2.78	2	
	9 48 6	20.34	−2.67	3	
	Measures from south limb.				
Nov. 26	7 12 52	21.81	−0.55	3	
	7 18 15	21.22	−1.15	3	
	7 22 30	22.22	−0.14	3	
	Measures from north limb.				
Nov. 26	7 27 17	23.63	−1.27	3	
	7 30 22	26.37	−1.01	3	
	7 32 27	23.41	−0.05	2	
	7 34 40	23.76	−1.29	2	
	7 41 5	35.03	58.88	3	From fol. limb
	7 43 16	35.13	58.98	3	
	7 46 18	35.17	59.03	3	
	7 48 59	34.83	58.68	4	
	7 52 47	83.07	59.22	3	From pr. limb
	7 54 58	82.17	58.32	3	
	7 57 35	82.13	58.28	4	
	8 0 3	33.74	57.60	3	From fol. limb
	8 1 53	33.40	57.25	2	
	8 4 7	33.21	57.10	3	
	8 7 41	32.91	56.77	4	
	Measures from south limb.				
Nov. 26	8 14 20	19.32	−3.04	3	
	8 18 52	19.89	−2.47	1	

1904	Central Stand. Time	From limb	From center	Comp.	
	Measures from north limb.				
	h m s	"	"		
Nov. 26	8 22 37	25.54	−3.17	2	
	8 24 50	25.28	−2.92	2	
	8 29 7	25.51	−3.14	3	
	8 32 28	24.68	−2.31	3	
	Measures from south limb.				
Dec. 5	5 58 27	23.74	+1.94	4	
	6 24 3	23.39	+1.59	3	
	Measures from north limb.				
Dec. 5	6 39 3	21.56	+0.24	3	
	6 55 13	21.97	−0.17	4	
	Measures from south limb.				
Dec. 5	6 59 16	22.41	+0.61	3	
	7 1 46	22.12	+0.32	3	
	7 4 5	22.23	+0.43	2	
	7 6 38	21.79	+0.01	3	
	7 8 43	21.44	+0.36	3	
	7 11 21	21.57	+0.23	4	
	Satellite V and Satellite I (north).				
Dec. 5	7 17 13	214.34	. .	4	
	7 20 2	215.50	. .	3	
	7 23 32	. .	14.92	5	
	7 26 47	. .	16.61	5	
	7 30 44	222.90	. .	4	
		"			
Dec. 5	7 34 44	29.45	52.70	3	From fol. limb
	7 37 18	29.96	53.20	3	
	7 39 35	75.66	52.40	3	From pr. limb
	7 41 50	74.51	51.26	3	
	7 44 20	27.55	50.81	3	From fol. limb
	7 46 3	27.29	50.55	2	
	7 47 55	26.48	49.74	4	
	7 50 16	26.21	49.49	3	
	7 52 28	71.96	48.70	3	From pr. limb
	7 55 6	71.28	48.03	3	
	Measures from south limb.				
Dec. 5	7 59 52	20.78	−1.02	3	
	8 1 51	20.51	−1.29	2	
	Measures from north limb.				
Dec. 5	8 4 21	22.62	−0.86	3	
	8 6 28	22.82	−1.02	2	
Dec. 10	5 53 30	32.46	55.36	6	From fol. limb

On this date the satellite was momentarily glimpsed for a few minutes, and then was lost in bad seeing.

Following are the apparent semi-diameters of *Jupiter* used in the reductions. They are derived from my measures of the diameters of *Jupiter* (*A.J.* 325).

1903		Apparent Semi-Equat. Diameter ″	Apparent Semi-Polar Diameter ″
July	21	23.143	. . .
Aug.	11	24.421	. . .
	17	24.701	. . .
	24	24.942	. . .
	31	25.161	23.587
Sept.	1	25.167	. . .
	21	25.157	23.583
	22	25.149	. . .
	28	24.996	. . .
Oct.	13	24.324	. . .

1903–4		Apparent Semi-Equat. Diameter ″	Apparent Semi-Polar Diameter ″
Oct.	20	23.930	22.433
	26	23.543	21.781
1904 Aug.	22	22.905	. . .
	27	23.237	21.783
Sept.	3	23.689	22.207
	5	23.815	22.324
	12	24.196	22.676
	24	24.808	. . .
Oct.	1	25.020	23.455
	3	25.112	. . .

1904		Apparent Semi-Equat. Diameter ″	Apparent Semi-Polar Diameter ″
Oct.	15	25.281	22.699
	17	25.281	23.700
	29	25.133	23.561
	31	25.080	23.511
Nov.	5	24.917	. . .
	12	24.630	23.089
	14	24.538	23.003
	26	23.855	22.362
Dec.	5	23.555	21.800
	10	22.901	. . .

Position of Micrometer Wires at Latitude Measures.

	°		°		°
1903 Aug. 31	64.62	1904 Sept. 5	66.80	1904 Oct. 31	67.60
Sept. 21	65.17	12	68.50	Nov. 12	68.70
Oct. 20	65.16	Oct. 1	67.30	14	66.50
26	65.60	15	67.62	26	66.02
1904 Aug. 27	68.25	17	68.34	Dec. 5	67.30
Sept. 3	69.54	29	67.37		

Position-Angle of Belts.

	h m	°	
1903 July 20	13 20	66.57	5
27	13 40	64.83	5
Aug. 17	12 30	65.81	4
31	11 15	64.87	6
Sept. 8	9 40	65.74	5
21	9 30	64.98	5
Oct 26	7 0	65.82	5
27	10 30	64.00	5

	h m	°	
1904 July 4	15 40	69.58	6
23	15 30	69.25	5
Aug. 6	15 45	69.70	5
22	15 0	70.09	5
Sept. 5	16 0	67.28	5
Oct 22	14 0	67.58	5
Nov. 26	9 0	67.53	6
Dec. 5	8 30	67.37	5

Though these results are frequently discordant, they have been carefully made, as will be seen from the number of settings. The conditions have not been the best for observations of this kind because the belts have not been symmetrical — the heaviest portion of the equatorial belt system being south of the equator.

East Elongation Times of the Satellite.

	h m	O — C m
1903 July 21	13 58.0	−1.6
Aug. 17	11 33.5	+1.5
31	10 13.5	−1.1
1904 Aug. 27	15 50.0	−1.8
Oct. 29	10 5.0	−1.2

The quantities in the column O — C are comparisons with the elongation times published in *Connaisance des Temps* for 1903 and 1904, which are based on Dr Cohn's value of the period of the satellite.

The following transits of the great Red Spot were observed during the observations of the satellite.

	h m
1903 Sept. 21	7 51.5
Oct. 26	11 10.0
1904 Aug. 15	15 25.0
Oct. 17	12 16.0
Dec. 10	6 50.0

Though the spot has been faint, it has been distinctly outlined, and retains the size and shape of former years.

1904 Aug. 15. The great Red Spot in transit. It seems to be an ill-defined, *luminous* spot, with the general form of the old Red Spot, but decidedly brighter than the surrounding region.

1904 Oct. 17. The outline of the great Red Spot is fairly well seen, but it seems to join into the dusky belt south, and perhaps to the red belt or bay north. There seems to be several light markings on the spot.

Yerkes Observatory, 1905 *January*.

Measures of Some of the other Satellites of *Jupiter*.

Satellites I and II (Satellite I is north).

	h m s	°	″	
1903 July 27	13 28 40	222.44	. .	4
	13 32 48	. .	14.56	4
	13 35 55	. .	14.42	4

Satellites I and II (Satellite I is north).

	h m s	°	″	
1903 Aug. 3	15 37 27	16.25	. .	6
	15 42 25	. .	6.98	4
	15 45 39	. .	6.93	4

1904 Oct. 17   11ʰ 34ᵐ 2ˢ Satellite I ½ on disc at transit
                  11 59 32   "   II ½ on disc at transit

Satellites II and III (Satellite III is north).

	h m s	°	″	
1904 Oct. 22	13 36 59	152.85	. .	5
	13 40 47	. .	7.67	4
	13 42 46	. .	7.46	4

SATELLITES OF SATURN,

WITH THE LEANDER McCORMICK OBSERVATORY 26-INCH REFRACTOR,

By GEORGE FREDERIC PADDOCK.

Tethys — Dione.

1905	Charl'ville M.T.	p	Charl'ville M.T.	s
	h m s	°	h m s	″
Aug. 24	12 40 9	275.51	12 49 46	103.55
28	12 21 21	81.04	12 34 51	17.71
28	12 45 21	81.56	. . . .	. .
31	12 29 59	104.35	12 44 52	76.58
31	12 58 26	105.70	. . . .	. .
Sept. 8	11 3 3	106.84	11 19 39	26.40
8	11 30 3	107.54	. . . .	. .
18	11 17 23	290.82	11 27 12	60.51
18	11 36 4	291.86	. . . .	. .
26	7 59 2	289.62	8 8 27	34.47
26	8 15 41	290.45	. . . .	. .
28	12 19 31	117.88	12 41 7	23.15
28	12 49 37	119.53	. . . .	. .
Oct. 4	9 54 28	257.77	10 4 40	25.19
4	10 13 28	158.85	. . . .	. .

Tethys — Rhea.

1905	Charl'ville M.T.	p	Charl'ville M.T.	s
	h m s	°	h m s	″
Aug. 27	10 55 21	89.26	11 10 29	99.25
27	11 32 8	90.31	. . . .	. .
28	12 58 24	106.12	13 15 3	27.58
31	11 12 45	78.39	11 25 33	30.94
31	11 37 0	78.92	. . . .	. .
Sept. 6	13 28 32	117.92	13 43 23	39.01
6	13 53 20	119.83	. . . .	. .
18	10 45 26	308.62	10 55 59	39.39
18	11 6 20	311.98	. . . .	. .
26	8 21 16	281.60	8 32 13	54.12
26	8 38 51	281.71	. . . .	. .
27	11 19 34	288.50	11 28 47	54.00
27	11 35 37	289.47	. . . .	. .
Oct. 3	8 57 28	96.75	9 4 27	37.11
3	9 11 26	96.87	. . . .	. .
4	9 1 8	127.86	9 13 0	36.80
4	9 25 3	131.57	. . . .	. .

Dione — Rhea.

1905	Charl'ville M.T.	p	Charl'ville M.T.	s
	h m s	°	h m s	″
Aug. 27	11 48 4	93.76	12 3 38	103.06
27	12 15 14	94.04	. . . .	. .
28	11 41 59	131.98	11 54 46	15.21
28	12 6 23	135.33	. . . .	. .
31	11 49 54	295.84	12 5 41	54.91
31	12 19 46	297.79	. . . .	. .
Sept. 6	14 10 30	94.08	14 24 22	65.46
6	14 33 1	92.85	. . . .	. .
18	11 45 6	84.63	11 55 29	32.27
18	12 5 27	85.05	. . . .	. .
19	10 48 10	83.51	11 0 12	20.33
19	11 9 44	83.62	. . . .	. .
26	8 48 16	267.63	8 56 3	23.79
26	9 3 19	267.59	. . . .	. .
27	11 43 27	286.44	11 53 7	62.05
27	12 2 17	286.89	. . . .	. .
28	12 3 9	87.43	12 13 22	98.68
28	12 23 31	87.86	. . . .	. .
Oct. 3	9 18 28	91.82	9 26 8	28.94
3	9 33 10	91.50	. . . .	. .
4	9 33 19	110.94	9 40 25	50.11
4	9 47 40	111.62	. . . .	. .

The position angles are the mean of four readings. The distances are the mean of four double distances, except that on Aug. 24, which is the mean of three single distances. Refraction corrections have been applied.

THE BIELA METEORS, 1905,

By EDWIN F. SAWYER.

It was reasonably expected that a repetition of the meteor shower observed on November 23, 1885, would manifest itself this year on or about November 20, and accordingly preparations were made for observing the same. But for reasons not at present determined, the shower failed to make its appearance, so far as the observations of the writer show. Observations at intervals were permissible on every evening, from November 14 to 30 inclusive, except November 15 and 28, which remained overcast throughout. While the general paucity of meteors was noticeable, those radiating from the vicinity of *Gamma Andromedæ* were remarkably so, only two of 3d and 4th magnitudes respectively, with characteristic features of the Bielas being observed, and these on the 24th, during an hour's watch. These paths were well observed, and intersected at a point near *Gamma Andromedæ*, or at R.A. 26° +42½°. This remarkable absence of the Bielas on the dates of its expected appearance would indicate that the disturbing influence of the planet *Jupiter* in 1901–2, as pointed out by Dr. SCHULHOF in 1894, has greatly deflected the stream, and perhaps thrown it into an entirely new orbit not intersecting that of the earth. If not observed in 1911 or 1912, when again in the vicinity of the earth's orbit, the interesting story of BIELA's comet and its debris would appear to be closed. Center of observation *Gamma Andromedæ*. View restricted to one-sixth of visible heavens.

Brighton, 1905 *Dec.* 3.

NEW COMET (*SCHAER, Nov. 17*).*

Cable dispatches of Nov. 18 and 20, from Dr. KREUTZ, announce the discovery of a bright comet at Geneva, by SCHAER, on Nov. 17; and give the positions at discovery, and also one on Nov. 18 obtained by Dr. E. HARTWIG at Bamberg, as follows:

1905 Greenw. M.T.	α	δ	
	h m s	° ′ ″	
Nov. 17.322	4 22 32.	+86 0 0	Geneva
18.2549	0 58 19.5	+80 40 5	Bamberg

The comet is described as circular, about 8′ in diameter, of seventh magnitude, with some central condensation, and no tail.

* *From Supplement to No. 579.*

ELEMENTS AND EPHEMERIS OF COMET *b* 1905 (*SCHAER*),

BY ELEANOR A. LAMSON.

[Communicated by the Superintendent Naval Observatory.]

The following elements were deduced from observations made at Washington, on November 21, 23, 26.

ELEMENTS.

$$T = \text{Oct. } 25.734508$$
$$\pi = 355°\ 35'\ 24''.8$$
$$\Omega = 222\ 55\ 37.7$$
$$i = 140\ 36\ 10.5$$
$$q = 1.05197$$

Residuals $(O - C)$: $\cos\beta.\Delta\lambda = +1''.9$
$\Delta\beta = +1.5$

HELIOCENTRIC COORDINATES.

$$x = r\,[9.955085]\sin(\ \ 6°\ 57'\ 18''.9 + v)$$
$$y = r\,[9.832045]\sin(\ 65\ 45\ 46.6 + v)$$
$$z = r\,[9.930305]\sin(114\ \ 6\ 25.6 + v)$$

EPHEMERIS.

G.M.T. 1905	α	δ	Light.
	h m s	° ′	
Dec. 3.5	23 30 8	+ 0 25.8	0.20
7.5	30 19	− 4 47.4	0.10
11.5	31 5	− 8 11.2	0.07
15.5	32 15	−10 33.1	0.05
19.5	33 41	−12 15.9	0.04
23.5	35 21	−13 32.6	0.03
27.5	37 13	−14 31.3	0.02
31.5	23 39 15	−15 17.1	0.01

Brightness on November 21 taken as unit.

NEW COMET *c* 1905 (*GIACOBINI, Dec. 6*).

A cable despatch from Dr. KREUTZ announces the discovery of a bright comet by GIACOBINI, at Nice, in the following position:

1905 Gr. M.T. Dec. 6.6837. R.A. = $14^h\ 21^m\ 39^s.4$; Decl. = $+20°\ 59'\ 29''$.

Daily motion in α $+4^m\ 32^s$, δ $-26'$.

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ON THE DARWINIAN THEORY OF THE GENESIS OF THE MOON, AND ON TIDAL EVOLUTION,

BY JOHN N. STOCKWELL.

When the rigorous solution of a problem is difficult or impossible, speculation becomes easy and extravagant. But it sometimes happens that the wildest speculations in regard to a physical problem may suggest other problems whose solution may throw important light upon the original problem and indirectly facilitate its solution. Such seems to be the case with modern cosmogonies, or theories of world making. The rigorous solution of the problem of the construction of the solar system by purely mechanical forces, from the crude materials of a nebula, surpasses the power of mathematical analysis; and yet it gives rise to other problems whose solution is comparatively easy, and which serve to confirm or refute the theories advanced concerning their origin. Professor DARWIN's theory of the genesis of the moon is a problem of this character.

It will be remembered that while Professor GEORGE HOWARD DARWIN accepts the *Nebular Hypothesis* of LAPLACE as a correct explanation of the general features of the solar system, he also considers that there are special reasons why the moon had a different origin. The principal reason he gives is the comparatively large mass of the moon, amounting to one-eightieth of the mass of the earth; whereas the mass of *Saturn* is equal to 4600 times the mass of its satellite *Titan*, which is by far the largest satellite in the solar system. For this reason he conceives the material of which the moon was formed, instead of being thrown off in the form of a ring, was thrown off in the form of huge fragments which ultimately became our moon. Just how this was accomplished, the following extracts from Professor DARWIN's book on "*The Tides*," will sufficiently explain. He says:

"We have grounds for conjecturing that the moon is composed of fragments of the primitive planet which we now call the earth, which detached themselves when the planet spun very swiftly, and afterwards became consolidated. It surpasses the power of mathematical calculation to trace the details of this rupture and subsequent consolidation, but we can hardly doubt that the system would pass through a period of turbulence, before order was re-established in the formation of a satellite."

"May we not then conjecture that as the rotation of the primitive earth was gradually reduced by solar tidal friction, the period of the solar tide was brought into closer and closer agreement with the free period, and that consequently the solar tide increased more and more in height? In this case the oscillation might at length become so violent that in co-operation with the rapid rotation, it shook the planet to pieces, and that huge fragments were detached which ultimately became our moon."

"There is nothing to tell us whether this theory affords the true explanation of the birth of the moon, and I say that it is only a wild speculation, incapable of verification."

These extracts from Professor DARWIN's book are sufficient to show the nature of the Darwinian theory of the genesis of the moon, and since I have no desire to criticise the book in this place, no more copious extracts seem to be necessary.

Accepting then Professor DARWIN's theory, we must remember that the moment a satellite is born it becomes an individual member of the system, and is subject to inevitable perturbations from the sun; and the problem then arises as to how near to the surface of a planet can a satellite revolve without being brought into contact with the surface by reason of the sun's attraction? In this form the problem becomes sharply defined and its solution is simple and rigorous.

We shall here consider the most favorable case that can arise, viz., that the satellite at the moment of its birth moves in a circular orbit, and in the plane of the ecliptic. With these limitations the problem becomes a simplified form of the lunar theory, the solution of which has already been elaborately worked out by mathematicians. If we designate the mean distance of the satellite from the center of the earth by a, and its mean longitude by nt; also the mean distance, mean longitude and mass of the sun by a', $n't$ and m', n and n' being the mean motions of the

satellite and sun respectively; the perturbations of the radius-vector, δr, of the satellite will be given by the equation

(1)
$$\delta r = -a\frac{m'}{\mu}\frac{a^3}{a'^3}\frac{n}{n-n'}\left\{\frac{9}{8}\frac{n}{n-2n'}-\frac{3}{8}\frac{n}{3n-2n'}\right\}\cos 2(nt-n't)$$

in which μ denotes the sum of the masses of the earth and its satellite.

The above equation shows the least distance above the earth and its atmosphere at which a satellite could revolve round the earth without getting tangled in its atmosphere by reason of the sun's attraction, and thereby be compelled to fall to the earth.

If we now suppose the moon's horizontal parallax to be $\pi = 3422''.7$, and the sun's to be $\pi' = 8''.8$, we shall have

$$a' \div a = \sin\pi \div \sin\pi' = 388.9252$$

and in the case of the earth and moon we have $\mu = 1.0125$, and the sun's mass, $m' = 332448$, the mass of the earth being unity. We also have $n' = 0.0748013\,n$, and equation (1) will become

(2) $$\delta r = -a\,[0.1098044]\,\frac{m'}{\mu}\left(\frac{a}{a'}\right)^3\cos 2\,(nt-n't)$$

If we now put

$$\overline{m}^2 = m'\left(\frac{a}{a'}\right)^3$$

we shall have $\log\frac{\overline{m}^2}{\mu} = 97.7478173$; and equation (2) will become

(3) $$\delta r = -a\,[97.8576217]\cos 2\,(nt-n't)$$

the numbers in brackets being logarithms.

Now the above parallax gives the moon's distance $a = 238\,824$ miles, and we get finally

(4) $$\delta r = -1720.68 \text{ miles} \times \cos 2\,(nt-n't)$$

Now at the time of new and full moon we have $\cos 2\,(nt-n't) = 1$, and equation (4) becomes

$$\delta r = -1720.68 \text{ miles}$$

Whence it follows that if a fragment were thrown off the earth when its diameter was equal to that of the moon's orbit, according to the Darwinian theory, the sun's action would cause it to describe an orbit which would pass 1720 miles below the earth's surface at each conjunction and opposition with the sun. This calculation also shows that if the earth's surface or atmosphere ever extended to the distance of 237 100 miles from its present center, the moon would sink into it to the depth of more than a thousand miles twice in the course of each revolution around the earth.

Equation (1) gives the whole perturbation of the radius-vector arising from the first power of the disturbing force, that is independent of the eccentricities; and is sufficiently accurate for the purpose of this investigation.

In corroboration of the accuracy of the above formula for δr we may refer to the lunar theory of DELAUNAY, who gives the correction of the moon's parallax for the terms which are independent of the eccentricities as equal to $+24''.6$, which corresponds to $\delta r = -1700$ miles, which agrees very nearly with the value previously found.

If a satellite were revolving very near the earth's surface, as existing at present, so that n' would be very small in comparison with n, we may neglect n' in the coefficient of $\cos 2\,(nt-n't)$, and equation (1) will be greatly simplified, and at the same time sufficiently accurate for our purpose. It will then become

$$\delta r = -a\frac{\overline{m}^2}{\mu}\cos 2\,(nt-n't) \qquad (5)$$

According to Professor DARWIN's idea, the moon was thrown off the earth at a time when the earth rotated on its axis in about three hours. Now a satellite revolving around the earth in three hours would have a mean distance of 6521 miles; and equation (5) would give $\delta r = -4.07$ feet. It would therefore follow that a satellite moving at that distance from the earth would pass four feet below the earth's surface twice in the course of each revolution, and would therefore require to be "born again" at the rate of sixteen times in every twenty-four hours, in order to insure its continued existence.

Now at whatever distance from the earth's center a fragment may be thrown off the earth's surface by the centrifugal force of the earth's rotation, it follows from equation (1) that the sun's action on such fragment would at once precipitate it again upon the earth, and its existence as a satellite could not continue during a single revolution; and we therefore conclude that Professor DARWIN's theory of the genesis of the moon is wholly untenable.

We shall now consider the subject of tidal evolution. It is the object of the theory of tidal evolution to explain how the planets and their satellites came into their present relations to each other, the genesis of the satellites having been brought about in the manner already explained. According to the Darwinian theory the matter of which the moon is composed was thrown off the earth when the earth rotated on its axis in about three hours, and the moon consolidated and revolved in its orbit at a distance of about 6520 miles in the same time. At present the earth rotates on its axis in a period of twenty-four hours, while the moon revolves in its orbit in a period of 27.32 days, at a distance of about 240 000 miles; and it is the object of the theory of tidal evolution to explain how these changed relations between the earth and moon were brought about by the operation of purely mechanical forces.

The agency by which these changed conditions between the moon and earth were brought about is the tides generated by the moon in the waters of the oceans which cover the greater part of the surface of the earth. It is claimed that these tides have the effect of driving the moon further and further from the earth at each revolution until its distance now amounts to about 240 000 miles; while the reaction of the moon on the earth through the friction of the tides has had the effect of increasing its period of rotation from three hours to twenty-four hours. More than a century ago, before the acceleration of the moon's mean motion had been explained by LAPLACE, it had been suggested that the acceleration was only apparent, and might be fully accounted for by supposing the length of the day to be gradually increasing. But this explanation could not be admitted, because if the day were really becoming longer there would be apparent secular changes in the motions of the other heavenly bodies, a conclusion which was not borne out by observation.

About sixty years ago, however, the idea of a tidal retardation of the earth's rotation was revived by Dr. J. R. MAYER, in a dissertation on *Celestial Dynamics*, in which he estimated that the length of the day had increased by one-sixteenth of a second during the last 2500 years, by reason of tidal friction. A few years later Dr. HERMAN LUDWIG FERDINAND HELMHOLTZ, developed the same idea in a lecture which was delivered at Königsberg in 1854. About the same time the late WILLIAM FERREL undertook an elaborate mathematical investigation of the problem in a paper which was published in this Journal in December, 1853. The conclusion at which Mr. FERREL arrived was that the earth's rotation was retarded to the extent of 37.44 miles by the moon's attraction, and 7.01 miles by the sun's attraction, making a total of 44.45 miles as the amount by which a point on the earth's equator would be retarded in 100 years by reason of tidal friction. This would produce an apparent secular acceleration of the moon, amounting to 84″ in the course of a century. But as this effect was incompatible with observation it was inferred that this effect of the tides was neutralized by an equivalent shrinkage of the earth's volume by reason of its secular cooling, so that the time of the earth's rotation remained constant.

Since the above investigations were published, the idea that the tides have a remarkable influence on the development of planetary systems has been constantly growing in favor with physicists and astronomers, until it has become, chiefly through the labors of Professor DARWIN, a generally accepted doctrine of science. However, if we carefully compare the results of the calculations of MAYER and FERREL, we shall find the discrepancies between them are so great as to justify serious doubts as to the accuracy of either. These discrepancies may result from imperfection of the data of the problem, or from imperfect or perhaps inadequate methods of treating such an extremely intricate problem. According to FERREL, a point on the earth's equator falls behind its mean place at the rate of 44.45 miles in the course of a century; and this would be equivalent to losing a whole revolution on its axis in 560 centuries, or in other words it would lose a whole day in 56 000 years. According to MAYER the day grows longer by one-sixteenth of a second in a period of 2500 years; and at this rate it would lose a whole day in a period of 3 456 000 000 years; which is about 61 700 times the period according to FERREL's calculations.

I therefore propose to give an exact solution of the problem of the tidal retardation of the earth's rotation by attacking it indirectly; that is, by an exact calculation of the effect of the tides on the motion of the moon; and then by means of the principle that action and reaction are equal and in *opposite directions* determine the effect of the moon on the tidal forces on the earth's surface. In this way we are able to obtain an accurate knowledge of the *nature* of the effects of the tidal forces which act upon the earth, although we may still be unable to determine their exact amount.

The first consideration to be attended to relates to the amount of the tidal disturbing forces. HELMHOLTZ cites BESSEL as authority for the statement that the quarter of the earth covered by the flood tide contains 25 000 cubic miles of water more than the quarters which contain the ebb tide; and we shall assume that 25 000 cubic miles of water is the mass of the tidal force. Now 25 000 cubic miles of water is equivalent to a sphere of water 36.278 miles in diameter; and this again is equivalent to a sphere of 20.552 miles in diameter, and having a density equal to the mean density of the earth. We may therefore represent the total tidal forces which act upon the moon, by means of an inflexible rod passing through the earth in the plane of the equator, each end of which hold a sphere of 20.552 miles in diameter just outside the earth's surface. We shall also suppose the radius of the earth to be 4000 miles, and the moon's distance to be 240 000 miles from the earth's center, or sixty times the earth's radius.

Now if we suppose the longitude of the moon to be denoted by nt, and the longitude of the tidal sphere which is nearest the moon by $n't$, and put $\beta = nt - n't$, we must compute the equations,

$$u^3 = \{3601 - 120\cos\beta\}^{-\frac{3}{2}} \quad , \quad u'^3 = \{3601 + 120\cos\beta\}^{-\frac{3}{2}} \qquad (6)$$

and the effective tidal tangential force acting on the moon will be given by the equation

$$\left(\frac{dR}{dv}\right) = 60\, m' \{u^3 - u'^3\} \sin\beta \qquad (7)$$

in which m' denotes the mass of a sphere 20.552 miles in diameter whose density is equal to that of the earth.

After having found the value of $\left(\frac{dR}{dv}\right)$ for any assumed values of β, we may find the corresponding perturbations of the radius-vector and longitude of the moon by the equations

$$\delta r = -2a^2\frac{n}{\mu}\left(\frac{dR}{dv}\right).t \quad ; \quad \delta_0 v = -\frac{1}{2}\,a\,\frac{n^2}{\mu}\left(\frac{dR}{dv}\right)t^2$$
$$\delta_1 v = 2a\,\frac{n^2}{\mu}\left(\frac{dR}{dv}\right)t^2 \quad ; \quad \delta v = \delta_0 v + \delta_1 v = \frac{3}{2}\,a\,\frac{n^2}{\mu}\left(\frac{dR}{dv}\right)t^2\,. \tag{8}$$

Table Showing the Effect of Assumed Tidal Forces on the Motion of the Moon.

β	u^3	u'^3	$\log\left(\frac{dR}{dv}\right)$	$\frac{\delta r}{t}$	$\frac{\delta v}{t^2}$	log. R	D
° ′				ft.	″		
+ 0 30	5] 486904	5] 440566	85.62829	−0.0530	+ 5.496	6.33407	839.73
0 0	5] 486905	5] 440565	Inf. neg.	0.0000	0.000	6.33405	839.74
− 0 30	5] 486904	5] 440566	85.62829n	+0.0530	− 5.496	6.33407	839.73
1 0	5] 486901	5] 440569	85.92925n	0.1060	10.991	6.33411	839.70
1 30	5] 486896	5] 440573	86.10523n	0.1590	16.483	6.33420	839.64
2 0	5] 486889	5] 440578	86.23002n	0.2119	21.969	6.33431	839.57
3 0	5] 486871	5] 440594	86.40567n	0.3175	32.921	6.33464	839.36
4 0	5] 486843	5] 440617	86.52998n	0.4228	43.831	6.33511	839.05
5 0	5] 486809	5] 440647	86.62609n	0.5275	54.688	6.33571	838.68
10 0	5] 486523	5] 440890	86.92046n	1.0389	107.712	6.34072	835.45
15 0	5] 486048	5] 441293	87.08535n	1.5187	157.453	6.34916	830.06
20 0	5] 485390	5] 441854	87.19441n	1.9522	202.401	6.36115	822.45
25 0	5] 484555	5] 442570	87.27056n	2.3263	241.174	6.37690	812.57
30 0	5] 483551	5] 443436	87.32378n	2.6296	272.636	6.39670	800.31
35 0	5] 482387	5] 444448	87.35919n	2.8530	295.791	6.42092	785.57
40 0	5] 481073	5] 445599	87.37949n	2.9895	309.960	6.45009	768.18
45 0	5] 479622	5] 446882	87.38608n	3.0353	314.690	6.48492	747.92
50 0	5] 478047	5] 448290	87.37936n	2.9886	309.860	6.52641	724.48
55 0	5] 476362	5] 449812	87.35894n	2.8513	295.625	6.57593	697.45
60 0	5] 474582	5] 451440	87.32345n	2.6276	272.423	6.63560	666.23
65 0	5] 472721	5] 453168	87.27011n	2.3240	240.945	6.70867	629.90
70 0	5] 470796	5] 454970	87.19387n	1.9497	202.148	6.80063	586.97
75 0	5] 468822	5] 456847	87.08473n	1.5164	157.229	6.92172	534.87
80 0	5] 466816	5] 458782	86.91980n	1.0373	107.546	7.09507	468.24
85 0	5] 464794	5] 460761	86.62549n	0.5277	54.612	7.39437	372.14
86 0	5] 464388	5] 461161	86.52935n	0.4222	43.767	7.49110	345.51
87 0	5] 463983	5] 461562	86.40498n	0.3170	32.869	7.61593	313.94
88 0	5] 463579	5] 461964	86.22937n	0.2116	21.936	7.79188	274.28
88 30	5] 463376	5] 462165	86.10455n	0.1587	16.457	7.91682	249.20
89 0	5] 463174	5] 462366	85.92870n	0.1059	10.977	8.09275	217.73
89 30	5] 462972	5] 462568	85.62756n	0.0529	5.487	8.39394	172.79
89 40	5] 462905	5] 462635	85.45158n	0.0353	3.659	8.56992	150.96
89 50	5] 462837	5] 462703	85.15056n	0.0177	1.830	8.87095	119.81
89 55	5] 462804	5] 462736	84.84985n	+0.0088	− 0.915	9.17166	95.12
90 0	5] 462770	5] 462770	Inf. neg.	0.0000	0.000	Inf.	0.00
−90 5	5] 462736	5] 462804	84.84985n	−0.0088	+ 0.915	9.17166	95.12

In these equations μ denotes the sum of the masses of the earth and moon, and $\mu = 1.0125$. The mean motion of the moon in a *Julian* year is $n = 17325593''.54$; and if we multiply this by sin 1″, we shall have $\log . n = 1.9242630$, in which n is given in parts of the radius as unity. Therefore we shall have $\log \frac{n}{\mu} = 1.9188680$. Now we have $m' = \left(\frac{10.276}{3956.17}\right)^3$, which gives $\log . m' = 92.2433626$. The moon's distance $a = 60$, in terms of the earth's radius as unity. Therefore we shall have $\log 2a^2\frac{n}{\mu} = 13.0961093$, which gives the value of δr in feet per year. We also have $\log . \frac{3}{2} . a\frac{n^2}{\mu} = 11.1117987$, which gives the value of δv in seconds per year. The above table contains the values of the various quantities which arise in connection with the problem, all of which are self-explanatory, with the exception of the last two columns of the table. In these columns R denotes the ratio of the effective tangential forces due to the two tidal spheres on the opposite sides of the earth to the actual tidal tangential force due to either sphere acting by itself alone. In other words,

$$R = \frac{1-u^3}{u^3-u'^3} = \frac{1-u'^3}{u^3-u'^3}$$

correctly to five decimals; and D denotes the diameter of a sphere in feet, of the mean density of the earth, which, acting alone in place of the tidal sphere nearest the moon, would be the exact equivalent of both. It should also be mentioned in this connection that in the values of u^3 and u'^3 the figure which precedes the bracket, denotes the number of ciphers between the decimal point and the first significant figure of the given number. The argument β, of the table is entirely arbitrary, and denotes the angular distance between the moon and the tidal sphere that is nearest the moon, as viewed from the center of the earth. When the tidal sphere is to the west of the moon β is positive; and when to the east of the moon it is negative.

Having computed the values of $\left(\frac{dR}{dv}\right)$, the logarithms of which are given in the table, we find the logarithms of $\frac{\delta r}{t}$ by adding the number 13.09611 to it, and we get the logarithms of the corresponding numbers in the table. In like manner by adding 15.11180, we get the logarithms of the values of $\frac{\delta v}{i^2}$ in which i denotes centuries.

By means of this table of exact values of the functions which are required in the solution of the problem, we can easily determine the action of the tidal forces as concentrated in the two spheres, upon the motion of the moon, when the force acts at any given angle with respect to the moon.

As a first example, we may suppose the tidal sphere to be situated 0° 30′ to the westward of the moon, in which case $\beta = +0° 30'$. In this case we find in the first line of the table that the moon would approach the earth at the rate of 0.0530 feet annually, and would also be accelerated in its motion by $5''.496 \,.\, i^2$; or in other words, it would be subjected to a secular acceleration of $5''.496$ per century, in longitude, and the amount would increase as the square of the number of centuries. If the motion of the tidal sphere were perfectly uniform the effect of its action on the moon would be to accelerate its motion in its orbit, and thereby increase the effectiveness of the tidal force by increasing the angle β.

If the angle $\beta = 0° 0'$, the earth, moon, and tidal force would be situated in the same straight line, and no change of position would result from their mutual attraction. If the tidal force were situated 0° 30′ to the eastward of the moon we should have $\beta = -0° 30'$, and the effects of the first case would be reversed; and the moon, instead of being accelerated, would suffer a secular retardation of $5''.496 \; i^2$. In either case the moon would be carried further from the position of equilibrium; which shows that when the three bodies are in the same straight line the moon is in a position of unstable equilibrium. This case is analogous to that of two planets moving at the same mean distance from the sun, which was discussed in No. 557 of this Journal; where it was shown that when they were in the same straight line, passing through the sun, they were in a position of mutual instability.

In Professor FERREL's investigation already alluded to, he assumed that the tidal wave was situated at a distance of 30° to the eastward of the moon, and was two feet in height. If we suppose $\beta = -30° 0'$, we shall have the analogous case of the tidal action on the moon, and from the table with the argument $\beta = -30°$, we find $\delta r = 2.6296 \,.\, t$, and $\delta v = -272''.636 \,.\, i^2$; which show that the assumed tidal forces, acting at that angle would increase the moon's distance from the earth by 2.63 feet per year, and cause it to have a secular equation in longitude of $-272''.636 \,.\, i^2$. Now since action and reaction are equal, and in opposite directions, it follows that a *negative* secular equation of the moon's motion from this cause must produce a *positive* secular equation of the tidal sphere, which is supposed to be attached to the earth's surface, and thereby *increase* the earth's velocity of rotation. In other words, *the day is becoming shorter* by reason of tidal friction. This conclusion is exactly contrary to Professor DARWIN's views, who regards an *increasing* length of the day as one of the most fundamental conceptions of the theory of tidal evolution.

Professor DARWIN gives no specific examples by means of which the effect of a tidal force of any magnitude acting under specified conditions upon the moon could be determined. Nor does he intimate at what angle the tidal wave follows the moon in its orbit; but he roughly estimates the time at which the moon was in close proximity to the earth as sixty millions of years. We find by means of our table that the angle of 45° between the moon and the tidal wave is the best angle for efficient tidal action. If we take that as the proper value of β, we find that the moon's distance is now increasing at the rate of 3.0353 feet per year. At that rate of variation the moon's distance would have varied only to the extent of about 34 500 miles; while if we take the angular distance of 30° which was employed by FERREL, the change of distance would amount to less than 30 000 miles in sixty millions of years. It follows from this calculation that sixty million years ago the moon was more than 200 000 miles from the earth, instead of being very close to its surface, as Professor DARWIN supposes.

Again, we see by the table, that when $\beta = -45°$ the assumed tidal tangential forces would produce a negative secular equation in the moon's longitude equal to $\delta v = -314''.690 \, i^2$, which would cause the moon to fall behind its mean place by more than 314″ during the first hundred years. These figures are not speculative, but exact; and since there is no evidence that the moon is subject to any secular retardation whatever, it necessarily

follows that the forces whose effects we have tabulated, either have no existence, or else they are wholly neutralized by other forces of a less tangible nature. It also follows from the table that there can be no forces of the character here considered, which operate within 90° to the eastward of the moon, which would produce a secular *acceleration* of that body. Now it is a fundamental conception of the theory of tidal evolution that the moon's motion should be *accelerated*, and the earth's rotation should be *retarded* by the action of the tidal forces; and as these are motions or conditions which most certainly do not exist, the theory on which they are based can have no valid foundation.

We have already observed that the moon would be in a position of unstable equilibrium when placed directly over the disturbing body; and we see by the table that it would move more and more rapidly to the westward until it reached an angular distance of 45° from the tidal force, when its motion would begin to become slower, and gradually diminish, until when it reached the angle of 90° it would cease entirely. And if from any cause it should pass beyond 90° the secular equation would be reversed, and bring it back to the 90° point, where it would permanently remain. This shows that the point of stable equilibrium is not in the line joining the two tidal spheres, but in a line which is perpendicular to the middle of that line.

Transferring our analysis to the moon it would show that the longest axis of the lunar spheroid is not directed towards the center of the earth, but is at right angles to the line joining the centers of the earth and moon. It seems strange that Sir Isaac Newton should have mistaken the point of unstable equilibrium for that of stable equilibrium; and perhaps stranger still that his statement should have been accepted as correct by all succeeding astronomers during a period of more than two hundred years. For it is easy to see, independently of analysis, that when the moon is situated in the straight line joining the centers of the earth and the two tidal spheres, it is unsymmetrically situated with respect to the forces which act upon it; whereas, it is symmetrically situated when those forces are at right angles to that line.

This fact, however, does not invalidate Newton's statement that the same face of the moon is kept constantly towards the earth by reason of the moon's spheroidal form; but it shows that the moon, like an egg, or a prolate spheroid, cannot be balanced upon its end; and the phenomena concerning the moon's libration remain the same. It is not the purpose of this article to discuss the action of the moon upon the tides; but rather the action of the tides upon the moon. There is, however, one feature of the theory of the tides that is so universally accepted, and at the same time so obviously erroneous, that I may incidentally speak of it in this place; and that is, that the necessary effect of the attraction of the sun or moon upon the waters of the ocean is to heap them up directly under the attracting body. In this case, if the bodies were fluid and without rotation, they would obviously assume the form of a prolate spheroid whose longer axis would be directed towards each other. But we have already seen that such a position would be one of unstable equilibrium; and consequently could not exist in nature. The form which the surface of the water would assume under the given conditions would be that of an oblate spheroid, whose shorter axis would be directed towards the attracting body; and consequently it ought to be low water under the attracting body. This may be proved as follows:—

We have already seen that the perturbations of the radius-vector of a body moving around the earth and near its surface, would be given by equation (5). Now when the attracting body is directly over or under the attracted body we have $\cos 2(nt-n't) = -1$, and $\delta r = +a\dfrac{m^2}{\mu}$. On the other hand, when the two bodies form a right angle at the center of the earth we have $\cos 2(nt-n't) = -1$, and $\delta r = +a\dfrac{\overline{m}^2}{\mu}$; and these results show that the longer axis of the orbit described would be at right angles with the radius-vector of the attracting body. If we now suppose the orbit to revolve around its shorter axis it would describe the surface of equilibrium, which would be that of an oblate spheroid whose shorter axis would be directed towards the attracting body.

We shall now examine the question of the reality of the forces whose effects we have computed and tabulated. If they are found to be real, and not neutralized by other forces of a less tangible nature, the results we have found must take place.

For this purpose let us assume the earth to be a perfect solid sphere, and free from all inequalities of surface. On this supposition there would be no tangential forces to act upon the moon, and we should have $\left(\dfrac{dR}{dv}\right) = 0$. We will further suppose that the earth rotates on its axis at the same rate as the moon revolves in its orbit, so that the moon remains constantly over the same point of the earth's surface. Now suppose we build upon the earth's surface a pyramid, or construct a solid globe having the same mean density as the earth, and situated at an angular distance of 45° to the eastward of the moon. We find by the table for $\beta = -45°$, that $D = 747.92$ feet. Now this is the diameter of a sphere which, acting alone, would have the effective force of the two tidal spheres of water of more than thirty-six miles in diameter, and acting at the same angle upon the moon. If such a sphere of 748 feet in diameter creates a *positive* tangential force, the excavation from which the materials of the pyramid or globe were taken must create an equal *negative* tangential force, and the two forces would exactly neutralize each other. Now

in the case of nature every tidal wave or elevation of water above the general level of the ocean is at the expense of a precisely equal amount from below the general level; and if the elevated water creates a positive tangential force the shallows of the ebb must create a precisely equivalent negative tangential force, and the two would exactly neutralize each other.

If this reasoning is correct, it would necessarily follow that the tidal tangential forces which have occupied the attention of philosophers and physicists during the past sixty years are imaginary rather than real; and the conclusions which have been correctly drawn from the theory of tidal friction will be as imaginary as the forces which produce them. We therefore conclude that the theory of tidal evolution has no logical foundation; and that the splendid structures of world-making which have been based upon it must be regarded as mere philosophical romances.

Cleveland, Ohio, 1905 *March* 2.

MICROMETRICAL OBSERVATIONS OF THE SATELLITE OF *NEPTUNE* AT THE OPPOSITIONS OF 1903–1904,

MADE WITH THE 40-INCH REFRACTOR,

BY E. E. BARNARD.

The following are all the observations I have been able to secure of the Satellite of *Neptune* at the past two oppositions. They are a continuation of previous observations printed in the *Astronomical Journal*, No. 538 and previously.

Fewer observations have been obtained than at previous oppositions because less time has been available for the work and the conditions have been worse.

The measures refer to the center of the disc and have been uniformly made with a power of 700. The parallel for the position-angles has always been obtained with the planet itself at the observations. No refraction corrections have been applied. The micrometer threads were reversed in the seconds sets of distances.

Sometimes I have thought the Satellite was brighter in the past few oppositions.

Date	Central Stand. Time	Pos. Ang.	Dist.	Comp.	
1903	h m s	°	″		
Aug. 31	16 40 21	119.97	. .	6	
	16 44 29	. . .	13.99	2	Very uncertain.
Sept. 21	15 6 56	283.56	. .	5	Satellite bright.
	15 11 7	. . .	15.61	5	Small star 16″ n.p. *Neptune* $\frac{2}{10}$M less than Satellite.
	15 15 3	. . .	15.63	5	
28	14 39 48	235.59	. .	6	
	14 45 9	. . .	13.43	4	
	14 48 42	. . .	13.53	4	
Oct. 13	13 20 32	308.99	.	6	Satellite faint from thick sky.
	13 26 28	. . .	12.36	5	
	13 30 6	. . .	12.30	5	
19	14 42 13	24.00	. .	6	Satellite fairly well seen.
	14 48 40	. . .	11.71	5	
	14 52 39	. . .	11.50	5	
20	11 37 56	312.51	. .	6	Satellite difficult.
	11 43 7	. . .	13.10	5	
	11 47 11	. . .	13.07	5	
Oct. 26	14 51 40	296.46	. .	5	Difficult—seeing bad.
	14 57 16	. . .	14.45	5	
	15 1 18	. . .	14.64	5	
27	13 37 0	256.16	. .	6	
	13 42 17	. . .	15.81	5	
	13 46 38	. . .	15.96	5	
Nov. 9	12 45 38	171.58	. .	6	Excessively difficult.
	12 52 19	. . .	11.07	4	
	12 56 59	. . .	10.78	4	
21	14 8 43	146.41	. .	7	Satellite very difficult.
	14 15 47	. . .	12.16	5	
	14 19 35	. . .	12.06	5	
24	12 28 28	328.18	. .	5	Satellite very faint from bad seeing.
	12 34 2	. . .	12.15	5	
	12 38 10	. . .	12.20	5	
Dec. 14	10 34 33	203.89	. .	6	Excessively difficult.
	10 40 4	. . .	11.39	4	
	10 43 38	. . .	11.28	5	
21	11 38 48	113.14	. .	5	Satellite fairly well seen.
	11 43 11	. . .	15.18	4	
	11 45 59	. . .	15.31	4	
22	9 35 52	105.70	. .	5	Seeing excessively bad. But observation good.
	9 40 20	. . .	16.85	4	
	9 42 56	. . .	17.00	4	
1904 Jan. 3	13 48 47	54.80	. .	5	Excessively difficult from moonlight and bad seeing. Temp. —7°.
	13 53 31	. . .	14.31	4	
	13 57 0	. . .	13.99	4	
19	7 32 29	149.70	. .	6	Excessively difficult.
	7 40 38	. . .	10.96	6	
	7 48 18	. . .	11.03	6	

1904	Central Stand. Time h m s	Pos. Ang. °	Dist. ″	Comp.	
Jan. 26	11 52 10	81.54	. .	7	Temp. = − 11°.0 F.
	11 55 46	. . .	17.08	4	
	11 58 1	. . .	17.14	5	
Feb. 2	8 56 5	28.46	. .	7	Satellite well seen.
	9 2 19	. . .	12.04	5	
	9 6 1	. . .	11.99	5	
15	11 36 23	284.49	. .	6	Satellite faint and difficult, but obsn. good. Temperature = − 11°.
	11 41 26	. . .	15.05	4	
	11 44 29	. . .	15.74	4	
16	11 36 49	242.44	. .	5	
	11 41 22	. . .	15.41	5	Temperature = −5°.
	11 45 47	. . .	15.42	5	
April 4	8 0 20	186.43	. .	6	
	8 5 13	. . .	10.84	4	
	8 7 50	. . .	10.88	4	
18	8 49 57	54.70	. .	5	
	8 55 2	. . .	13.94	4	
	8 58 7	. . .	13.90	4	
19	9 22 5	335.18	. .	5	
	9 26 11	. . .	10.91	4	
	9 29 5	. . .	10.32	4	
May 3	8 8 2	36.2	. .	3	Seen by glimpses, too low. The angle may be 216°.2.
Oct. 15	13 58 26	153.39	. .	6	
	14 5 26	. . .	11.69	5	
	14 13 16	. . .	11.61	7	
17	13 43 42	231.25	. .	6	Satellite is bright.
	13 48 39	. . .	14.03	4	
	13 52 17	. . .	13.88	5	
22	16 35 19	88.41	. .	6	Satellite bright and easy.
	16 40 16	. . .	16.83	4	
	16 43 7	. . .	16.88	4	
29	13 2 4	38.20	. .	6	Satellite fairly bright, but moon near.
	13 6 28	. . .	12.25	5	
	13 9 52	. . .	12.26	5	
31	13 30 7	266.71	. .	6	Satellite quite bright in spite of the bad seeing.
	13 35 34	. . .	16.14	4	
	13 38 53	. . .	16.34	4	

1904	Central Stand. Time h m s	Pos. Ang. °	Dist. ″	Comp.	
Nov. 5	11 20 44	311.32	. .	8	Satellite bright.
	11 28 54	. . .	13.20	5	
	11 33 8	. . .	13.43	5	
12	13 22 39	257.72	. .	5	
	13 26 27	. . .	15.97	4	
	13 29 22	. . .	16.17	4	
14	13 42 50	115.02	. .	5	
	13 48 21	. . .	15.89	4	
	13 50 8	. . .	15.34	5	
21	11 49 15	71.06	. .	6	
	11 54 10	. . .	15.77	4	
	11 57 7	. . .	15.96	4	
26	11 48 19	107.28	. .	5	
	11 52 28	. . .	16.02	5	
	11 55 53	. . .	16.21	5	
28	13 17 39	343.27	. .	5	
	13 21 30	. . .	11.46	4	
	13 24 7	. . .	11.42	4	
Dec. 5	11 10 31	280.25	. .	5	
	11 14 38	. . .	16.15	4	
	11 17 38	. . .	16.45	4	
10	11 33 37	328.81	. .	5	
	11 36 48	. . .	12.08	4	
	11 40 1	. . .	12.44	4	
12	12 22 11	226.08	. .	6	
	12 27 26	. . .	13.60	4	
	12 31 38	. . .	13.06	4	
31	9 20 39	125.21	. .	7	Satellite bright but difficult from bad seeing.
	9 25 5	. . .	14.41	4	
	9 28 17	. . .	14.39	5	

Observations of *Neptune* and a 10ᴹ.0 Star.

1904	h m s	°	″		
Feb. 15	11 48 7	59.85	. .	5	Star follows. Estimated mag. 10.8.
	11 51 44	. . .	30.78	4	
	11 55 11	. . .	29.99	-	
16	11 50 21	81.76	. .	5	Same star as last night. Estimated mag. 9.9.
	11 54 47	. . .	79.07	4	
	11 58 9	. . .	78.96	4	

Observations of *Neptune* and a 7ᴹ Star.

1904	h m s	°	″		
Dec. 5	11 37 55	282.84	. .	5	Star preceding.
	11 43 43	. . .	175.96	5	
	11 47 18	. . .	175.42	5	

Yerkes Observatory, 1905 *Jan.* 1.

CONTENTS.

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OBSERVATIONS OF THE SATELLITES OF *SATURN* IN 1905,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by the Superintendent.]

In the following observations, the position-angles are taken about the inner satellite of each pair. Usually eight settings in position-angle, and eight measurements of distance were made. A magnifying power of 400 diameters was used unless otherwise stated. When *Iapetus* is near elongation, its distance from *Titan* makes it necessary to slide the eye-piece in measuring position-angle and distance, and it was thought preferable at these times to observe differences in right-ascension and declination by the method of transits. The printed differences in α and δ are in the sense *Iapetus* minus *Titan*. They have been corrected for the motion of *Saturn* during the interval between the transits of the satellites.

The micrometer was not disturbed, except on July 14, when it was removed to replace a broken thread. The parallel was determined at the end of each night's work, and was constant, except for the change due to the removal. The following values of the correction for parallel were adopted:

From July 9 to July 11	+0.060	(2 determinations)
" " 16 to Oct. 30	+0.401	(31 ")

Bright wire illumination was used during the entire series of observations. The value of one revolution of the screw is the same as that hitherto used:

$$1 \text{ rev.} = 9''.9316 - 0''.00005\,(t - 50^\circ \text{ Fahr.})$$

In the last column is given the condition of the seeing, the following abbreviations being used: b = bad, p = poor, f = fair, g = good, and e = excellent seeing.

Tethys–Rhea.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
	1905	h m s	°	h m s	″			1905	h m s	°	h m s	″	
1	July 9	14 11 0	278.81	14 11 8	112.92	*e*	19	Sept. 15	10 42 47	280.46	10 42 48	117.19	*e*
2	17	13 19 6	195.77	13 19 12	6.23	*f*	20	16	9 50 42	84.00	9 50 58	56.20	*g*
3	18	13 7 55	273.60	13 7 50	109.16	*f*	21	18	11 14 49	132.64	11 14 56	37.81	*e*
4	25	12 26 6	98.96	12 26 8	119.56	*b*	22	24	8 57 49	273.53	8 58 2	91.92	*f*
5	26	12 50 37	266.32	12 50 43	57.70	*g*	23	25	9 35 51	62.89	9 37 5	42.15	*p*
6	Aug. 1	14 7 42	296.42	14 7 46	46.63	*g*	24	26	9 10 6	102.08	9 10 11	51.82	*e*
7	3	12 59 49	94.99	12 59 49	99.86	*e*	25	27	9 5 38	105.80	9 5 55	57.06	*f*
8	5	13 16 51	282.20	13 16 41	45.54	*g*	26	28	9 21 33	270.69	9 21 4	114.49	*g*
9	19	12 17 10	276.68	12 17 18	80.80	*p*	27	30	9 2 44	56.06	9 2 51	14.72	*f*
10	21	12 0 56	106.56	12 0 59	54.42	*e*	28	Oct. 1	8 43 21	100.16	8 43 43	113.62	*f*
11	23	11 18 9	272.34	11 18 20	118.23	*e*	29	4	8 38 50	305.00	8 38 48	41.12	*f*
12	27	10 27 57	268.52	10 28 42	97.21	*p*	30	5	8 20 1	97.23	8 20 3	113.56	*p*
13	31	10 43 50	257.07	10 43 43	30.91	*f*	31	7	8 35 30	259.07	8 35 45	52.13	*e*
14	Sept. 6	11 9 36	290.49	11 9 55	58.39	*f*	32	8	8 19 50	292.84	8 19 42	68.30	*e*
15	7	10 26 0	9.21	10 26 3	7.18	*f*	33	14	7 50 44	90.60	7 51 12	107.89	*g*
16	8	10 3 13	92.70	10 3 31	108.57	*g*	34	16	7 41 11	265.10	7 41 11	25.67	*f*
17	9	10 40 58	238.78	10 41 8	34.57	*e*	35	17	7 35 58	280.74	7 36 1	98.97	*e*
18	14	10 50 47	192.36	10 50 47	14.56	*p*							

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
					Tethys–Dione.								
	1905	h m s	°	h m s	″			1905	h m s	°	h m s	″	
1	July 9	13 48 36	263.26	13 48 40	24.74	e	17	Sept. 15	10 31 40	93.65	10 31 52	15.92	e
2	17	12 36 28	96.10	12 37 4	59.83	f	18	16	9 41 18	194.24	9 41 29	10.99	g
3	18	13 18 29	272.55	13 18 41	87.62	f	19	18	11 4 18	109.66	11 4 23	64.99	e
4	25	12 14 2	98.47	12 14 15	100.41	b	20	24	9 9 20	244.35	9 9 19	35.96	f
5	26	13 1 29	275.21	13 1 35	89.10	g	21	25	9 21 35	344.18	9 21 37	18.76	p
6	Aug. 1	13 56 35	301.69	13 56 22	31.80	g	22	26	9 0 5	113.02	9 0 11	30.08	e
7	3	12 48 42	266.66	12 48 47	9.80	e	23	30	8 38 38	274.36	8 38 48	102.09	p
8	5	13 29 18	96.29	13 29 20	89.22	g	24	Oct. 1	8 32 9	88.80	8 32 46	77.47	f
9	21	11 52 11	98.99	11 52 13	23.33	e	25	4	9 2 34	72.18	9 2 51	19.44	g
10	23	12 28 10	279.24	12 28 18	76.56	e	26	5	8 56 44	153.86	8 57 26	18.97	p
11	27	11 8 28	151.40	11 8 47	7.22	g	27	7	8 25 12	103.56	8 25 20	77.36	e
12	31	11 6 22	281.76	11 6 32	88.87	p	28	8	7 59 56	276.90	8 0 4	60.15	g
13	Sept. 3	10 34 5	278.13	10 34 28	25.45	g	29	14	7 30 48	305.90	7 30 57	24.68	g
14	7	10 47 42	126.75	10 47 57	28.17	f	30	16	7 31 3	114.33	7 31 43	19.65	f
15	8	9 52 44	283.99	9 52 37	32.10	g	31	17	8 7 59	283.09	8 7 43	66.53	e
16	9	11 12 44	314.22	11 12 50.	6.19	e	32	30	7 0 53	265.24	7 1 9	69.05	e
					Dione–Rhea.								
1	July 9	14 0 16	282.65	14 0 46	89.96	e	19	Sept. 16	10 0 16	75.05	10 0 22	62.95	g
2	17	12 24 54	270.43	12 25 10	61.72	f	20	18	11 32 38	263.91	11 32 34	32.30	e
3	18	12 43 37	278.79	12 43 41	22.73	f	21	21	9 50 6	71.24	9 50 17	50.64	f
4	25	13 7 27	100.27	13 7 28	19.93	f	22	22	9 38 10	103.30	9 38 21	118 33	p
5	26	13 14 2	110.57	13 14 9	33.03	g	23	24	8 48 0	288.88	8 47 50	66.53	p
6	Aug. 1	12 37 15	284.27	12 37 32	16.20	g	24	25	9 10 13	87.44	9 10 27	40.07	p
7	3	13 11 1	94.38	13 11 28	110.10	e	25	26	9 20 1	87.86	9 19 51	24.06	e
8	5	13 39 55	278.51	13 40 2	133.44	g	26	27	9 20 28	103.40	9 19 55	66.44	p
9	18	12 0 54	262.26	12 1 9	83.03	f	27	30	8 49 34	89.70	8 49 15	113.78	p
10	19	12 1 8	286.52	12 1 10	96.15	p	28	Oct. 1	8 54 23	123.30	8 54 23	38.29	f
11	21	12 9 36	112.05	12 9 49	31.14	e	29	4	8 22 45	288.74	8 22 55	53.69	f
12	23	11 6 43	261.74	11 6 43	41.26	e	30	5	8 31 36	88.40	8 30 15	99.59	p
13	27	10 37 20	272.50	10 37 37	102.58	f	31	7	8 46 57	274.01	8 47 2	125.71	e
14	31	10 54 48	113.01	10 55 15	62.60	f	32	8	8 30 4	350.64	8 29 55	19.25	e
15	Sept. 7	10 37 4	315.89	10 37 36	33.53	f	33	9	8 16 26	94.80	8 16 19	46.03	g
16	8	10 14 20	95.46	10 14 19	139.67	e	34	14	7 40 16	96.89	7 40 19	126.82	g
17	9	10 51 9	230.22	10 50 51	34.03	e	35	16	7 52 6	277.97	7 51 57	43.00	f
18	15	10 53 54	279.82	10 54 10	132.86	e	36	17	7 20 46	277.29	7 20 53	30.44	e
					Enceladus–Tethys.								
1	July 10	13 49 10	277.28	13 48 59	60.26	g	11	Sept. 9	11 2 57	85.72	11 2 47	60.10	e
2	17	13 30 52	271.01	13 30 38	60.31	f	12	14	10 13 15	276.27	10 12 53	82.02	p[1]
3	Aug. 3	12 39 12	116.03	12 39 25	8.56	e	13	16	9 29 26	310.97	9 29 33	11.47	g
4	5	13 6 10	267.39	13 6 34	61.87	g	14	18	10 52 39	279.48	10 52 56	56.52	e
5	19	12 29 51	303.44	12 29 19	22.25	p	15	25	8 58 8	266.94	8 59 39	58.23	p
6	21	11 7 14	89.06	11 7 31	59.94	e	16	30	8 26 19	97.71	8 26 19	73.44	p[2]
7	23	11 27 25	21.58	11 27 45	3.67	e	17	Oct. 4	8 50 12	100.82	8 50 15	73.19	f
8	Sept. 3	10 24 32	282.64	10 24 49	49.51	g	18	8	8 9 38	102.48	8 9 42	47.65	e
9	7	10 58 30	304.06	10 58 43	6.28	f[1]	19	17	7 51 29	76.04	7 51 23	7.40	e
10	8	9 42 57	148.78	9 43 13	13.65	g	20	30	6 50 44	85.45	6 51 4	53.08	e

[1] *Enceladus* very faint; poor. [2] Power 175.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
					Mimas–Enceladus.								
1	July 9	13 17 46	254.93	13 18 30	7.44	e	5	Aug. 21	10 44 56	256.51	10 45 8	12.46	e
2	Aug. 1	12 25 41	91.93	12 25 52	59.73	g	6	27	11 18 43	335.50	11 19 5	5.90	g
3	3	12 5 41	172.43	12 6 9	6.31	e	7	Sept. 21	11 37 45	103.45	11 41 40	54.36	f[2]
4	19	11 47 54	98.40	11 47 38	65.67	p[1]	8	Oct. 8	8 42 45	269.26	8 42 50	7.57	e

[1] *Mimas* very faint. [2] Comparisons 4, 8.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
					Mimas–Tethys.								
1	July 9	13 32 29	96.48	13 33 3	11.38	e	3	Aug. 21	10 55 16	91.41	10 55 29	45.18	e
2	Aug. 3	12 17 2	142.28	12 17 28	12.44	e	4	27	11 28 32	93.20	11 28 35	16.98	g
					Mimas–Dione.								
1	Aug. 3	12 27 33	202.32	12 27 41	10.97	e	4	Aug. 27	11 39 47	111.05	11 38 11	20.43	g[2]
2	18	12 49 18	87.24	12 49 1	56.77	f	5	Sept. 21	11 28 28	116.94	11 28 18	37.84	f[3]
3	21	11 14 20	94.42	11 17 50	69.74	e[1]							

[1] Comparisons 4, 8. [2] *Mimas* very faint. [3] Power 175.

Mimas–Rhea.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
	1905	h m s	°	h m s	″			1905	h m s	°	h m s	″	
1	Aug. 1	13 39 15	292.85	13 39 7	31.17	g	3	Aug. 19	11 34 18	286.22	11 34 38	40.84	p
2	18	11 48 18	258.57	11 48 31	30.64	f	4	Sept. 21	11 16 48	91.10	11 17 1	95.33	f^1

[1] Power 175.

Rhea–Titan.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
1	July 9	14 25 40	98.52	14 25 15	258.06	e^1	21	Sept. 14	10 29 26	123.78	10 29 11	50.73	p
2	17	12 49 31	283.51	12 49 26	154.60	f	22	15	11 15 37	185.51	11 15 22	40.23	e
3	18	12 55 22	293.57	12 54 59	58.29	f	23	18	11 43 51	281.09	11 43 52	192.06	e
4	25	12 37 42	99.08	12 37 50	106.95	b	24	21	9 35 29	287.02	9 35 40	121.48	f
5	26	13 28 1	98.82	13 28 44	164.32	g	25	22	9 48 54	320.14	9 48 49	65.29	p
6	31	12 38 43	272.49	12 38 45	103.49	p	26	24	8 36 51	91.08	8 37 28	225.27	p
7	Aug. 1	12 48 16	270.73	12 48 29	124.60	g	27	26	9 31 8	98.19	9 31 5	110.54	e
8	3	13 23 48	283.08	13 23 59	210.47	e	28	27	8 54 48	96.57	8 54 57	151.37	f
9	5	14 2 54	77.57	14 2 53	95.31	g	29	28	9 9 29	102.05	9 9 40	204.74	f
10	18	12 13 0	286.75	12 13 25	116.88	f	30	30	9 12 28	236.61	9 12 32	76.68	f
11	19	11 22 46	289.55	11 23 6	64.08	p	31	Oct. 1	9 5 56	265.75	9 5 48	142.10	f
12	21	11 43 0	302.63	11 43 31	80.90	e	32	1	9 13 28	271.52	9 13 41	187.83	g
13	23	12 15 28	90.00	12 15 49	229.41	e	33	5	8 45 37	279.00	8 46 2	214.11	p
14	27	11 53 1	100.32	11 53 5	193.54	g	34	7	8 57 35	28.43	8 57 22	14.91	e
15	29	11 5 13	208.62	11 5 36	53.72	b	35	8	8 52 27	77.47	8 52 11	74.45	e
16	31	10 33 12	271.40	10 33 32	134.12	f	36	9	8 27 25	69.11	8 27 24	43.11	g
17	Sept. 6	10 33 28	78.08	10 33 48	91.72	f	37	14	8 0 53	129.49	8 0 59	57.93	g
18	7	11 9 57	81.63	11 10 25	66.02	f	38	16	8 7 1	121.40	8 11 11	67.39	f^2
19	8	10 34 47	75.58	10 35 49	68.65	e	39	17	8 26 19	210.54	8 26 28	49.75	e
20	9	11 22 52	87.81	11 22 58	182.73	e							

[1] Power 175. [2] Comparisons 4, 8.

Titan–Hyperion.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
1	July 25	12 54 18	318.96	12 55 18	36.85	b	11	Sept. 22	9 22 26	272.27	9 22 1	185.84	p^2
2	31	13 12 59	98.14	13 13 24	310.50	p^1	12	26	8 46 46	120.43	8 46 55	96.68	e
3	Aug. 6	12 38 4	263.14	12 39 11	265.04	p^1	13	27	8 40 51	173.05	8 40 43	46.89	f^2
4	21	11 29 24	119.92	11 29 27	171.58	e	14	28	8 51 29	65.00	8 55 24	88.18	f
5	27	10 52 48	273.96	10 53 29	306.36	g^1	15	30	8 8 19	88.83	8 8 7	234.15	p
6	29	10 45 46	281.55	10 45 48	230.60	p	16	Oct. 1	8 11 5	93.43	8 11 50	290.22	f^1
7	30	10 11 23	287.88	10 11 56	161.05	b	17	3	9 58 7	101.32	9 59 40	319.97	p^1
8	31	10 19 5	304.90	10 18 51	81.74	f	18	21	7 13 54	98.84	7 15 9	390.28	f^1
9	Sept. 18	10 26 37	194.29	10 27 1	30.17	e	19	30	7 18 24	271.03	7 18 55	270.90	e^1
10	21	9 19 10	266.86	9 19 36	148.71	f							

[1] Power 175 [2] *Hyperion* difficult.

Titan–Iapetus.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
1	July 17	13 3 48	261.12	13 4 5	372.75	f^1	13	Aug. 31	11 19 55	86.59	11 20 18	542.18	p^1
2	18	13 29 44	260.78	13 29 58	403.16	f^1	14	Sept. 6	10 50 44	96.88	10 51 3	175.66	f
3	25	13 22 16	270.09	13 22 55	548.43	f^1	15	7	11 21 24	101.39	11 21 51	58.28	f
4	26	13 44 54	270.94	13 46 29	472.98	g^1	16	8	10 24 3	271.02	10 24 1	39.82	e
5	31	12 50 20	121.88	12 50 22	41.35	p	17	9	11 34 17	277.22	11 34 14	123.78	e
6	Aug. 1	12 59 40	109.01	12 59 39	103.14	g	18	14	10 39 51	282.88	10 40 8	151.68	p
7	3	13 36 56	155.82	13 36 59	178.25	e	19	15	11 4 2	280.38	11 4 9	119.59	e
8	5	13 51 6	113.74	13 51 15	92.57	g	20	18	10 39 24	256.84	10 39 32	127.61	e
9	23	12 39 9	87.44	12 39 22	401.87	e^1	21	21	10 4 56	261.56	10 5 11	356.71	f^1
10	27	12 15 37	80.16	12 15 37	376.43	g^1	22	22	10 3 33	264.06	10 3 27	458.75	p^1
11	29	11 20 52	82.80	11 21 2	461.95	b^1	23	Oct. 17	8 43 46	287.02	8 43 47	76.15	e
12	30	10 28 51	81.60	10 29 23	508.29	b^1	24	21	7 32 8	221.84	7 32 27	159.52	f

[1] Power 175.

Titan–Iapetus — (rectangular).

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
1	Aug. 18	12 33 43	+47.50	12 33 43	− 9.4	f^1	9	Sept. 30	9 30 4	−37.38	9 30 4	+19.1	f^1
2	19	11 3 24	+45.55	11 3 24	−14.0	p^2	10	Oct. 1	10 13 6	−32.46	10 13 6	+ 7.8	f^1
3	21	12 23 57	+36.68	12 23 57	− 7.8	e^1	11	4	9 31 44	−24.14	9 31 44	−37.2	g^1
4	Sept. 24	9 25 50	−43.21	9 25 50	−23.4	f^1	12	5	9 12 35	−24.59	9 12 35	−18.9	p^1
5	25	9 58 21	−46.98	9 58 21	− 7.0	p^1	13	7	9 12 50	−29.80	9 12 50	−54.3	e^1
6	26	9 47 53	−18.60	9 47 53	+ 6.9	e^1	14	8	9 11 22	−33.33	9 11 22	−47.6	e^1
7	27	9 40 49	−48.17	9 40 49	+17.6	p^1	15	9	8 42 17	−36.27	8 42 17	−35.6	g^1
8	28	9 37 42	−45.77	9 37 42	+24.0	g^1	16	14	8 16 41	−27.00	8 16 41	+34.5	g^1

[1] Power 175; comparisons 25, 5.

PHENOMENA OF *JUPITER'S* SATELLITES,

OBSERVED AT THE AMHERST COLLEGE OBSERVATORY,

By ROBERT H. BAKER.

The observations were made with the 6-inch Pope Reflector, mounted temporarily north of the New Observatory, longitude $4^h\ 50^m\ 6^s.0 \pm$ W. from Greenwich. A power of 125 was used, except in three instances noted below, when power 175 was substituted. The mirror was in excellent condition until the last of the observations, when it had become somewhat tarnished and scratched. The instrument was without clockwork or slow motion. No occulting bar was used, and to secure the best definition the planet was in every case kept in the field.

Time was taken by the eye-and-ear method from a chronometer of fairly constant rate, corrected at frequent intervals by transit observations. Except for eclipses of the first and second satellites, time was taken to the nearest second. The quality of the seeing was denoted by scale 1–5; 5, excellent; 4, good; 3, fair; 2, poor; 1, very poor.

	1904	Sat.			Amherst Mean Time of Observation h m s	Am. Ephem. Time red. to Meridian of Amherst h m s	Def.	Remarks
1	July 14.6	III	Ec. D.	Last seen	14 53 51	14 52 51	4+	
2	16.6	I	Ec. D.	Last seen	14 39 42.3	14 39 58	5	
3	18.5	III	Tr. E.	First contact	12 43 12	12 40	3+	
4	18.5	I	Oc. R.	First contact	12 45 17	12 43	3+	
5	29.5	II	Ec. D.	Last seen	13 59 20.7	13 57 45	3+	Rather unsteady
6	29.6	II	Ec. R.	First seen	16 23 43.3	16 24 12	4	
7	29.6	II	Oc. D.	Last contact	16 45 25	16 43	2	Strong twilight
8	Aug. 8.6	I	Ec. D.	Last seen	14 50 47.1	14 50 54	5	
9	9.5	I	Tr. I.	Last contact	13 30 12	13 29	2	Very thick
10	9.6	I	Tr. E.	First contact	15 37 33	15 41	3	Hazy
11	14.6	II	Tr. I.	Last contact	16 5 16	16 2	3	Perhaps late
12	15.6	I	Ec. D.	Last seen	16 44 25.4	16 45 7	3+	Slight haze
13	17.6	I	Oc. R.	First contact	14 38 45	14 40	4	
14	18.4	I	Tr. E.	First contact	11 56 19	11 58	4	
15	23.4	II	Ec. D.	Last seen	11 1 59.1	11 0 13	4	
16	23.6	II	Oc. R.	First contact	15 42 3	15 47	4	
17	24.5	I	Ec. D.	Last seen	13 7 58.1	13 7 56	4	
18	24.6	I	Oc. R.	First contact	16 30 32	16 29	2	Clouds passing
19	25.5	I	Tr. I.	Last contact	11 36 23	11 35	3	Late
20	26.4	I	Oc. R.	First contact	10 55 36	10 56	2+	Rather unsteady
21	26.6	III	Ec. D.	Last seen	15 0 51	15 0 56	5	
22	26.7	III	Ec. R.	First seen	16 59 25	16 59 55	5	
23	30.4	III	Tr. E.	First contact	11 12 12	11 18	2	Unsteady
24	Sept. 6.4	III	Sh. E.	Last contact	11 18 55	11 15	4	
25	6.5	III	Tr. I.	Last contact	13 35 23	13 28	4+	Power 175
26	6.6	III	Tr. E.	First contact	14 42 40	14 48	3	Power 175
27	6.6	II	Ec. D.	Last seen	16 11 9	16 9 50	5	
28	7.7	I	Ec. D.	Last seen	16 56 40.2	16 56 47	5	
29	16.5	I	Ec. D.	Last seen	13 20 3.7	13 20 4	5	1^s or 2^s late
30	17.4	I	Sh. I.	First contact	10 42 28	10 34	5	Poor obsn.
31	17.4	I	Tr. I.	Last contact	11 14 35	11 22	5	
32	17.4	II	Oc. R.	Last contact	11 58 41	11 57	5	
33	Oct. 1.4	III	Ec. D.	Last seen	11 11 9	11 10 7	5	
34	1.4	III	Ec. R.	First seen	12 59 55	13 1 43	3	
35	1.5	II	Ec. D.	Last seen	13 13 24.6	13 11 51	3	
36	1.5	III	Oc. D.	Last contact	13 21 26	13 11	3	
37	1.6	III	Oc. R.	First contact	14 24 28	14 32	2+	
38	1.6	I	Tr. I.	Last contact	14 51 55	14 50	2	
39	1.6	II	Oc. R.	First contact	16 24 39	16 27	2	
40	1.7	I	Tr. E.	First contact	16 56 44	17 1	1	Very unsteady

	1904		Sat.			Amherst Mean Time of Observation h m s	Am. Ephem. Time red. to Meridian of Amherst h m s	Def.	Remarks
41	Oct.	2.4	I	Ec. D.	Last seen	11 38 35.0	11 38 15	4	
42		2.5	I	Oc. R.	First contact	14 10 39	14 13	4	
43		3.4	I	Tr. E.	First contact	11 23 24	11 27	4	
44		4.3	I	Oc. R.	First contact	8 42 42	8 40	4	
45		19.3	I	Tr. E.	First contact	9 18 31	9 19	2	Clouds passing
46		19.3	III	Tr. I.	Last contact	9 26 14	9 22	2	Clouds passing
47		23.7	I	Oc. D.	Last contact	17 13 11	17 13	2+	Fuzzy
48		26.3	I	Tr. I.	Last contact	8 42 43	8 41	2+	Fuzzy
49		26.3	I	Sh I.	First contact	9 4 17	9 3	2+	Fuzzy
50		26.4	I	Tr. E.	First contact	11 1 3	11 2	3+	
51		26.5	II	Ec. R.	First seen	12 37 49.8	12 39 17	4	
52		26.5	III	Tr. I.	First contact	12 39 6	12 36	3+	Late
53		26.5	III	Sh. I.	First contact	13 24 24	13 15	4	
54		26.5	III	Tr. E.	First contact	14 3 40	14 9	5	
55	Nov.	1.5	I	Oc. D.	Last contact	13 25 5	13 23	5	
56		1.6	I	Ec. R.	First seen	15 56 35.9	15 56 31	3+	Slight haze
57		2.5	II	Oc. D.	Last contact	12 6 24	12 2	3	Hazy
58		2.5	I	Tr. E.	First contact	12 44 10	12 46	2	Rather thick
59		3.3	I	Oc. D.	Last contact	7 50 11	7 49	3+	Slight haze
60		3.4	I	Ec. R.	First seen	10 26 26.4	10 25 19	3	Hazy
61		9.5	II	Oc. D.	Last contact	14 19 55	14 17	4+	
62		9.6	I	Tr. E.	First contact	14 26 28	14 31	4+	
63		10.4	I	Oc. D.	Last contact	9 37 11	9 34	2	Clouds passing
64		11.2	I	Tr. I.	Last contact	6 47 56	6 45	5	
65		11.3	I	Sh. I.	First contact	7 25 29	7 22	4	Power 175
66		11.3	I	Tr. E.	First contact	8 55 11	8 57	4	
67		11.3	II	Tr. I.	Last contact	9 4 21	9 2	4	
68		11.3	I	Sh. E.	Last contact	9 30 7	9 34	3+	
69		11.4	II	Tr. E.	First contact	11 23 13	11 28	5	
70		12.2	I	Ec. R.	First seen	6 49 34.6	6 49 42	5	
71		16.5	I	Tr. I.	Last contact	14 7 48	14 4	4	Poor obs.
72		17.4	I	Oc. D.	Last contact	11 20 47	11 20	4	
73		18.3	I	Tr. I.	Last contact	8 33 42	8 30	5	
74		18.3	I	Sh. I.	First contact	9 16 51	9 17	5	
75		18.4	I	Tr. E.	First contact	10 41 23	10 43	5	Late
76		18.4	II	Tr. I.	Last contact	11 25 37	11 21	5	
77		18.4	I	Sh. E.	Last contact	11 27 7	11 29	5	
78		18.5	II	Sh. I.	First contact	12 55 52	12 54	5	
79		18.5	II	Tr. E.	First contact	13 44 5	13 48	3+	
80		27.5	II	Ec. R.	First seen	12 17 30	12 20 3	4	
81		28.2	I	Ec. R.	First seen	5 9 40.6	5 9 45	4	
82	Dec.	1.2	III	Tr. I.	Last contact	5 29 14	5 19	4	
83		1.3	III	Tr. E.	First contact	7 12 46	7 18	4	
84		3.5	I	Ec. R.	First seen	12 36 41.7	12 36 36	4	
85		4.3	I	Tr. E.	First contact	8 44 30	8 45	5	
86		4.4	II	Oc. D.	Last contact	10 26 0	10 23	5	
87		6.2	II	Tr. I.	Last contact	5 23 0	5 18	5	
88		6.3	II	Sh. I.	Last contact	7 34 37	7 31	5	
89		6.3	II	Tr. E.	First contact	7 42 45	7 47	5	
90		8.3	III	Tr. I.	Last contact	8 59 50	8 52	5	

	1904	Sat.			Amherst Mean Time of Observation h m s	Am. Ephem. Time red. to Meridian of Amherst h m s	Def.	Remarks
91	Dec. 8.4	III	Tr. E.	First contact	10 50 25	10 55	2+	Rather unsteady
92	11.3	I	Tr. I.	Last contact	8 25 1	8 21	5	
93	11.3	I	Sh. I.	First contact	9 32 20	9 31	4	
94	11.4	I	Tr. E.	First contact	10 32 7	10 34	5	
95	11.4	I	Sh. E.	Last contact	11 47 4	11 44	3	Poor obs.
96	13.2	I	Tr. E.	First contact	4 59 20	5 2	3+	
97	15.2	II	Ec. R.	First seen	6 51 12.9	6 51 1	1	Thick
98	18.4	I	Tr. I.	Last contact	10 16 36	10 12	3+	
99	18.4	I	Sh. I.	Last contact	11 27 14	11 27	3+	
100	18.5	I	Tr. E.	First contact	12 23 58	12 26	2	Unsteady
101	19.3	I	Oc. D.	Last contact	7 32 39	7 31	5	
102	19.3	III	Ec. D.	Last seen	7 36 8	7 36 53	5	
103	19.3	III	Ec. R.	First seen	9 12 28	9 13 52	3	Hazy
104	19.4	I	Ec. R.	First seen	10 57 30.1	10 57 2	4	
105	21.2	I	Ec. R.	First seen	5 26 21.2	5 26 1	3+	Trifle unsteady

SUNSPOT OBSERVATIONS,

MADE AT THE AMHERST COLLEGE OBSERVATORY,

BY ROBERT H. BAKER.

1905	d	h	New Gr.	New Spots	Disapp. Gr.	Disapp. Spots	Reapp. Gr.	Reapp. Spots	Total Gr.	Total Spots	Def.
Oct.	3	5	1	35	–	–	1	3	4	46	4
	5	4	2	13	1	4	1	6	5	40	4
	6	5	–	3	–	–	–	–	5	30	4
	7	0	–	4	–	–	–	–	5	28	3
	8	2	1	12	1	5	1	1	5	23	3
	9	0	–	2	–	–	–	1	5	20	5
	10	0	–	2	1	1	–	–	3	15	3
	12	3	–	2	–	–	–	–	3	14	4
	13	0	1	2	1	2	1	2	3	12	5
	14	0	–	4	1	5	–	3	2	10	3
	15	0	–	9	–	–	–	–	2	18	4
	15	23	–	30	–	–	–	–	2	48	5
	16	21	–	–	–	–	–	–	2	42	4
	17	23	–	29	–	–	–	–	2	76	5
	19	0	–	–	–	–	–	–	2	21	2
	20	22	1	1	1	1	1	1	2	56	5
	21	23	–	1	–	–	–	1	2	40	3
	22	20	–	2	–	–	–	–	2	42	4
	23	20	–	3	–	–	–	–	2	29	4
	24	20	–	4	–	–	–	–	2	22	4
	25	23	–	–	1	13	–	–	1	9	3
	26	23	1	7	–	–	1	1	2	16	5
	29	2	–	4	–	–	–	–	2	13	5
	29	22	–	17	–	–	–	–	2	30	4
	31	0	–	–	–	–	–	–	2	19	4
	31	22	–	4	–	–	–	–	2	20	3
Nov.	1	21	1	17	–	–	1	1	3	36	4
	2	22	–	1	1	8	–	–	2	30	3
	3	21	1	4	–	–	1	3	3	24	2
	4	20	–	1	–	–	–	–	3	34	5
	6	21	1	1	–	7	1	2	4	20	3
	8	22		19	–	–	–	–	4	39	5
	9	22	1	31	1	3	1	4	4	67	5
	10	22	1	13	–	–	–	–	5	75	4
	11	21		1		–		–	5	57	4
Nov.	12	21	1	10	–	–	–	–	6	62	3
	13	21	–	5	–	–	–	–	6	54	4
	14	21	–	1	1	3	–	–	5	45	3
	16	22	1	5	2	12	1	4	4	24	4
	17	22	1	9	–	–	1	2	5	40	5
	18	21	–	8	1	6	–	–	4	34	4
	19	21	–	9	–	–	–	–	4	40	4
	20	22	1	10	–	–	1	2	5	36	4
	21	22	1	1	1	8	1	1	5	19	3
	22	21	1	7	–	2	–	–	6	30	5
	23	22	1	15	1	1	1	12	5	32	5
	24	23	–	13	–	–	–	–	5	44	5
	26	21	–	33	1	1	–	–	4	76	5
	30	22	–	–	2	32	–	–	2	29	4
Dec.	3	21	3	12	1	1	3	12	4	27	4
	4	21	–	2	–	–	–	–	4	19	3
	5	21	–	–	–	–	–	–	4	15	2
	6	21	1	1	1	4	1	1	4	10	4
	7	21	–	2	–	–	–	2	4	12	3
	10	4	–	–	–	–	–	–	3	4	1
	11	22	1	2	–	–	1	2	3	5	2
	12	22	–	6	–	–	–	4	3	11	5
	13	22	–	4	–	–	–	–	3	13	3
	17	2	–	–	1	2	–	–	1	10	4
	17	22	–	–	–	–	–	–	1	9	3
	18	21	–	–	–	–	–	–	1	8	3
	19	21	–	2	–	–	–	–	1	7	3
	22	3	1	14	–	–	–	–	2	21	3
	23	23	2	10	–	3	1	2	4	25	3
	24	22	–	–	1	3	–	–	3	19	4
	25	23	1	13	1	1	1	6	3	25	4
	26	23	–	–	–	–	–	–	3	18	2
	27	22	–	–	–	4	–	–	3	15	5
	31	22	–	16	2	9	–	–	1	22	4

Observed with 6-inch Reflector

SUNSPOT OBSERVATIONS,

MADE AT BERWYN PENN., WITH A $4\frac{1}{2}$-INCH REFRACTOR,

BY A. W. QUIMBY.

1905	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
July 1	6	1	4	25	3	fair
2	6	..	1	2	..	v.poor
3	6	1	4	9	1	good
4	6	2	6	10	3	good
5	6	1	7	15	3	good
6	6	..	4	9	2	poor
7	6	1	5	34	3	fair
8	6	..	3	40	3	fair
9	6	..	3	65	2	fair
10	6	1	4	82	3	fair
11	6	1	4	134	3	fair
12	6	2	5	116	4	fair
13	6	2	8	168	4	fair
14	6	1	5	158	4	poor
15	6	..	4	134	5	poor
16	6	..	4	108	4	fair
17	6	1	5	60	3	poor
18	6	..	4	86	2	fair
19	6	2	6	81	3	fair
20	6	..	6	77	4	fair
21	6	1	7	50	4	fair
22	6	..	6	51	3	fair
23	7	..	4	41	2	fair
24	6	..	4	26	2	poor
*25	8	..	1	13	1	fair
26	6	..	1	2	1	fair
27	6	..	..	..	1	fair
28	6	..	..	..	2	fair
29	6	1	1	1	2	fair
30	6	2	2	2	3	fair
31	6	1	3	10	3	fair
Aug. 1	6	..	3	10	3	fair
2	6	..	2	9	2	fair
3	6	1	3	13	2	fair
4	6	1	5	18	4	fair
5	6	1	6	23	1	fair
6	7	1	4	10	1	poor
7	6	2	7	39	2	fair
8	7	1	5	15	..	poor
9	4	1	6	33	1	fair
10	6	..	6	37	1	fair
11	6	..	5	40	3	fair
12	6	..	4	20	2	fair
13	6	..	3	20	2	fair
14	6	..	3	40	2	fair
*15	7	..	2	25	1	poor
16	6	1	3	17	5	fair
17	6	1	4	17	2	fair
18	6	2	6	17	5	fair
19	6	..	5	15	3	fair
20	8	1	6	16	3	fair
21	6	1	7	27	5	fair
22	6	..	6	27	2	fair
23	1	1	7	25	5	fair
*24	7	..	3	10	3	fair
*25	5	..	2	5	..	poor
26	6	..	2	11	3	fair
27	6	..	2	6	2	fair
28	6	1	3	7	2	fair
29	5	2	4	17	2	fair
Aug. 30	6	1	5	14	2	fair
31	7	..	5	21	3	fair
Sept. 1	7	..	5	30	3	fair
2	7	..	4	20	2	poor
3	5	1	5	26	2	fair
4	7	1	6	28	3	fair
5	7	1	7	20	3	fair
6	7	1	6	13	4	fair
7	7	1	6	9	3	fair
8	7	1	7	11	4	fair
9	7	1	6	12	5	fair
10	7	1	6	34	3	fair
*12	7	..	6	12	2	poor
*13	7	..	6	17	2	fair
14	4	1	4	32	3	fair
15	7	..	4	34	3	fair
16	7	..	4	25	2	fair
17	7	..	4	12	2	poor
18	2	1	5	20	3	fair
19	8	..	4	12	2	fair
20	11	..	2	3	1	poor
21	7	..	1	2	1	fair
22	7	..	1	1	3	fair
23	7	..	..	..	2	fair
24	7	1	1	10	2	fair
25	7	..	1	10	1	fair
*26	7	..	1	10	..	poor
*27	7	3	4	12	3	fair
*28	9	..	4	10	2	poor
*29	7	..	3	10	2	poor
30	3	2	5	25	4	good
Oct. 1	7	..	5	31	5	good
2	11	..	3	19	2	poor
3	3	2	6	62	3	good
4	7	..	6	50	3	good
5	7	..	5	60	3	good
6	7	..	5	49	3	fair
7	7	..	5	50	3	fair
8	7	1	6	50	3	fair
9	7	..	5	26	3	fair
10	7	..	5	32	5	good
12	7	..	3	19	3	fair
13	7	1	3	10	3	fair
14	7	1	3	17	2	good
15	7	..	2	32	3	good
16	7	..	2	88	2	good
17	7	..	2	82	2	good
18	2	..	2	124	1	fair
19	8	..	2	103	1	fair
20	2	..	1	60	..	poor
21	7	1	2	122	1	fair
22	7	0	2	96	1	fair
23	7	0	2	64	1	fair
24	9	0	2	54	2	poor
26	7	..	1	11	2	fair
27	7	..	1	10	1	fair
28	7	3	4	15	2	fair
29	7	..	3	16	3	fair
30	7	1	4	22	2	fair
Oct. 31	7	..	3	26	3	fair
Nov. 1	7	..	2	23	2	fair
2	7	1	4	26	3	fair
3	7	2	6	40	3	fair
4	7	..	5	34	3	fair
5	8	..	4	26	2	fair
6	10	..	4	11	2	poor
7	8	1	5	36	4	fair
8	2	..	4	58	3	fair
9	8	..	4	56	3	fair
10	8	1	5	58	3	fair
11	8	2	7	76	3	fair
12	8	..	7	63	3	fair
13	8	..	7	103	2	fair
14	8	..	7	64	2	fair
15	8	..	5	80	1	fair
16	11	..	3	26	1	poor
17	8	1	5	19	2	fair
18	8	1	6	22	2	fair
19	9	..	4	16	2	poor
20	8	..	4	21	2	poor
21	8	..	4	19	3	poor
22	8	2	6	40	3	fair
23	8	1	6	25	1	poor
24	8	1	7	38	1	fair
25	8	..	7	90	2	fair
26	8	..	7	96	2	fair
27	8	..	4	106	2	fair
28	8	..	4	100	2	fair
29	3	..	2	41	1	poor
30	8	..	2	97	3	fair
Dec. 1	8	..	2	49	1	fair
2	8	..	1	10	..	v.poor
3	4	..	1	10	..	v.poor
4	8	3	4	19	3	fair
5	8	..	4	16	3	fair
6	8	..	4	11	3	fair
7	8	1	5	12	4	fair
8	9	..	3	5	..	poor
10	2	..	4	4	..	fair
11	8	..	3	3	2	fair
12	8	2	4	6	3	fair
13	8	..	4	11	3	fair
14	9	..	4	7	2	fair
16	9	..	3	3	..	poor
17	8	..	3	7	1	poor
18	8	1	3	13	3	poor
19	4	2	5	14	..	poor
20	9	..	5	11	..	poor
22	8	..	5	17	2	poor
23	8	..	3	8	..	poor
24	8	1	6	22	2	poor
25	8	..	6	16	3	fair
26	8	1	4	12	2	poor
27	4	..	3	11	1	poor
28	8	1	3	15	2	fair
29	2	..	3	12	2	poor
30	8	..	3	15	2	fair
31	8	..	2	12	..	poor

* $2\frac{1}{2}$-inch refractor.

OBSERVATIONS OF COMET *b* 1905,*

MADE AT THE GOODSELL OBSERVATORY WITH THE 16-INCH EQUATORIAL AND FILAR MICROMETER,

BY H. C. WILSON.

[Communicated by the Director, W. W. PAYNE.]

1905 Northfield M.T.	*	Comp	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Nov. 21 7 19 21	1	6, 2	−5 7.07	−5 48.4	23 45 41.34	+52 7 3.7	8.869*n*	0.068*n*	+3.65	+30.4
22 8 10 24	2	6, 4	−0 41.60	−4 55.6	23 40 53.91	+43 36 0.5	8.946	9.183	+3.20	+29.6
Dec. 15 6 49 6	3	9, 6	−0 29.44	−0 55.1	23 32 15.01	−10 35 8.9	8.989	0.854	+1.97	+11.1
	4	. .	. . . .	. . . .	. . . . .	. . . . .	. .	. .	+1.97	+11.1
26 7 47 22	5	12, 6	−0 38.90	−0 53.1	23 36 45.07	−14 19 59.9	9.421	0.856	+1.83	+ 8.9
26 7 47 22	6	6, 2	−2 14.92	+1 32.8	23 36 45.13	−14 19 57.4	9.421	0.856	+1.83	+ 8.8

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	23 50 44.76	+52 12 21.7	Harv. A.G. Zones 8532	4	23 32 47.66	−10 30 1.1	Schj. 9748–9, Mun. 32611, Gottingen 6720, Bruxelles 10581
2	23 41 32.31	+43 41 26.4	Lalande 46580	5	23 37 22.14	−14 19 15.7	S DM. −14°6529, comp. with No. 6
3	23 32 42.44	−10 34 24.9	10ᴹ comp. with No. 4	6	23 38 58.22	−14 21 39.0	Munich$_2$ 13066

* From *Supplement to No. 581.*

NOTE ON PONTECOULANT'S LUNAR THEORY,

BY A. HALL.

There is a small error in this theory, p. 625, that may be worthy of notice. For the argument, $2\xi - 2\phi + \phi'$, the residual should be $3''.677$, instead of $1''.123$. The author intimates that observation is wrong, apparently because he agrees with PLANA. Differentiating $\sin(2\xi - 2\phi + \phi')$ with respect to the time, and comparing with the corresponding term, p. 515, I find

$$-\frac{45}{16}m + \frac{193}{32}m^2 + \frac{196739}{1536}m^3$$

for the coefficient, agreeing with PONTECOULANT. These terms give

$$\begin{array}{r} '' \\ -2.182 \\ +0.350 \\ +0.555 \\ \hline -1.277 \end{array}$$

BURCKHARDT and DAMOISEAU have $+2''.4$, and $+2''.55$. HANSEN gives $+2''.52$.

1905 *September* 1.

NEW COMET a 1906 (BROOKS).

A moderately bright comet was discovered by W. R. BROOKS, of Hobart College, on Jan. 26. The following positions have been communicated by the Harvard College Observatory. The observer at Lick Observatory was MADDRILL, and at Princeton, BUGAN.

1906 Greenw. M.T.	α	δ	Obs'y
	h m s	° ′ ″	
Jan. 26.82	16 19 30	+47 10 –	Smith
27.8920	16 18 49.7	+48 51 15	Lick
28.8257	16 18 7.6	+50 23 58	Princeton
28.9598	16 17 57.4	+50 37 12	Lick

Also, references of the comet to neighboring *Durchmusterung* stars, by Prof. MORGAN, at Glasgow, Missouri, give some uncertainties, due to telegraphic obscurities, as indicated below:

Jan. 27.935 ? —— ? foll. 1′ 4″ north of DM. 49°2494
Jan. 28.935 ? $2^m\ 36^s.6$ prec. 1′ 17″ north of DM. 50°2282

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FOUNDED BY B. A. GOULD.

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A NEW THEORY OF INDIRECT OBSERVATIONS,

By J. MIDZUHARA.

§1. New Hypothesis of Errors and the Results of its Development.

From a number of indirect observation-equations which contain any number of independent unknown quantities, to find the best values of the unknown quantities we must necessarily assume some standard law which governs the theory of the errors; if we were not free to assume such standard law it is impossible to apply the theory of the probability for the solution of the problem. For example, the method of least-squares which is still used universally among astronomers, physicists, geodesists, &c., to solve indirect observations, was derived, by Gauss, from the assumption that: "The arithmetical mean of direct observations of equal weights is the most probable value of that unknown quantity."

Hitherto many scholars have endeavored to demonstrate the truth of this law; but it is certain that they have not yet obtained any reliable results; that is, from the assumption of the law: "The most probable error of the arithmetical mean is zero." But this assumption may be expected to be true only when the number of observations is infinite; whereas, in reality, we have never such infinite number of observations, and therefore practically we must assume that this error is not zero (let us call this error by u_0). This assumption, however, was adopted by Gauss in his formula for finding the mean error of a single observation as follows:

$$\epsilon^2 = \frac{[vv]}{m-\mu}$$

where

$[vv]$ = the sum of the squares of the residuals
m = the number of observations
ϵ = the mean error of single observation
μ = the number of the unknown quantities

the symbol μ in this formula having been indeed derived from the assumption that u_0 is equal to that mean error; though we can not adopt this formula without some improvements (see *Astron. Journ.*, No. 535), it is certain that the value of μ can not be put equal to zero; that is to say, u_0 is not zero. As a specimen of the demonstration of this fact we may apply the theorem

$$[vv]_1 > [vv]_2$$

where $[vv]_1$ and $[vv]_2$ denote the values of $[vv]$ deduced from two hypothetical observation-equations depending on the same system of observations, both forms of the equations being identical, except that the latter contains one or more additional terms of unknown quantities; but it being now universally accepted that μ is not zero, I think, we have not here a necessity of demonstrating it. Thus it being ascertained that the value of u_0 is not zero, we are now desirous to assume a law different from that of the method of least-squares, in which u_0 is put equal to zero, for solving the indirect observation-equations without the aid of the theory of probability. After several examinations I found the following law of errors:

$$[au]^2 = [aa]\epsilon^2 + F\{[a]\}\epsilon^2 \qquad (1)$$

where

a = a known quantity
u = the true error of observation
$F\{[a]\}$ = undetermined function of $[a]$

The function F seems nearly to possess my proposed properties, satisfying some necessary conditions among functions of the errors of the indirect observations in consistence with the results of the direct observations. I shall now describe the most remarkable results which have been derived from this hypothesis:

1. The best values of unknown quantities found by solving indirect linear observation-equations of equal weights, containing any number of unknown quantities, will be nearly equal to the unknown quantities found by solving the following normal-equations:

New Normal Equations.

$$
\left.
\begin{array}{l}
\left\{[aa]+\frac{[a]^2}{m}\right\}x+\left\{[ab]+\frac{[a][b]}{m}\right\}y+\left\{[ac]+\frac{[a][c]}{m}\right\}z+\ldots+\left\{[ah]+\frac{[a][h]}{m}\right\}=0 \\
\left\{[ab]+\frac{[a][b]}{m}\right\}x+\left\{[bb]+\frac{[b]^2}{m}\right\}y+\left\{[bc]+\frac{[b][c]}{m}\right\}z+\ldots+\left\{[bh]+\frac{[b][h]}{m}\right\}=0 \\
\left\{[ac]+\frac{[a][c]}{m}\right\}x+\left\{[bc]+\frac{[b][c]}{m}\right\}y+\left\{[cc]+\frac{[c]}{m}\right\}z+\ldots+\left\{[ch]+\frac{[c][h]}{m}\right\}=0 \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{array}
\right\} \quad (2)
$$

where h = the absolute term of the observation-equations; for, the numerical values of the true errors (this means the errors derived from the development of the function of the new hypothesis; hereafter it will be called the likely errors) of the unknown quantities found from these normal-equations are generally less than those of the likely errors found from the unknown quantities arbitrarily assumed.

2. The likely error of one of the unknown quantities found from the normal-equations (2), for example, that of x, will be nearly equal to

$$\sqrt{\frac{D_a}{\Delta}}\cdot\epsilon$$

where

Δ = the determinant formed from all the coefficients of the unknown quantities in (2).

D_a = the minor corresponding to the constituent

$$[aa]+\frac{[a]^2}{m} \text{ of } idem.$$

3. The degree of precision of the best values of unknown quantities found by solving the observation-equations, containing any number of unknown quantities, may be measured by the magnitudes of the likely errors.

Note. Since the functions of the true errors are not generally proportional to the corresponding mean errors or probable errors,* to compare the degrees of the precision of the best values of the unknown quantities in the indirect observations we can not adopt the mean errors or the probable errors of them.

4. In the beginning of the present treatise I have remarked that, in the method of the least-squares, there is the theorem:

$$[rr]_1 > [rr]_2$$

but this is not the case in the new method, for, in the new method the quantity corresponding to $[rr]_1$ is sometimes less than that corresponding to $[rr]_2$, probably demonstrating the excellence of the new method.

5. The value of $[au]^2$ generally varies when the signs of any number of a_i are changed.

Note. This result is evidently true, for, a_i being independent to u_i, we may change the sign of a_i without changing that of u_i.

6. The best value of the unknown quantity in direct observations of equal precision is the arithmetical mean of the observations (this may be easily seen from the normal equations (2)), and its likely error u_0 (say) is equal to

$$\pm\sqrt{\frac{[uu]}{2}} \quad \text{that is to} \quad \pm\frac{0.7071\,\epsilon}{\sqrt{m}}$$

From this result we may say that:

A. To compare the degrees of the precision of single observations in direct or indirect observations of different systems we may use the value of u_0 corresponding to the case $m = 1$, that is, $0.7071\,\epsilon$,

B. The weights of the observations in the same system are proportional to the inverse squares of the likely errors.

C. The value of the likely error of single observation is between the values of the "mean of errors" ($0.8453\,\epsilon$), and of "probable error" ($0.6745\,\epsilon$), and is rather close to the latter.

7. If, all the numerical values of a_i be equal to each other, and each of $\frac{1}{n}$ parts of them has a different sign from each of the remaining $\frac{n-1}{n}$ parts, we have

n	$[au]$	n	$[au]$
1	$0.707\sqrt{[aa]}\,.\,\epsilon$	10	$0.825\sqrt{[aa]}\,.\,\epsilon$
2	1.000 .	20	0.771 .
3	0.972 .	30	0.751 .
4	0.935 .	40	0.741 .
5	0.906 .	50	0.734 .
6	0.882 .	100	0.721 .
7	0.863 .	.	. .
8	0.848 .	.	. .
9	0.835 .	.	. .
10	0.825 .	∞	0.707 .

Note. Comparing this small table with the equations (2) we see that, in the case of single unknown quantity, if we have

A. $a_i = 1$, *B.* $a_i = -1$, *C.* $a_i = \pm 1$, $[a] = 0$

then the resulting value of the unknown quantity in each

* See my paper in *Astron. Journ.*, No. 535.

case is the same as that obtained from the method of the least-squares; while their likely errors are respectively,

$$\frac{0.707\,\epsilon}{\sqrt{m}}\ ,\quad \frac{0.707\,\epsilon}{\sqrt{m}}\ ,\quad \frac{1.000\,\epsilon}{\sqrt{m}}$$

§ 2. Development of the Hypothetical Function (1).

The quantity a in the equation (1) being independent of n or ϵ, we must have

$$(3)\quad \begin{cases} [an]^2 = [aa]\,\epsilon^2 + F\{[a]\}\,\epsilon^2 \\ [bn]^2 = [bb]\,\epsilon^2 + F\{[b]\}\,\epsilon^2 \\ [(a+b)n]^2 = [(a+b)^2]\,\epsilon^2 + F\{[a+b]\}\,\epsilon^2 \end{cases}$$

or

$$(4)\quad \begin{cases} F\{[a+b]\}\,\epsilon^2 = F\{[a]\}\,\epsilon^2 + F\{[b]\}\,\epsilon^2 - 2\,[ab]\,\epsilon^2 \quad * \\ +2\cdot\dfrac{[ab]}{[ab]_0}\sqrt{([aa]\,\epsilon^2+F\{[a]\}\,\epsilon^2)([bb]\,\epsilon^2+F\{[b]\}\,\epsilon^2)} \\ \quad \ldots\ldots\ldots \end{cases}$$

which, if we suppose that

$$a_i = b_i$$

becomes

$$(5)\qquad F\{[2a]\} = 4\,F\{[a]\}$$

and therefore, if p be a constant quantity, we have generally

$$(6)\qquad F\{[pa]\} = p^2 F\{[a]\}$$

Now let us change the sign of each of $\left(\frac{1}{n}\right)$ parts of all number of the supposed quantity b_i; then (4) becomes

$$(7)\quad \begin{cases} F\{[a+b]\} = F\{[a]\} + F\{[b]\} - 2\left(\dfrac{n-2}{n}\right)[aa] \\ \qquad + 2\,\dfrac{[ab]}{[ab]_0}\sqrt{([aa] + F\{[a]\})\,([bb] + F\{[b]\})} \end{cases}$$

which, when

$$a_i = a_j$$

is identical with

$$(8)\qquad F\left\{\left[\frac{n-1}{n}\,(a+b)_{b_i=a_i}\right]\right\} = 4\left(\frac{n-1}{n}\right)^2 F\{[a]\}$$

where

$$F\left\{\left[\frac{n-1}{n}\,(a+b)_{b_i=a_i}\right]\right\} = \text{the value of } F\left\{\left[\frac{n-1}{n}\,(a+b)\right]\right\}$$

in the case $b_i = a_i$.

Therefore, if we suppose that

$$(9)\qquad F\{[b]\} = F\{[a]\}$$

we must have

$$4\left(\frac{n-1}{n}\right)^2 F\{[b]\} = 4F\{[b]\} + 2\left\{1 - \frac{n-2}{n}\right\}[aa]$$

or

$$F\{[b]\} = \frac{-[aa]}{2-\dfrac{1}{n}} \qquad (10)$$

that is

$$[bn]^2 = \frac{\left(1-\dfrac{1}{n}\right)[aa]\,\epsilon^2}{2-\dfrac{1}{n}} \qquad (11)$$

which, when $n=\infty$, becomes free from the supposition (9), and gives

$$[an]^2 = \frac{\left(1-\dfrac{1}{\infty}\right)[aa]\,\epsilon^2}{2-\dfrac{1}{\infty}} = \frac{[aa]\,\epsilon^2}{2} \qquad (12)$$

or

$$[an] = \pm 0.7071\,\sqrt{[aa]}\cdot\epsilon \qquad (13)$$

Therefore we have the following principle:

"The likely error of the arithmetical mean of the direct observations of equal precision is

$$\pm\,\frac{0.7071\,\epsilon}{\sqrt{m}}\text{"}$$

The above is only the discussion of the expression $[an]$ in the special case $a_i = a_j$. I shall now proceed to consider the case in which the coefficient a_i possesses more general value.

Let us suppose that

$$a_i = Ca'$$
$$b_i = \pm a'$$

(where C and a' are constants for series of i) and each of $\frac{1}{n}$ parts of all number of b_i has the different sign from each of the remaining parts of it; then we have

$$(14)\quad \begin{cases} F\{[a]\}\,\epsilon^2 = F\{[Ca']\}\,\epsilon^2 = C^2F\{[a']\}\,\epsilon^2 \\ F\{[b]\}\,\epsilon^2 = F\left\{\left[\dfrac{n-2}{n}\,a'\right]\right\}\epsilon^2 = \left(\dfrac{n-2}{n}\right)^2 F\{[a']\}\epsilon^2 \\ [ab]\,\epsilon^2 = C\left(\dfrac{n-2}{n}\right)[a'a']\,\epsilon^2 \\ [aa]\,\epsilon^2 = C^2\,[a'a']\,\epsilon^2 \\ [bb]\,\epsilon^2 = [a'a']\,\epsilon^2 \\ [(a+b)^2]\epsilon^2 = \left\{\dfrac{(C+1)^2(n-1)+(C-1)^2}{n}\right\}[a'^2]\,\epsilon^2 \end{cases}$$

Also let us put

$$F\{[a+b]\}\,\epsilon^2 = -\frac{[(a+b)^2]}{2}\,\epsilon^2 + X\epsilon^2 \qquad (15)$$

$$= -\left\{\frac{(C+1)^2(n-1)+(C-1)^2}{2n}\right\}[a']^2\epsilon^2 + X\epsilon^2 \qquad (16)$$

then since we have, from (4),

* $[ab]_0$ is the numerical value of $[ab]$.

$$F\{[a+b]\}\,\epsilon^2 = C^2F\{[a']\}\,\epsilon^2 + \left(\frac{n-2}{n}\right)^2 F\{[a']\}\,\epsilon^2 - 2C\left(\frac{n-2}{n}\right)[a'a']\,\epsilon^2$$

(17)

$$+2\cdot\frac{[ab]}{[ab]_0}\sqrt{\left(C^2[a'a']\,\epsilon^2 + C^2F\{[a']\}\,\epsilon^2\right)\left([a'a']\,\epsilon^2 + \left(\frac{n-2}{n}\right)^2 F\{[a']\}\,\epsilon^2\right)}$$

$$= \left\{-\frac{C^2}{2} - \left(\frac{n-2}{n}\right)^2\cdot\tfrac{1}{2} - 2C\left(\frac{n-2}{n}\right) + 2\cdot\frac{[ab]}{[ab]_0}\sqrt{\frac{C^2}{2}\left\{1-\left(\frac{n-2}{n}\right)^2\cdot\tfrac{1}{2}\right\}}\right\}[a'a']\,\epsilon^2$$

comparing (16) and (17) we have

(18)

$$X = \left\{C\left(\pm\sqrt{1+\frac{4}{n}-\frac{4}{n^2}} - 1 + \frac{2}{n}\right) + \frac{2}{n}\left(1-\frac{1}{n}\right)\right\}[a'a']$$

we have

(19)
$$P = 16\left(1-\frac{1}{n}\right)\left(\frac{1}{n}\right)a'^4\,.\,mC'^2$$

Also since

(20)
$$[a+b] = \left\{(C+1)\left(1-\frac{1}{n}\right) + (C-1)\left(\frac{1}{n}\right)\right\}.\,a'm$$

(21)
$$[(a+b)^2] = \left\{(C+1)^2\left(1-\frac{1}{n}\right) + (C-1)^2\left(\frac{1}{n}\right)\right\}.\,a'^2.\,m$$

if we put

(21)′
$$M = [(a+b)^2] - \frac{[a+b]^2}{m}$$

we have

(22)
$$M = 4\left(1-\frac{1}{n}\right)\left(\frac{1}{n}\right).\,a'^2.\,m$$

and therefore

(23)
$$\frac{P}{M} = 4a'^2.\,C^2$$

(24)
$$X = \frac{M}{4}\left\{2 + \frac{C\left(\pm\sqrt{1+4\left(1-\frac{1}{n}\right)\left(\frac{1}{n}\right)} - 1 + \frac{2}{n}\right)}{\left(1-\frac{1}{n}\right)\left(\frac{1}{n}\right)}\right\}$$

in which the values of C and n are still to be determined.

Now, since we have, from (20), (21), (21)′ and (22)

$$\frac{m\,[(a+b)^2]}{[a+b]^2} = \frac{(C+1)^2\left(1-\frac{1}{n}\right) + (C-1)^2\left(\frac{1}{n}\right)}{\left\{(C+1)\left(1-\frac{1}{n}\right) + (C-1)\left(\frac{1}{n}\right)\right\}^2}$$

or

(25)
$$\frac{[(a+b)^2]}{M} = \frac{m\,[(a+b)^2]}{m\,[(a+b)^2] - [a+b]^2}$$

$$= \frac{(C+1)^2\left(1-\frac{1}{n}\right) + (C-1)^2\left(\frac{1}{n}\right)}{4\left(1-\frac{1}{n}\right)\left(\frac{1}{n}\right)}$$

$$= \frac{(C+1)^2 - 4C\left(\frac{1}{n}\right)}{4\left(1-\frac{1}{n}\right)\left(\frac{1}{n}\right)}$$

Put

$$P = [(a+b)^4] - \frac{[(a+b)^2]^2}{m}$$

then since

$$[(a+b)^4] = (C+1)^4a'^4\left(\frac{n-1}{n}\right)m + (C-1)^4a'^4\left(\frac{1}{n}\right)m$$

$$\frac{[(a+b)^2]^2}{m} = \left\{\frac{(C+1)^4(n-1)^2 + 2(C+1)^2(C-1)^2(n-1) + (C-1)^4}{n^2}\right\}a'^4.\,m$$

comparing these with

$$\frac{P}{M^2} = \frac{C'^2}{m\left(1-\frac{1}{n}\right)\left(\frac{1}{n}\right)} \qquad (26)$$

we have

$$\frac{1}{n} = \frac{(C+1)^2}{4C} - \frac{M[(a+b)^2]C}{mP} \qquad (27)$$

Also from (20) and (21) we have

$$[(a+b)^2] - (C+1)\,a'[a+b] = -2(C-1)\left(\frac{1}{n}\right)a'^2.\,m \qquad (28)$$

$$[(a+b)^2] - (C-1)\,a'[a+b] = 2(C+1)\left(1-\frac{1}{n}\right)a'^2.\,m \qquad (29)$$

or

$$[(a+b)^2]^2 + (C+1)(C-1)\,a'^2.\,[a+b]^2 - 2Ca'[a+b]\,[(a+b)^2]$$
$$= -4\,(C+1)\,(C-1)\left(\frac{1}{n}\right)\left(1-\frac{1}{n}\right)a'^4.\,m^2 \qquad (30)$$

by (19)

$$= -\frac{(C+1)\,(C-1)\,mP}{4C^2}$$

and from (23)

$$2a' = \pm\frac{1}{C}\sqrt{\frac{P}{M}}$$

Therefore

$$[(a+b)^2]^2 + \frac{(C^2-1)\,P\,[a+b]^2}{4C^2M} \mp \sqrt{\frac{P}{M}}\cdot[a+b]^2.\,[(a+b)^2]$$

$$= -\frac{\left(1-\frac{1}{C^2}\right)mP}{4}$$

that is

(31)

$$C'^2 = \frac{P\,\{[a+b]^2 + mM\}}{4M[(a+b)^2]^2 + P[a+b]^2 \mp 4\sqrt{PM}\,[a+b]^2.[(a+b)^2] + mPM}$$

$$= \frac{mP}{4M\,[(a+b)^2] + mP \mp 4\sqrt{PM\,.\,[a+b]^2}}$$

Thus, from (27) and (31), we may find the values of C and n, and therefore these being substituted in the equation (24), we may determine the value of X. Now summing up the above results we may put

$$(32)\quad \left\{\begin{aligned} &[uu]^2 = \frac{[aa]\,\epsilon^2}{2} + X.\epsilon^2 \\ &\text{where} \\ &X = \frac{M}{4}\left\{2 + \frac{C\left(\pm\sqrt{1+4\left(1-\frac{1}{n}\right)\left(\frac{1}{n}\right)} - 1 + \frac{2}{n}\right)}{\left(1-\frac{1}{n}\right)\left(\frac{1}{n}\right)}\right\} \\ &M = [aa] - \frac{[a]^2}{m} \\ &C = \pm\sqrt{\left\{\frac{mP}{4[aa]M + mP \mp 4\sqrt{MP}[a]^2}\right\}} \\ &\frac{1}{n} = \frac{(1+C)^2}{4C} - \frac{MC[aa]}{mP} \\ &P = [a^4] - \frac{[aa]^2}{m} \end{aligned}\right.$$

This formula, when $a_i = \pm a_j$, becomes very simple, as follows:

$$(33)\qquad [uu]^2 = [aa]\,\epsilon^2 - \frac{[a]^2}{2m}\,\epsilon^2$$

§ 3. Application of the New Hypothesis of Errors to the Solution of Indirect Observations.

We have now seen that, when the numerical values of all of the coefficients a_i in $[uu]$ are equal to each other, the formula (33) may be adopted as the rigorous law of errors. Though, when they are not equal to each other, the formula (33) requires more or less corrections, its application to the practical problem being little complicated, we must now be contented to apply the formula (33) for the solution of indirect observations by supposing that it is always true, no matter what the value of the coefficient a_i may be. The new solution of the indirect observations is as follows:

Let

$$(34)\quad \left\{\begin{aligned} a_1x + b_1y + c_1z + \ldots + h_1 &= u_1 \\ a_2x + b_2y + c_2z + \ldots + h_2 &= u_2 \end{aligned}\right.$$

represent the given observation-equations, and let us confine our attention, for instance, to the discussion of the value of x. Suppose that multiplying the equations (34) respectively by $t_1, t_2, t_3, \ldots$ the arbitrary numbers, and adding together we have

$$\begin{aligned} [at] &= 1 \\ [bt] &= [ct] = [dt] = \ldots = 0 \end{aligned} \qquad (35)$$

$$x = -[th] + [tu] \qquad (36)$$

then the likely error of the unknown quantity x, by (33), must satisfy the equation

$$[tu]^2 = [t^2]\epsilon^2 - \frac{[t]^2\,\epsilon^2}{2m} \qquad (37)$$

Therefore to find the best value of x let us differentiate (35) and (37), and put (since (37) must be a minimum)

$$\begin{array}{ccccc} \left(t_1 - \frac{[t]}{2m}\right)\delta t_1 + & \left(t_2 - \frac{[t]}{2m}\right)\delta t_2 + & \ldots + & \left(t_m - \frac{[t]}{2m}\right)\delta t_m & = 0 \\ a_1\delta t_1 + & a_2\delta t_2 + & + & a_m\delta t_m & = 0 \\ b_1\delta t_1 + & b_2\delta t_2 + & + & b_m\delta t_m & = 0 \\ c_1\delta t_1 + & c_2\delta t_2 + & + & c_m\delta t_m & = 0 \\ \ldots & \ldots & \ldots & \ldots & \end{array}$$

which may be assumed to have the following relations:

$$\left.\begin{aligned} t_1 - \frac{[t]}{2m} &= \lambda_a a_1 + \lambda_{ab} b_1 + \lambda_{ac} c_1 + \ldots \\ t_2 - \frac{[t]}{2m} &= \lambda_a a_2 + \lambda_{ab} b_2 + \lambda_{ac} c_2 + \ldots \\ &\ldots \end{aligned}\right\} \qquad (38)$$

where λ_a, λ_{ab}, λ_{ac}, &c., are undetermined numbers; then we may easily deduce the following equations:

$$\left.\begin{aligned} 1 - \frac{[t]}{2m}[a] &= \lambda_a[aa] + \lambda_{ab}[ab] + \lambda_{ac}[ac] + \ldots \\ -\frac{[t]}{2m}[b] &= \lambda_a[ab] + \lambda_{ab}[bb] + \lambda_{ac}[bc] + \ldots \\ -\frac{[t]}{2m}[c] &= \lambda_a[ac] + \lambda_{ab}[bc] + \lambda_{ac}[cc] + \ldots \\ &\ldots \\ [t^2] - \frac{[t]^2}{2m} &= \lambda_a \\ \frac{[t]}{2} &= \lambda_a[a] + \lambda_{ab}[b] + \lambda_{ac}[c] + \ldots \end{aligned}\right\} \qquad (39)$$

that is

$$\left.\begin{aligned} 1 &= \lambda_a\left\{[aa] + \frac{[a]^2}{m}\right\} + \lambda_{ab}\left\{[ab] + \frac{[a][b]}{m}\right\} + \lambda_{ac}\left\{[ac] + \frac{[a][c]}{m}\right\} + \ldots \\ 0 &= \lambda_a\left\{[ab] + \frac{[a][b]}{m}\right\} + \lambda_{ab}\left\{[bb] + \frac{[b]^2}{m}\right\} + \lambda_{ac}\left\{[bc] + \frac{[b][c]}{m}\right\} + \ldots \\ 0 &= \lambda_a\left\{[ac] + \frac{[a][c]}{m}\right\} + \lambda_{ab}\left\{[bc] + \frac{[b][c]}{m}\right\} + \lambda_{ac}\left\{[cc] + \frac{[c]^2}{m}\right\} + \ldots \\ &\ldots \end{aligned}\right\} \qquad (40)$$

$$(41)\quad t_i = \lambda_a\left(a_i + \frac{[a]}{m}\right) + \lambda_{ab}\left(b_i + \frac{[b]}{m}\right) + \lambda_{ac}\left(c_i + \frac{[c]}{m}\right) + \ldots$$

$$(42)\quad x = -[th] \pm \sqrt{\lambda_a}\,.\,\epsilon = -\lambda_a\left([ah] + \frac{[a][h]}{m}\right) - \lambda_{ab}\left([bh] + \frac{[b][h]}{m}\right) - \lambda_{ac}\left([ch] + \frac{[c][h]}{m}\right) - \ldots\ldots \pm \sqrt{\lambda_a}\,.\,\epsilon$$

and by the same reasoning

$$y = -\mu_{ab}\left([ah] + \frac{[a][h]}{m}\right) - \mu_b\left([bh] + \frac{[b][h]}{m}\right) - \mu_{bc}\left([ch] + \frac{[c][h]}{m}\right) - \ldots\ldots$$

$$z = -\nu_{ac}\left([ah] + \frac{[a][h]}{m}\right) - \nu_{bc}\left([bh] + \frac{[b][h]}{m}\right) - \nu_c\left([ch] + \frac{[c][h]}{m}\right) - \ldots\ldots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

in which we have the following relations:

$$(43)\quad \left\{\begin{array}{llll} \Delta x\lambda_a = D_a & \Delta x\lambda_{ab} = D_{ab} & \Delta x\lambda_{ac} = D_{ac} & \&c. \\ \Delta x\mu_{ab} = D_{ab} & \Delta x\mu_b = D_b & \Delta x\mu_{bc} = D_{bc} & \&c. \\ \Delta x\nu_{ac} = D_{ac} & \Delta x\nu_{bc} = D_{bc} & \Delta x\nu_c = D_c & \&c. \\ \cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot & \end{array}\right.$$

where

Δ = the determinant formed from all the coefficients of λ's in the second side of (40)

D_a = the minor corresponding to the constituent

$$[aa] + \frac{[a]^2}{m} \text{ of idem}$$

D_{ab} = that corresponding to the constituent $[ab] + \dfrac{[a][b]}{m}$

&c. &c. &c.

Therefore the normal-equations which give the best values of x, y, z, &c., are as (2); and the likely errors of the values of x, y, z, &c., found from those normal-equations, by (41), are

$$\sqrt{\frac{D_a}{\Delta}}\,.\,\epsilon\ ,\quad \sqrt{\frac{D_b}{\Delta}}\,.\,\epsilon\ ,\quad \sqrt{\frac{D_c}{\Delta}}\,.\,\epsilon\ ,\quad \&c.$$

respectively.

As an example of practical application of the new normal-equations (2) let us take the following datum, which are in CHAUVENET's "Spherical and Practical Astronomy," Vol. II, page 206.

THE OBSERVATION-EQUATIONS.

$$\begin{array}{rlllll}
1 & +0.214\,a & +\ 1.296\,e\ + & \Delta\theta & +\ 2^{s}.10 & = 0 \\
2 & +1.032\,a & +\ 1.086\,e\ + & \Delta\theta & +\ 2.96 & = 0 \\
3 & +1.031\,a & +\ 1.085\,e\ + & \Delta\theta & +\ 3.17 & = 0 \\
4 & +1.135\,a & +\ 1.156\,e\ + & \Delta\theta & +\ 3.19 & = 0 \\
5 & -0.732\,a & +\ 2.056\,e\ + & 0.707\,\Delta\theta & +\ 0.15 & = 0 \\
6 & -0.732\,a' & -\ 2.056\,e\ + & 0.707\,\Delta\theta & -\ 0.97 & = 0 \\
7 & +2.606\,a' & +\ 2.993\,e\ + & \Delta\theta & +\ 2.22 & = 0 \\
8 & +1.879\,a' & +\ 1.984\,e\ + & \Delta\theta & +\ 1.91 & = 0 \\
9 & -1.322\,a' & -\ 3.319\,e\ + & \Delta\theta & -\ 0.58 & = 0 \\
10 & -0.229\,a' & -\ 1.802\,e\ + & \Delta\theta & +\ 0.58 & = 0 \\
11 & +2.264\,a' & +\ 2.508\,e\ + & \Delta\theta & +\ 2.18 & = 0 \\
12 & +2.016\,a' & +\ 2.166\,e\ + & \Delta\theta & +\ 1.94 & = 0
\end{array}$$

THE NORMAL-EQUATIONS.

$$\begin{array}{l}
3.998\,a + 0 + 2.325\,e + 2.894 + 10.283 = 0 \\
0 + 21.818\,a' + 27.881\,e + 6.697 + 19.569 = 0 \\
2.325\,a + 27.881\,a' + 51.969\,e + 9.153 + 36.352 = 0 \\
2.894\,a + 6.697\,a' + 9.153\,e + 11.000 + 19.090 = 0
\end{array}$$

Tokyo Astronomical Observatory, 1905 December.

THE RESULTING VALUES OF THE UNKNOWNS.

$$a = -1^{s}.681 \qquad e = -0^{s}.423$$
$$a' = -0.083 \qquad \Delta\theta = -0.891$$

viz., if we apply these datum to the new normal-equations (2) we have

THE NEW NORMAL EQUATIONS.

$$\begin{array}{l}
4.597\,a + 1.448\,a' + 4.369\,e + 5.443\,\Delta\theta + 14.493 = 0 \\
1.448\,a + 25.394\,a' + 32.825\,e + 12.863\,\Delta\theta + 29.751 = 0 \\
4.369\,a + 32.825\,a' + 58.950\,e + 17.859\,\Delta\theta + 50.730 = 0 \\
5.443\,a + 12.863\,a' + 17.859\,e + 21.857\,\Delta\theta + 37.020 = 0
\end{array}$$

THE RESULTING VALUES OF THE UNKNOWNS.

$$a = -1^{s}.682 \qquad e = -0^{s}.423$$
$$a' = -0.083 \qquad \Delta\theta = -0.880$$

Thus, in this example, the old and the new solutions give very nearly the same values of the unknown quantities except $\Delta\theta$, which has the difference $0^{s}.011$.

As another example, let us also take the following datum from the same treatise, Vol. II, page 381.

THE OBSERVATION-EQUATIONS.

$$\begin{array}{ll}
\xi + 0.90\,e + 14''.9 = 0 & \xi + 0.12\,e + 28''.1 = 0 \\
\xi + 0.89\,e + 20.4 = 0 & \xi - 0.13\,e + 48.1 = 0 \\
\xi + 0.83\,e + 12.4 = 0 & \xi - 0.45\,e + 58.6 = 0 \\
\xi + 0.72\,e + 13.7 = 0 & \xi - 0.88\,e + 67.9 = 0 \\
\xi + 0.36\,e + 23.3 = 0 & \xi - 0.90\,e + 72.4 = 0
\end{array}$$

THE NORMAL-EQUATIONS.

$$\begin{array}{l}
10.00\,\xi + 1.5500\,e + 359.8 = 0 \\
1.55\,\xi + 4.9273\,e - 92.712 = 0
\end{array}$$

THE RESULTING VALUES OF THE UNKNOWNS.

$$\xi = -40''.9\ , \qquad e = +31''.7$$

These data give

THE NEW NORMAL EQUATIONS.

$$\begin{array}{l}
20.00\,\xi + 3.1000\,e + 719.600 = 0 \\
3.10\,\xi + 5.1676\,e - 36.943 = 0
\end{array}$$

and

THE RESULTING VALUES OF THE UNKNOWNS

$$\xi = -40''.8\ , \qquad e = +31''.3$$

which are also identical with those of the old, nearly.

To complete the subject of the present investigation we must still find the value of ϵ. This investigation, however, will be postponed to the future.

OBSERVATIONS OF COMETS,

MADE WITH THE 16-INCH EQUATORIAL OF THE CINCINNATI OBSERVATORY,

By J. G. PORTER.

1905 Cincinnati M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Comet 1904 I.										
d h m s			m s	′ ″	h m s	° ′ ″			s	″
Jan. 8 10 49 30	1	7 , 9	−0 0.29	+ 5 43.4	11 59 29.52	+58 16 51.7	n9.938	0.452	−0.47	−16.5
14 10 15 10	3	12 , 6	−2 53.56	− 0 57.7	11 46 33.30	+59 51 18.2	n9.958	0.426	−0.08	−16.7
26 8 49 21	4	5 , 6	+0 4.51	− 9 27.2	11 13 31.89	+62 32 21.2	n9.998	0.406	+0.88	−15.9
27 9 25 49	5	6 , 8	+0 20.20	+ 4 16.8	11 10 15.97	+62 43 51.4	n9.990	0.137	+0.95	−15.7
Feb. 4 10 5 58	6	8 , 6	−0 44.57	− 3 31.6	10 42 55.39	+63 55 10.9	n9.928	n0.050	+1.55	−13.8
28 9 37 8	8	8 , 8	+0 55.01	+ 1 37.2	9 18 22.60	+63 59 0.4	n9.488	n0.575	+2.14	− 4.1
Mar. 3 14 11 26	9	8 , 8	+2 16.98	+ 4 26.5	9 8 48.23	+63 38 21.0	9.936	n9.942	+2.09	− 3.0
5 7 47 24	10	8 , 8	−0 25.85	− 3 0.4	9 3 53.35	+63 25 40.9	n9.777	n0.393	+2.05	− 2.3
10 8 42 50	11	8 , 8	−1 12.69	− 0 5.6	8 50 44.20	+62 43 12.4	n9.389	n0.524	+1.83	− 0.7
21 8 49 10	12	8 , 8	−0 8.65	+ 4 2.1	8 23 11.80	+60 17 55.3	9.134	n0.494	+1.23	+ 2.2
27 9 52 38	13	8 , 8	−0 51.24	+ 0 37.3	8 18 49.61	+59 43 36.7	9.631	n0.382	+1.12	+ 2.4
Apr. 3 9 34 47	14	8 , 8	−0 15.38	+ 1 9.2	8 10 39.16	+58 23 6.4	9.672	n0.299	+0.81	+ 3.2
Comet 1904 e.										
Jan. 1 7 56 20	15	12 , 12	−1 51.01	+ 1 28.9	1 17 42.71	− 7 23 17.4	9.207	0.799	−0.13	− 9.1
3 8 0 14	16	12 , 12	+1 7.10	+ 3 35.5	1 20 25.91	− 5 45 10.6	9.253	0.787	−0.14	− 8.7
6 7 37 35	18	12 , 12	−1 23.18	+ 5 0.7	1 24 41.80	− 3 19 34.0	9.167	0.770	−0.10	− 8.2
27 8 28 54	21	10 , 8	+1 1.15	+ 6 23.9	2 1 36.63	+12 44 26.1	9.508	0.633	−0.05	− 5.3
28 7 40 22	22	5 , 8	−0 3.35	− 3 11.2	2 3 34.67	+13 25 11.3	9.392	0.605	−0.04	− 5.2
Feb. 2 9 5 46	24	12 , 7	+0 27.11	− 0 33.8	2 14 16.54	+16 52 23.3	9.594	0.620	−0.02	− 4.5
4 8 43 37	26	8 , 8	−0 24.70	− 1 16.6	2 18 38.53	+18 10 41.6	9.570	0.591	0.00	− 4.3
6 9 15 12	28	8 , 6	−1 18.09	− 0 13.3	2 23 13.08	+19 28 47.0	9.620	0.608	−0.01	− 4.1
28 8 58 51	29	8 , 8	+2 4.24	− 1 8.6	3 19 37.87	+31 37 2.0	9.678	0.549	−0.04	− 2.6
Mar. 5 7 23 26	30	8 , 8	−0 6.27	+ 0 33.4	3 34 0.33	+33 48 57.9	9.551	0.230	−0.07	− 2.4
10 8 8 34	31	8 , 8	−0 55.54	− 0 53.1	3 49 17.72	+35 51 27.8	9.652	0.315	−0.09	− 2.4
22 7 28 29	32	8 , 8	+0 24.04	− 0 4.5	4 27 59.59	+39 53 44.4	9.624	0.055	−0.14	− 2.0
25 7 55 14	33	8 , 8	+0 52.26	− 5 39.8	4 38 12.88	+40 43 55.3	9.678	0.160	−0.16	− 2.0
25 8 27 7	33	8 , 8	+0 56.62	− 5 19.9	4 38 17.24	+40 44 15.2	9.719	0.292	−0.16	− 2.0
26 9 28 26	34	8 , 8	+0 59.81	− 2 19.0	4 41 51.22	+41 0 32.2	9.768	0.484	−0.16	− 2.0
27 8 46 20	36	8 , 8	−3 54.34	+ 0 3.6	4 45 11.66	+41 15 14.1	9.741	0.356	−0.13	2.0
30 8 13 10	37	8 , 8	+0 51.27	+ 5 35.0	4 55 31.83	+41 57 25.7	9.713	0.214	−0.16	− 1.9
30 8 33 46	38	8 , 8	−0 9.18	− 0 41.1	4 55 35.71	+41 57 38.4	9.736	0.301	−0.16	− 1.9
31 7 54 28	39	10 , 10	+0 30.64	− 0 10.5	4 58 59.55	+42 10 32.8	9.689	0.115	−0.16	− 1.8
Apr. 3 7 58 12	40	8 , 8	+1 0.80	− 0 44.2	5 9 35.25	+42 47 39.0	9.701	0.123	−0.21	− 1.8
4 8 26 17	41	8 , 8	−0 14.24	+ 0 0.1	5 13 13.16	+42 59 8.4	9.737	0.191	−0.22	− 1.8
Comet 1905 a.										
Apr. 3 8 43 49	42	8 , 8	−1 21.68	− 2 45.7	6 15 9.61	+21 0 6.5	9.567	0.551	+0.13	− 8.9
3 8 43 49	43	8 , 8	−1 33.50	+ 4 59.0	6 15 9.49	+21 0 9.6	9.567	0.551	+0.13	− 8.9
4 8 54 44	44	8 , 8	−0 9.89	+ 5 34.7	6 19 17.85	+22 11 57.3	9.587	0.547	+0.12	− 8.5
6 8 46 0	45	8 , 8	+0 23.52	+ 2 8.5	6 27 43.57	+24 31 44.4	9.581	0.504	+0.12	− 7.8
22 8 12 31	47	8 , 8	+0 1.05	+ 2 34.8	7 47 8.76	+40 13 19.0	9.553	9.840	+0.16	− 1.7
27 9 9 45	50	8 , 6	−0 16.01	+11 27.2	8 16 4.39	+43 41 2.6	9.676	9.949	+0.19	− 0.1
27 9 9 45	51	6 , 4	−2 21.35	−10 10.6	8 16 5.20	+43 40 53.5	9.676	9.949	+0.21	0.0
30 9 14 47	52	8 , 8	−0 0.02	+ 7 10.7	8 33 55.96	+45 21 42.5	9.688	9.832	+0.20	+ 0.8
30 9 56 25	53	8 , 8	−0 43.56	+ 2 5.8	8 34 6.70	+45 22 33.6	9.745	0.139	+0.20	+ 0.8
May 1 8 36 10	54	8 , 8	+2 23.76	+ 2 50.6	8 39 48.10	+45 50 37.9	9.607	7.952	+0.19	+ 1.1
3 8 24 51	55	8 , 8	+0 9.95	+ 1 21.2	8 51 51.46	+46 44 18.3	9.576	n9.491	+0.18	+ 1.7
7 9 53 23	56	8 , 8	−0 1.98	+ 2 17.8	9 16 33.56	+48 11 41.0	9.747	n9.875	+0.23	+ 2.6
Comet 1905 b.										
Dec. 25 7 14 24	58	12 , 12	−0 9.46	− 5 22.2	23 36 15.85	−14 5 18.0	9.351	0.833	+2.05	+11.0
27 6 10 5	59	8 , 8	−0 17.98	+ 3 19.0	23 37 11.93	−14 32 10.0	9.057	0.846	+2.03	+10.7

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	11 59 30.28	+58 11 24.8	Comp. with 2	32	4 27 35.69	+39 53 50.9	Lund, A.G. 2287
2	12 1 57.32	+58 7 47.1	Cin. 1900, 2177	33	4 37 20.78	+40 39 37.1	Bonn, A.G. 3795
3	11 49 26.94	+59 52 32.6	Hels. Gotha, A.G. 6942	34	4 40 51.57	+41 2 53.2	Comp. with 35
4	11 13 26.50	+62 42 4.3	" " A.G. 6710	35	4 40 25.10	+41 7 56.0	Bonn, A.G. 3832
5	11 9 54.82	+62 39 50.3	B.D. +62°1167 Comp. with 4	36	4 49 6.13	+41 15 12.5	" A.G. 3961
6	10 43 38.41	+63 58 56.3	Comp. with 7	37	4 54 40.72	+41 51 52.6	" A.G. 4040
7	10 41 23.64	+64 7 37.9	Hels. Gotha, A.G. 6480	38	4 55 45.05	+41 58 21.4	Comp. with 37
8	9 17 25.45	+63 57 27.3	" " A.G. 5933	39	4 58 29.10	+42 10 45.1	Bonn, A.G. 4103
9	9 6 29.16	+63 33 57.5	" " A.G. 5866	40	5 8 34.66	+42 48 25.0	" A.G. 4269
10	9 4 17.15	+63 28 43.6	" " A.G. 5849	41	5 13 27.62	+42 59 10.1	" A.G. 4350
11	8 51 55.06	+62 43 18.7	" " A.G. 5762	42	6 16 31.16	+21 3 1.1	Berlin B, A.G. 2332
12	8 23 19.22	+60 13 51.0	" " A.G. 5566	43	6 16 42.86	+20 55 19.5	" " A.G. 2334
13	8 19 39.73	+59 42 57.0	" " A.G. 5534	44	6 19 27.62	+22 6 31.1	" " A.G. 2360
14	8 10 53.73	+58 21 54.0	" " A.G. 5468	45	6 27 19.93	+24 29 43.7	Comp. with 46
15	1 19 33.85	− 7 24 37.2	Rad.3 327, Gr. 10, Y 216 2 Gr. 10, Y. 516	46	6 25 1.76	+24 26 58.2	Berlin B. A.G. 2424
16	1 19 18.95	− 5 48 37.4	B.D. −6°266 Comp. with 17	47	7 47 7.55	+40 10 45.9	Comp. with 48, 49
17	1 14 48.70	− 5 49 33.5	Par. 1674, War. 202	48	7 43 47.05	+40 0 35.0	Lund, A.G. 3995 Bonn A.G. 6238
18	1 26 5.08	− 3 24 26.5	War.228,com.with 19,20	49	7 46 25.40	+40 25 53.9	Bonn, A.G. 6257
19	1 27 17.05	− 3 37 7.1	Rüm. 331, Yar. 472	50	8 16 20.21	+43 29 35.5	B.J. 31 Lyncis
20	1 27 15.62	− 3 12 46.4	W. 1h421, Rüm. 330	51	8 18 26.34	+43 51 4.1	Bonn, A.G. 6573
21	2 0 35.53	+12 38 7.5	Leip. I, A.G. 622	52	8 33 55.78	+45 14 31.0	Bonn, A.G. 6711
22	2 3 38.06	+13 28 27.7	Comp. with 23	53	8 34 50.06	+45 20 27.0	Comp. with 52
23	2 4 49.98	+13 32 38.4	Leip. I, A.G. 639	54	8 37 24.15	+45 47 46.2	Bonn, A.G. 6747
24	2 13 49.45	+16 53 1.6	B.D. +16°279 Comp. with 25	55	8 51 41.33	+46 42 55.4	Bonn, A.G. 6861
25	2 11 50.43	+16 52 32.5	Berlin A, A.G. 634	56	9 16 35.31	+48 9 20.6	Comp. with 57
26	2 19 3.23	+18 12 2.5	Comp. with 27	57	9 20 40.71	+48 11 1.9	Bonn, A.G. 7115 Cin. 1900, 1759
27	2 17 23.36	+17 58 26.1	Berlin A, A.G. 655	58	23 36 23.26	−14 0 6.8	Mün$_2$ 13053
28	2 24 31.18	+19 29 4.4	Berlin A, A.G. 684	59	23 37 27.88	−14 35 39.7	B.D. −14°6530 Comp. with 60, 61
29	3 17 33.67	+31 38 13.2	Leiden, A.G. 1278	60	23 34 51.38	−14 44 50.4	Rad$_3$6316,2Gr.10,Y.6729
30	3 34 6.67	+33 48 26.9	Leiden, A.G. 1376	61	23 38 58.23	−14 21 38.1	Mün$_1$32715, Mün$_2$13066
31	3 50 13.35	+35 52 23.3	Lund, A.G. 2008				

OBSERVATIONS OF MINOR PLANETS AND COMETS,

MADE AT THE U.S. NAVAL OBSERVATORY,

By HERBERT L. RICE.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

1904 Wash'n M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				(2) *Pallas.*						
1904 h m s			m s	′ ″	h m s	° ′ ″			s	″
May 28 11 2 45	1	30 , 6	−0 37.38	+ 4 23.6	16 5 13.53	+26 19 18.1	n8.918	0.285	+2.22	+ 3.4
June 14 12 12 56	2	30 , 6	+2 40.91	− 0 42.2	15 52 19.82	+26 3 42.9	9.381	0.359	+2.20	+ 7.1
22 12 22 39	3	30 , 6	−0 39.11	− 1 37.0	15 47 47.42	+25 26 35.0	9.509	0.431	+2.18	+ 8.6
July 26 10 15 32	4	25 , 5	−3 4.92	− 0 42.2	15 43 13.09	+20 37 23.7	9.518	0.527	+1.89	+12.5
30 10 47 29	5	30 , 6	−1 31.38	− 4 5.2	15 44 14.99	+19 55 12.4	9.596	0.580	+1.84	+12.6
Aug. 6 9 53 56	6	25 , 5	+2 29.85	+ 6 9.8	15 46 44.93	+18 40 46.0	9.548	0.569	+1.74	+12.9
				(3) *Juno.*						
Aug. 3 13 31 55	7	35 , 7	+1 1.50	− 7 45.7	20 57 12.12	− 3 56 25.9	9.231	0.772	+3.02	+19.8
5 13 38 50	8	25 , 5	+1 43.29	+ 1 54.0	20 55 29.97	− 4 10 35.5	9.291	0.774	+3.04	+20.0
11 11 39 7	9	30 , 6	+1 21.99	− 1 49.4	20 50 26.67	− 4 55 51.3	8.279	0.783	+3.08	+20.5
12 11 21 20	10	30 , 6	+2 17.92	− 0 22.9	20 49 36.44	− 5 3 51.0	n7.850	0.784	+3.08	+20.5
15 11 54 58	11	30 , 6	+1 45.21	− 0 48.3	20 47 4.78	− 5 28 54.8	8.948	0.786	+3.09	+20.6
16 12 2 3	12	25 , 5	−2 1.47	− 6 24.7	20 46 15.15	− 5 37 26.2	9.049	0.787	+3.09	+20.7

1904-05 Wash'n M.T.	*	Comp	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(172) Baucis,					
1904 h m s			m s	′ ″	h m s	° ′ ″			s	″
Aug. 16 13 26 49	13	35 , 7	−1 42.73	+ 3 6.9	22 22 8.15	−11 19 49.1	8.964	0.827	+3.03	+19.8
Sept. 23 10 14 50	14	30 , 6	−1 13.78	− 1 38.3	21 47 24.16	−10 15 0.2	8.880	0.820	+3.06	+21.7
					ENCKE'S COMET.					
Nov. 11 8 44 8	*15	19	+0 11.47	. . . .	22 30 47.03	. . . . . .	9.299	. .		
11 8 52 16	*15	6	. . . . .	+ 2 41.9	. . . . . .	+20 15 10.0	. .	0.481	+2.74	+28.9
11 9 13 56	*15	7	+0 10.29	. . . .	22 30 42.85	. . . . . .	9.407	. .		
27 7 15 7	*16	27 , 9	−0 36.71	− 2 19.9	21 26 16.49	+11 36 23.3	9.413	0.627	+2.03	+25.5
28 7 36 41	*17	18 , 6	+1 41.92	+ 5 16.7	21 22 31.78	+11 1 37.8	9.485	0.644	+1.98	+25.1
30 6 30 5	*18	24 , 8	+1 44.58	+ 3 12.1	21 15 22.12	+ 9 54 14.6	9.339	0.641	+1.91	+24.4
Dec. 8 6 56 35	*19	21 , 7	+0 27.00	− 6 3.6	20 45 41.35	+ 5 5 32.3	9.546	0.706	+1.69	+21.5
13 7 36 51	20	25 , 5	−1 57.58	− 2 57.8	20 25 32.65	+ 1 45 48.6	9.633	0.734	+1.60	+19.3
14 6 55 15	21	25 , 5	+1 33.58	+ 0 14.1	20 21 24.05	+ 1 4 32.2	9.605	0.735	+1.57	+18.7
16 7 8 22	22	21 , 8	−0 37.52	+ 0 3.2	20 12 29.49	− 0 23 58.8	9.631	0.742	+1.55	+17.9
20 6 5 16	23	8	. . . .	− 4 8.8	. . . . . .	− 3 33 28.8	. .	0.755	+1.51	+16.1
20 6 15 42	23	8	−0 22.77	. . . .	19 53 24.12	. . . . . .	9.617	. .		
					COMET 1905 *b* (*Schaer*).					
1905										
Nov. 21 6 14 46	*24	15 , 3	−3 51.22	− 5 12.0	23 46 14.76	+52 53 28.7	*n*9.464	*n*0.238	+3.67	+30.6
21 11 52 1	25	8 , 6	−0 50.41	−12 52.9	23 44 50.44	+50 53 17.2	9.807	9.998	+3.52	+30.6
22 12 24 31	26	6 , 10	+0 44.84	− 1 58.6	23 40 23.96	+42 35 19.5	9.772	0.424	+3.16	+29.6
23 10 39 34	27	30 , 6	−1 54.42	− 0 20.2	23 37 39.65	+35 48 47.2	9.622	0.248	+2.99	+28.2
25 9 58 45	28	25 , 5	+2 49.84	− 3 35.1	23 34 3.11	+23 58 56.6	9.512	0.463	+2.72	+25.6
26 9 56 19	29	20 , 5	−2 35.59	+ 1 58.9	23 32 54.65	+19 13 36.8	9.504	0.543	+2.68	+24.1
29 10 42 54	30	20 , 4	+4 57.63	− 0 48.4	23 30 55.32	+ 8 36 28.2	9.585	0.687	+2.48	+21.0
30 7 16 37	31	35 , 7	−0 49.19	+ 2 53.6	23 30 38.17	+ 6 23 22.1	8.659	0.672	+2.48	+20.1
Dec. 1 8 41 18	32	20 , 4	+1 17.18	− 7 33.4	23 30 21.92	+ 3 59 38.8	9.336	0.704	+2.44	+19.4

The first observation of Comet 1905 *b* was made by Mr. HAMMOND. The observations marked with an asterisk were made with the 26-inch equatorial; the others with the 12-inch.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	16 5 48.69	+26 14 51.1	Camb., Eng., A.G. 7507	17	21 20 47.88	+10 55 56.0	Leipzig I, A.G. 8489
2	15 49 36.71	+26 4 18.0	" " " 7384	18	21 13 35.63	+ 9 50 38.1	Leipzig II, A.G. 10665
3	15 48 24.35	+25 28 3.4	" " " 7376	19	20 45 12.66	+ 5 11 14.4	½ {Leipzig II, A.G. 10391 + Albany, A.G. 7289.}
4	15 46 16.12	+20 37 53.4	Berlin A, A.G. 5128	20	20 27 28.63	+ 1 48 27.1	Albany, A.G. 7158
5	15 45 44.53	+19 59 5.0	½ {Berlin A, A.G. 5662 + Berlin B, A.G. 5425}	21	20 19 48.90	+ 1 3 29.4	½ {Albany, A.G. 7104 + Nicolajew, A.G. 5161}
6	15 44 13.34	+18 34 23.3	Berlin A, A.G. 5657	22	20 13 5.46	− 0 24 19.9	Nicolajew, A.G. 5122
7	20 56 7.60	− 3 49 0.0	Strassburg, A.G. Zones	23	19 53 45.38	− 3 29 36.1	Strassburg, A.G. Zones
8	20 53 43.64	− 1 12 49.5	" " "	24	23 50 2.31	+52 58 10.1	Camb., U.S., A.G. 8526
9	20 49 1.60	− 4 54 22.4	" " "	25	23 45 37.33	+51 5 39.5	" " " 8491
10	20 47 15.44	− 5 3 48.6	" " "	26	23 39 35.96	+42 36 48.5	Bonn, A.G. 18101
11	20 45 16.48	− 5 28 27.1	" " "	27	23 39 31.08	+35 48 39.2	Lund, A.G. 11309
12	20 48 16.53	− 5 31 22.2	" " "	28	23 31 10.55	+24 2 6.1	Berlin B, A.G. 9036
13	22 23 47.85	−11 23 15.8	Camb., U.S., A.G. Zones	29	23 35 27.56	+19 11 13.8	Berlin A, A.G. 9651
14	21 48 34.88	−10 13 43.6	" " " "	30	23 25 55.21	+ 8 36 55.6	Leipzig II, A.G. 11668
15	22 30 29.82	+20 11 56.2	Berlin A, A.G. 9236	31	23 31 24.88	+ 6 20 8.4	" " " 11701
16	21 26 51.17	+11 38 17.7	Leipzig I, A.G. 8535	32	23 28 32.30	+ 4 6 52.8	Albany, A.G. 8102

The star places from the Strassburg Zones, also the places of Nos. 13 and 14 from the Cambridge (U.S.) Zones, were furnished through the courtesy of the Directors of the Observatories at these places.

OBSERVATIONS, ELEMENTS AND EPHEMERIS OF COMET a 1906 (BROOKS).*

1906 Gr.M.T.	α	δ	Observer
	h m s	° ′ ″	
Jan. 29.8230	16 17 7.5	+52 5 18	Dugan (Princeton)
29.8299	16 17 7.0	+52 5 57	Maddrill (Lick)

These were received by telegraph from Harvard College Observatory; also the following correction to the Princeton position of Jan. 28, given on p. 50.

	α		δ
	h m s		° ′ ″
for	α = 16 18 7.6	.	δ = +50 23 58
put	16 18 4.3	,	+50 23 40

And the following elements and ephemeris, computed from observations on Jan. 28, 29 and 30, by CRAWFORD and CHAMPNEY, and telegraphed to the H. C. O. by Dr. LEUSCHNER.

ELEMENTS.

T = 1905 Dec. 19.47 Gr. M.T.

$$\left.\begin{aligned}\pi-\Omega &= 86^\circ\ 22'\\ \Omega &= 285\ 27\\ i &= 126\ 49\end{aligned}\right\}1906.0$$

q = 1.2826

EPHEMERIS, GREENWICH M.T.

1906	α	δ	Brightness
	h m s	° ′	
Jan. 31.5	16 15 8	+55 3	1.04
Feb. 4.5	16 7 12	+62 36	
8.5	15 50 24	+70 37	
12.5	15 5 52	+78 37	1.05

* *From Supplement to No. 582.*

OBSERVATIONS OF COMET a 1906 (BROOKS, *Jan. 26*),*

BY E. E. BARNARD.

Cent. Stand. Time	*	Comp.	Δα	Δδ	App. α	App. δ
1906 d h m s			m s	′ ″	h m s	° ′ ″
Jan. 27 17 7 0	1	6, 4	+0 47.60	+3 57.7	16 18 46.40	+48 58 19.8
27 18 7 23	2	6, 6	+0 7.04	−3 51.2	16 18 43.77	+49 2 15.7

Mean Places of Comparison-Stars for the beginning of the year.

*	α 1906.0	Red. to Appt.	δ 1906.0	Red. to Appt.	Authority
	h m s	s	° ′ ″	″	
1	16 18 0.21	−1.41	+48 54 30.8	−8.7	Bonn A.G.C. 10481
2	16 18 38.15	−1.42	+49 6 15.6	−8.7	Bonn A.G.C. 10490

With the second star, the Δα was measured direct. Δα = +69″.16 (6).

The comet is 9th magnitude. Large, round, and very diffused, but gradually brighter in the middle, to an ill-defined nucleus of 12½ magnitude. The measures were made with the 40-inch telescope.

* *From Supplement to No. 582.*

ELEMENTS AND EPHEMERIS OF COMET a 1906 (BROOKS),

BY ELEANOR A. LAMSON, U.S. NAVAL OBSERVATORY.

[Communicated by Rear-Admiral C. M. CHESTER, U.S.N., Superintendent.]

The following elements were deduced from observations made by Mr. RICE at Washington, on Jan. 29, 31; Feb. 1, 3. The observations of Jan. 29 and Feb. 3, were used directly, while the middle place, equidistant from these dates, was a normal derived from the four observations in question.

ELEMENTS.

T = Dec. 21.98542, 1905, G.M.T.

$$\left.\begin{aligned}\pi &= 15^\circ\ 47'\ 58''.3\\ \Omega &= 286\ 18\ 37.4\\ i &= 126\ 28\ 42.0\end{aligned}\right\}1906.0$$

q = 1.29432

Residuals (O−C): $\cos\beta\,\Delta\lambda = -0''.5$

$\Delta\beta = -1.3$

HELIOCENTRIC COORDINATES.

$$x = r\,[9.803432]\sin(243^\circ\ 16'\ 59''.4+v)$$
$$y = r\,[9.999813]\sin(331\ 14\ \ 6.5+v)$$
$$z = r\,[9.887776]\sin(\ 59\ 50\ 49.5+v)$$

EPHEMERIS.

G.M.T. 1906	α	δ	Light
	h m s	° ′ ″	
Feb. 14.5	14 2 8.9	+82 28 41	1.00
16.5	11 48 30.1	+84 43 9	0.96
18.5	8 56 38.0	+84 4 8	0.90
20.5	7 22 39.9	+81 19 38	0.85
22.5	6 39 45.7	+78 0 31	0.79
24.5	6 17 24.0	+74 37 52	0.73
26.5	6 4 24.3	+71 21 15	0.67
28.5	5 56 52.6	+68 14 23	0.61

Brightness of Feb. 14 taken as unit.

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DEFINITIVE ORBIT OF COMET 1819 II,

By HENRY A. PECK.

This comet appeared suddenly about the beginning of July, emerging from the rays of the sun a few days after passing perihelion. A controversy arose early in the comet's history as to the validity of the observations of certain astronomers who claimed to have seen the comet in transit across the solar disc. In discussing this question, HIND computed an orbit from observations by GAUSS, NICOLAI and STRUVE, and suggests the possibility that the orbit may be other than a parabola. The first recorded measurement of position was made at Milan, July 3. The comet was last seen by STRUVE and KNORR October 25, but so faint that it was impossible to use the micrometer. Up to the present time no one has collected and reduced the observations of this body, although for a considerable period it was an object of much interest in the scientific world. On account of the necessity of compressing the material within the limits of a single paper of moderate length, everything has been omitted that was not absolutely necessary to enable a future investigator to test the present work for systematic errors of computation.

With this object in view, the following skeleton of the ephemeris employed is appended for the dates of the normal places. The elements used as a basis for the discussion are essentially those of HIND, reduced to the epoch 1819.0. These elements of HIND may be found in *M.N.*, Vol. 36, and are as follows:

$$T = 1819 \text{ June } 27.71547 \text{ G.M.T.}$$

$$\left.\begin{aligned} \Omega &= 273^\circ\ 41'\ 32'' \\ i &= \ \ 80\ \ 44\ \ 38 \\ \omega &= \ \ 13\ \ 26\ \ 14 \end{aligned}\right\} 1819.0$$

$$\log q = 9.533233$$

EQUATORIAL COÖRDINATES.

$$x = [9.237921]\, r \sin [\ \ 35^\circ\ 17'\ 50''.8 + v]$$
$$y = [9.996725]\, r \sin [260\ \ 42\ \ 30.1 + v]$$
$$z = [9.996730]\, r \sin [349\ \ 50\ \ 18.2 + v]$$

The positions of the sun were found with the assistance of NEWCOMB'S "*Tables of the Sun*," as follows:

	True long. of ☉	log R	Lat. of ☉	Sid. Time
	° ′ ″			h m s
July 5	102 33 39.4	0.0072117	−0.21	6 50 34.6
11	108 16 45.6	71438	−0.08	7 14 14.0
19	115 54 42.5	69549	+0.78	45 46.4
27	123 33 21.1	66154	+0.30	8 17 18.9
Aug. 4	131 12 37.7	61109	−0.39	48 51.3
16	142 43 37.2	52694	+0.69	9 36 10.0
26	152 22 7.9	43516	−0.19	10 15 35.5
Sept. 14	170 48 16.6	22839	+0.54	11 30 30.1
24	180 35 12.3	0.0010744	−0.53	12 9 55.6
Oct. 13	199 19 5.5	9.9987028	+0.29	13 24 50.1

From these data the following coördinates of the sun were computed, using NEWCOMB'S values of the obliquity of the ecliptic. The rectangular coördinates are referred to the true equinox and equator. The equation of time is to be added to apparent time.

	Equation of Time	X	Y	Z
	m s			
July 5	+ 4 0.5	−0.221120	+0.910340	+0.395166
11	57.6	318852	885462	384368
19	5 49.8	444042	838404	363945
27	6 8.5	561234	776164	336927
Aug. 4	5 48.7	668209	699911	303822
16	4 8.0	805473	562310	244098
26	+ 1 47.2	894874	429709	186533
Sept. 14	− 4 18.4	0.992354	+0.147358	+0.063969
24	7 47.7	1.002424	−0.009416	−0.004089
Oct. 13	−13 33.1	−0.940880	−0.302551	−0.131335

For the reductions from the equinox and ecliptic of 1819.0 to that of the date of observation the following values of the independent star numbers were computed from BAUSCHINGER'S *Tafeln zur Theoretischen Astronomie*. These dates, as well as the preceding, are with reference to Greenwich Mean Noon.

	f	G	log g	H	log h	i
	″	° ′		° ′		
July 5	+19.60	315 51	1.0755	168 26	1.3094	+1.76
11	20.62	317 2	0888	163 7	3076	2.55
19	21.94	318 24	1063	155 58	3044	3.55
27	23.20	319 31	1232	148 40	3003	4.51
Aug. 4	24.38	320 25	1391	141 12	2956	5.37
16	26.02	321 29	1608	129 41	2882	6.48
26	27.23	322 12	1763	119 41	2824	7.19
Sept. 14	29.29	323 30	2005	99 59	2748	8.03
24	30.30	324 17	2110	89 21	2737	8.14
Oct. 13	+32.26	326 14	1.2278	69 5	1.2782	+7.68

The positions of the comet as given by Hind's elements for the dates of the normal places are,

	α app.	δ app.	log r	log Δ	Ab. time
	h m s	° ′ ″			
July 5	6 58 5.25	+45 48 31.0	9.6037	9.9408	0.00504
11	7 20 40.94	50 19 43.8	7053	0.0267	614
19	43 20.70	51 50 40.5	8265	1117	747
27	8 0 22.20	47 25.0	9224	1718	858
Aug. 4	13 53.01	20 49.2	9.9987	2157	950
16	29 51.14	50 40 22.7	0.0890	2621	1057
26	40 11.75	19 55.6	1492	2879	1121
Sept. 14	53 14.66	29 40.9	2395	3158	1195
24	56 22.43	51 3 51.4	2785	3222	1213
Oct. 13	8 52 59.31	+53 6 16.6	0.3412	0.3242	0.01219

Only a comparatively small number of observations are given which include the designation of the stars upon which the comet position depends. The places of these stars have in general been found by a comparison of Lalande and Groombridge with the A.G. Harvard and Bonn Catalogs. No. 2 and No. 19 belong to the Fundamental Catalog. No. 5 is discussed by Porter in the Cincinnati observations. The epoch of Groombridge is so near the time of the comet's appearance, that I have often given it more weight than it might receive in connection with more modern observations. The positions as used are:

	α 1819.0	δ 1819.0		α 1819.0	δ 1819.0		α 1819.0	δ 1819.0
	h m s	° ′ ″		h m s	° ′ ″		h m s	° ′ ″
1	6 21 54.04	+44 41 10.0	11	7 15 15.89	+48 16 39.5	21	8 23 39.01	49 59 23.5
2	33 41.06	43 44 43.6	12	16 9.56	50 2 1.2	22	30 2.36	50 37 45.4
3	44 23.67	45 18 58.5	13	19 25.78	51 41 20.7	23	50 18.00	39 3.4
4	54 5.48	43 8 1.2	14	25 34.51	50 55 41.1	24	53 41.63	. . . .
5	7 2 22.28	47 32 55.3	15	30 19.32	51 0.0	25	42.64	19 26.7
6	3 40.29	48 46 21.1	16	32 23.59	50 26 53.6	26	57 14.68	50 33 24.9
7	11 35.24	41 0 34.2	17	39 31.57	51 45 22.2	27	59 16.37	51 10 11.8
8	12 16.40	48 53 22.7	18	44 25.15	51 49 17.3	28	9 10 8.14	50 18 24.0
9	13 1.80	49 33 28.3	19	54 47.57	52 1 1.2	29	9 20 53.92	+50 13 46.4
10	7 14 7.61	+43 36 22.8	20	7 55 27.80	+51 49 47.2			

The following is a complete list of the published observations of this comet so far as they have come to my notice:

Berlin. A series by Bode, extending through July, and published in *Von Zach*, Vol. II, and the *B.J.* for 1822. The first five observations were made on the two-foot Troughton Circle, the comet being observed differentially. The remaining observations were made with a ring micrometer, on the 3.5-foot Dolland telescope.

Bremen. A series of observations by Olbers, made with a ring micrometer, from July 6 to Oct. 12. The observations for July are reduced from the Schur–Stichtenoth edition of Olbers's Works. The remainder are taken from the *B.J.* for 1823, and are of necessity mere copies, as no data are given for a new reduction.

Cracow. These observations are contained in the *B.J.* for 1822. No use has been made of them on account of their inaccuracy.

Dorpat. This series by Struve and his assistants is one of the most important in existence. The right-ascensions during July and August were observed in the meridian at lower culmination. The declinations for the same period are deduced from zenith observations made with a repeating circle. At first the absolute zenith distance of the

comet was measured. Later, on account of the failing light of the comet, the difference of its zenith distance from that of the Pole Star was observed. The September and October observations were made with the ring micrometer. The star for October 12 can not with certainty be identified. In the *B.J.* for 1823 OLBERS states that he and STRUVE used the same star, and then gives the position of the comet as deduced by STRUVE. This position is the one used. Otherwise the observations have been reduced from the originals in *Dorpat Observations*, Vol. II, using the clock corrections given by STRUVE in Vol. III. The declinations contain the parallax.

Florence. Two observations recorded by VON ZACH. No details are given, but they were evidently transits at lower culmination.

Genoa. A series of July observations given by VON ZACH. Neither the observer's name, nor any other particulars, have been given. No use has been made of the series on account of the inexactness of the observations.

Göttingen. The first observation is by VON ZACH. During the interval July 19–26, GAUSS observed on the Repsold Circle at lower culmination. After July 26, GAUSS observed the transits on the Reichenbach instrument, and HARDING secured the declination with the Repsold Circle. The observations are published by VON ZACH, and in the *B.J.* for 1822.

Greenwich. A series of observations by POND. The meridian observations are published in the *Greenwich Observations* for 1819. The remainder are given by VON ZACH and the *B.J.* for 1822.

Hamburg. A single observation by REPSOLD, of which no use has been made.

Kremsmünster. A series of July and August observations, made with a three-foot Dolland telescope supplied with a net. The right-ascensions have been used, but not the declinations. These latter are evidently affected by some systematic error. The observations are published in the *B.J.* for 1822.

Mannheim. Nine observations by NICOLAI, given by VON ZACH and the *B.J.* for 1822. Except the last two all were made in the meridian.

Milan. These observations have been completely revised, using the material furnished by VON ZACH. The observer was CARLINI, and the instrument was an equatorial sector. After the observations had been reduced, they were found to contain a residual that varied its sign according as the transit of the comet had been observed above or below that of the comparison-star. Fortunately the observations had been made on a few days with two stars, so that this error could be computed and eliminated. An average inclination of the vertical thread of the instrument used by CARLINI, amounting to a quarter of a degree, seems to explain the discrepancies.

Munich. The observations were made by SOLDNER upon a variety of instruments. They are to be found in *Von Zach*, and the declinations are not of very great worth.

Padua. A long series of observations published anonymously in *Von Zach*.

Palermo. Another long series published by CACCIATORE in the *B.J.* for 1823.

Paris. Two series published in the *Connaissance des Temps* and the first volume of the *Paris Observations*. The first series consists of meridian observations between July 5 and August 15. The other series are extra meridian observations extending throughout the months of July and August.

Prag. The observations of HALLASCHKA have not been used on account of their inexactness. The observations of DAVID and BITTNER have been reduced from the *B.J.* for 1823. They were made with a seven-foot Dolland equatorial. No account has been taken of the declinations, and the temptation to exclude the right-ascensions was very strong.

Seeberg (Gotha). A series of July and August observations by LINDEMAN and ENCKE. In general they are made with a ring micrometer, though throughout July the observation was often repeated on the meridian instrument. The series is to be found in *Von Zach* and the *B.J.* for 1822.

Vienna. These observations are to be found in the *B.J.* for 1823. A portion of the observations were made with a Dolland telescope, in which the sight field was used as a ring micrometer. The remainder were made upon a transit instrument. No account has been taken of the declinations.

Viviers. According to the *Bulletin Astronomique* there are observations of this comet by FLAUGERGUES, extending from July 5 to September 2, which have never been published. They would, however, be of comparatively small importance, as this portion of the orbit is well covered.

Wilna. In the *B.J.* for 1823 a series by SNIADECKI from July 6 to August 10 is recorded. No particulars are given, but the observations were probably made upon a meridian instrument.

In comparing the observations with the ephemeris, the times of observation have been corrected for aberration, and also reduced to Greenwich Mean Time.

Date	Place	α apparent	π	O – C Δα cos δ	Wt.	δ apparent	π	O – C Δδ	Wt.	*
July		h m s	s	″		° ′ ″	″	″		
2.46258	Berlin	6 47 0.00	. .	+21.9	1	+41 56 0.0	+11.0	+23.7	1	Mer.
3.35638	Milan	51 1.41	+0.43	+38.3	0	43 28 26.9	9.3	– 7.9	3	7
3.36607	Padua	50 53.00	.41	–81.2	0	14 40.0	9.7	–14′ 53″.0	0	. .
3.37753	Milan	51 3.62	.43	+ 5.3	2	30 35.8	9.3	– 2.1	3	10
3.38125	Vienna	4.51	.24	– 1.5	1	. . . . .	. .	. .	.	4
3.38220	Munich	5.00	.34	+ 2.2	3	31 0.0	10.0	7.2	1	. .
3.42581	Dorpat	17.53	. .	8.1	3	35 39.7	. .	+11.2	2	Mer.
3.43431	Paris	20.10	.24	14.8	2	35 59.1	10.0	– 7.6	2	2
3.44897	Seeberg	23.20	+0.08	+ 4.6	2	37 40.0	10.6	+ 9.0	2	. .
3.47023	Seeberg	28.13	. .	– 1.0	2	. . . . .	. .	. .	.	Mer.
3.47467	Göttingen	29.40	. .	3.7	3	. . . . .	. .	. .	.	Mer.
3.47653	Mannheim	30.07	. .	1.1	3	40 0.0	10.6	– 8.1	1	Mer.
3.50003	Greenwich	35.60	. .	8.7	2	41 13.0	10.7	68.7	0	Mer.
3.56151	Paris	51 53.23	–0.27	+3.1	2	47 51.1	10.0	–19.8	2	3, 1
3.63174	Palermo	52 11.20	–0.68	– 7.5	2	55 14.0	7.6	+28.3	2	. .
4.34319	Padua	55 24.13	+0.48	+79.0	0	44 54 36.0	8.8	–2′ 27″	0	. .
4.34947	Milan	19.19	.45	8.5	2	57 50.3	8.6	+18.3	3	7
4.36638	Milan	28.53	.45	62.4	0	58 55.7	8.6	0.3	3	7
4.37342	Munich	26.53	.37	20.3	3	59 56.0	9.5	+24.6	1	. .
4.38462	Milan	30.77	.45	36.2	2	45 0 11.1	8.6	–18.1	3	9
4.40207	Paris	32.43	.35	4.4	2	1 56.1	9.5	+ 4.6	2	2
4.40421	Seeberg	35.13	.25	26.0	2	2 24.0	9.8	21.7	2	. .
4.42595	Dorpat	39.23	. .	7.4	3	4 20.6	. .	22.5	2	Mer.
4.42746	Vienna	38.47	+0.10	– 7.3	1	. . . . .	. .	. .	.	. .
4.46287	Berlin	49.27	. .	+12.9	1	7 24.0	10.2	+36.1	1	Mer.
4.47035	Seeberg	50.47	. .	+ 5.2	2	6 54.0	10.3	–31.1	0	Mer.
4.47646	Palermo	51.43	–0.03	– 0.6	2	8 1.0	7.8	+14.9	2	. .
4.47665	Mannheim	51.87	. .	+ 2.8	3	20.0	10.3	+85.0	0	Mer.
5.34405	Padua	59 42.40	+0.47	+1′ 54″.5	0	46 12 14.0	8.5	–48.9	0	. .
5.34881	Milan	33.59	.47	10.7	2	13 26.0	8.0	+ 0.6	3	9
5.36306	Milan	37.27	.47	11.8	2	14 33.9	8.0	9.0	3	9
5.36565	Munich	37.80	.39	10.3	3	49.0	9.0	+19.9	1	. .
5.42594	Dorpat	53.29	. .	10.3	3	18 43.7	. .	– 6.9	2	Mer.
5.43493	Seeberg	56.07	+0.12	+17.1	2	19 22.0	9.7	+ 2.7	2	. .
5.45441	Vienna	58.40	. .	–10.1	2	. . . . .	. .	. .	.	Mer.
5.46937	Palermo	7 0 2.73	–0.02	– 3.2	2	21 51.0	7.6	16.7	2	. .
5.47036	Seeberg	3.80	. .	+ 4.6	2	56.0	9.9	11.3	2	Mer.
5.47666	Mannheim	5.20	. .	+ 2.8	3	22 20.0	9.8	+10.1	1	Mer.
5.49995	Paris	10.20	. .	– 5.4	3	23 43.0	9.9	– 1.6	1	Mer.
5.51991	Greenwich	9.00	–0.08	69.4	0	25 2.0	9.7	4.1	2	. .
6.32231	Milan	3 30.79	+0.48	10.7	2	47 17 17.5	7.5	–1′ 40″	0	9
6.33616	Milan	21.08	.48	–2′ 23″.5	0	16 34.9	7.5	+ 3.6	3	9
6.33882	Padua	37.33	.49	+15.5	2	15 21.0	8.2	–1′ 16″.9	0	. .
6.35249	Munich	38.60	.42	– 5.8	3	17 19.0	8.4	– 6.7	1	. .
6.37350	Munich	45.80	.35	+15.1	3	18 50.0	8.8	+11.2	1	. .
6.37620	Vienna	39.00	.30	–61.3	0	. . . . .	. .	. .	.	. .
6.38830	Bremen	48.00	.30	+ 2.3	3	20 29.8	9.1	– 5.4	2	5
6.40386	Seeberg	52.20	.20	4.5	2	29.0	9.4	+ 4.9	2	. .
6.42573	Prague	1 1.63	+0.02	35.6	0	. . . . .	. .	. .	.	5
6.42584	Dorpat	3 58.45	. .	+12.2	3	22 9.4	. .	+20.1	2	Mer.
6.42960	Wilna	53.27	. .	–49.8	0	21 44.0	9.6	–1′ 48″.8	0	Mer.
6.45452	Vienna	4 4.33	. .	+ 1.9	1	. . . . .	. .	. .	.	Mer.
6.46278	Berlin	7.33	. .	+12.3	1	24 26.0	9.4	+38.7	1	Mer.
6.46934	Palermo	7.07	–0.01	– 5.6	2	18.0	7.6	13.3	2	. .
6.47027	Seeberg	8.67	. .	+ 7.5	2	33.0	9.7	19.5	2	Mer.
7.32305	Milan	7 29.74	+0.48	9.4	2	48 9 37.9	7.2	11.7	3	9
7.33689	Milan	33.38	.48	+13.7	2	10 20.0	7.2	13.1	3	9
7.34914	Kremsmünster	33.20	.39	–18.1	1	. . . . .	. .	. .	.	. .
7.35999	Munich	37.73	.38	+ 2.6	3	11 34.0	8.3	12.9	1	. .
7.37241	Vienna	41.81	.31	14.0	1	. . . . .	. .	. .	.	6
7.41560	Seeberg	7 7 52.47	+0.38	+21.4	2	+48 13 54.0	+ 8.3	+ 1.1	2	. .

Date July	Place	α apparent h m s	π s	O − C Δα cos δ ″	Wt.	δ apparent ° ′ ″	π ″	O − C Δδ ″	Wt.	*
7.42566	Dorpat	7 7 54.35	. .	+13.1	3	+48 14 53.6	. .	+23.6	2	Mer.
7.44937	Prague	8 0.18	+0.01	23.9	1	. . . .	. .	. .	.	11
7.45432	Vienna	7 59.67	. .	0.0	2	. . . .	. .	. .	.	Mer.
7.47280	Palermo	8 3.93	−0.02	0.8	2	16 50.0	+ 7.2	15.0	2	. .
7.48966	Greenwich	9.50	+0.04	19.0	2	17 41.0	8.8	15.8	2	. .
7.56766	Padua	33.07	−0.39	+1′ 9″.0	0	21 16.0	8.4	+11.8	1	. .
8.33792	Milan	11 19.35	+0.49	− 2.4	2	. . . .	. .	. .	.	9
8.34697	Milan	22.41	.49	+ 7.9	2	55 7.5	6.9	− 1.6	3	9, 12
8.35391	Milan	23.73	.49	5.7	2	33.1	6.9	+ 8.8	3	12
8.36519	Padua	27.33	.39	+15.3	2	56 21.0	8.1	32.6	1	. .
8.37214	Vienna	27.46	.36	− 5.0	1	. . . .	. .	. .	.	8
8.44750	Greenwich	45.20	.31	+11.2	2	59 15.0	8.1	8.8	2	. .
8.45069	Paris	45.39	+0.15	+ 4.0	2	31.3	8.7	+ 1.7	2	9
8.45399	Vienna	44.47	. .	−13.6	2	. . . .	.	. .	.	Mer.
8.47097	Palermo	52.60	−0.02	+30.1	2	58.0	6.8	− 2.0	2	. .
8.49309	Paris	56.30	. .	17.4	3	57.3	10.0	+ 2.6	1	Mer.
9.32158	Milan	14 55.40	+0.49	11.8	2	49 31 20.7	6.6	5.8	3	9, 12
9.33679	Milan	58.82	.49	13.5	2	59.2	6.6	+13.7	3	9, 12
9.33740	Padua	15 0.40	.46	27.1	2	44.0	7.2	− 1.4	1	. .
9.39132	Paris	10.69	.35	14.1	2	33 44.8	6.4	+ 9.8	2	9, 12
9.42496	Dorpat	18.68	. .	19.0	3	35 10.1	. .	22.1	2	Mer.
9.42874	Wilna	20.00	. .	24.0	1	19.1	8.4	32.3	1	Mer.
9.43817	Bremen	20.82	+0.12	13.5	3	8.0	8.5	1.6	2	9, 12
9.47901	Palermo	29.83	−0.04	16.2	2	36 36.0	6.5	11.7	2	. .
9.49267	Paris	32.68	. .	15.2	3	56.3	8.6	3.1	1	Mer.
10.32109	Milan	18 24.88	+0.49	14.7	2	50 2 10.0	6.3	13.6	3	9, 12
10.33540	Milan	27.66	.49	+13.4	2	32.1	6.2	+12.9	3	12
10.33619	Padua	25.60	.45	− 8.5	2	1 58.0	6.9	−22.6	1	. .
10.35906	Paris	33.13	.43	+18.7	2	3 5.9	6.9	+ 8 1	2	12
10.35964	Padua	26.60	.38	−45.7	0	3 13.0	7.4	14.1	1	. .
10.39565	Prague	39.14	.04	+ 0.5	1	. . . .	. .	. .	.	12
10.40872	Vienna	41.87	+0.30	4.0	1	. . . .	. .	. .	.	. .
10.42821	Wilna	47.93	. .	21.1	1	5 14.8	8.1	23.5	1	Mer.
10.45819	Prague	55.52	. .	35.4	1	. . . .	. .	. .	.	Mer.
10.47929	Palermo	58.03	−0.04	18.7	2	6 37.0	6.1	23.9	2	. .
11.32182	Milan	21 46.49	+0.47	23.3	2	27 19.9	6.2	12.1	3	12
11.33786	Milan	49.22	.47	19.3	2	44.5	6.2	10.3	3	12
11.34835	Padua	49.80	.40	4.1	2	59.0	7.0	10.9	1	. .
11.39076	Seeberg	59.80	.25	18.7	2	29 15.0	7.5	+29.8	2	. .
11.42776	Wilna	22 9.15	. .	36.5	1	28 31.4	7.9	−1′ 3″.3	0	Mer.
11.45028	Paris	12.73	+0.15	+30.0	2	30 4.3	7.9	− 0.1	2	12
11.46090	Berlin	9.60	. .	−21.0	0	50.0	8.0	+30.7	1	Mer.
11.47454	Mannheim	16.00	. .	+14.4	3	31 20.0	8.0	42.7	0	Mer.
11.47922	Palermo	18.03	−0.03	25.3	2	30 56.0	5.9	13.3	2	. .
11.49155	Paris	20.22	. .	23.3	3	31 2.7	8.1	3.4	1	Mer.
11.49802	Greenwich	20.00	. .	9.5	2	22.0	8.0	13.6	2	Mer.
12.32666	Milan	24 59.48	+0.43	20.5	2	48 18.9	6.1	21.2	3	12
12.34008	Palermo	25 1.93	.45	19.6	2	25.0	6.6	13.0	2	. .
12.39360	Seeberg	13.07	.24	28.0	2	49 20.0	7.4	9.6	2	. .
12.42250	Bremen	16.24	+0.16	+ 5.8	3	51.9	7.5	9.8	2	14
12.42697	Wilna	16.56	. .	− 0.6	1	48.4	7.7	1.7	1	Mer.
12.46011	Berlin	24.13	. .	+12.2	1	50 55.0	7.7	+31.9	1	Mer.
12.52777	Padua	42.20	−0.23	+1′ 1″.4	0	43.0	7.5	−53.1	0	. .
13.32953	Milan	28 4.97	+0.44	25.1	2	51 4 59.9	6.1	+15.4	3	12
13.34909	Munich	8.33	.37	+23.2	3	5 37.0	6.7	31.8	1	. .
13.36479	Vienna	8.27	+0.28	− 4.8	1	. . . .	. .	. .	.	. .
13.42237	Dorpat	21.91	. .	+23.2	3	6 26.8	. .	12.7	2	Mer.
13.44973	Vienna	25.80	. .	13.5	2	. . . .	. .	. .	.	Mer.
13.47333	Palermo	31.80	−0.05	30.2	2	7 15.0	5.8	+ 6.2	2	. .
13.49005	Paris	34.07	. .	23.2	3	0.0	7.6	− 5.9	1	Mer.
13.49384	Padua	7 28 34.00	−0.11	+15.1	2	+51 7 3.0	+ 7.6	− 6.0	1	. .

Date	Place	α apparent	π	O — C Δα cos δ	Wt.	δ apparent	π	O — C Δδ	Wt.	*
July		h m s	s	″		° ′ ″	″	″		
13.49653	Greenwich	7 28 34.50	. .	+16.2	2	+51 7 31.0	+ 7.6	+19.7	2	Mer.
13.55239	Paris	45.88	−0.21	26.1	2	8 12.1	7.3	13.3	2	15
14.32871	Milan	31 2.40	+0.44	30.8	2	18 26.7	6.1	+14.9	3	12
14.34229	Padua	1.67	.39	+ 1.1	2	13.0	6.6	− 8.3	1	. .
14.38570	Vienna	8.80	.21	− 3.8	1	. . . . .	. .	. .	.	. .
14.40429	Berlin	16.17	.16	+37.4	1	19 28.0	7.1	+22.9	1	. .
14.40904	Paris	14.17	.26	9.1	2	28.7	6.9	21.4	2	15
14.41497	Bremen	15.15	+0.17	10.7	3	32.1	7.1	19.9	2	13, 15, 16
14.42525	Wilna	17.17	. .	11.3	1	56.5	7.3	+36.0	1	Mer.
14.46446	Florence	26.47	. .	32.1	1	46.0	7.5	− 1.1	1	Mer.
14.48361	Palermo	28.97	−0.06	21.3	2	20 18.0	5.7	+16.8	2	. .
14.48919	Paris	29.83	. .	23.7	3	19 56.6	7.4	− 8.0	1	Mer.
15.33187	Milan	33 53.06	+0.43	+31.7	2	29 8.7	+5.8	+13.9	2	12
15.34708	Munich	55.40	.35	+29.3	3	31.0	6.3	27.6	1	. .
15.36061	Vienna	53.87	.28	− 6.8	1	. . . . .	. .	. .	.	. .
15.41139	Bremen	34 3.77	.17	+ 5.8	3	30 2.1	6.9	24.0	2	15, 16
15.42432	Wilna	33 59.90	. .	−51.9	0	31 10.6	7.0	+1′ 24″.6	0	Mer.
15.45797	Greenwich	34 19.40	+0.11	+78.9	0	30 11.0	7.1	6.4	2	. .
15.48347	Palermo	17.80	−0.06	23.5	2	37.0	5.6	18.3	2	. .
15.48823	Paris	18.53	. .	+22.9	3	47.0	7.2	25.9	1	Mer.
16.36529	Kremsmünster	36 37.60	+0.27	− 9.6	1	. . . .	. .	. .	.	. .
16.38722	Vienna	42.87	+ .19	+13.2	1	. . . .	. .	. .	.	. .
16.44821	Vienna	53.20	. .	18.0	2	. . . .	. .	. .	.	Mer.
16.46393	Seeberg	57.80	. .	29.1	2	38 34.0	6.8	22.9	2	Mer.
16.47022	Mannheim	57.93	. .	21.0	3	40.0	7.0	26.5	1	Mer.
16.47730	Seeberg	37 0.40	−0.04	33.0	2	34.0	6.8	17.3	2	. .
16.48721	Paris	0.97	. .	24.1	3	50.7	7.1	30.1	1	Mer.
17.32897	Milan	39 12.70	+0.41	25.3	2	43 50.4	5.6	15.4	3	12, 19
17.33904	Palermo	14.60	.40	28.1	2	51.0	5.8	+12.6	2	. .
17.36401	Padua	27.07	.30	+1′ 47″.2	0	3.0	6.3	−43.1	0	. .
17.46913	Mannheim	34.53	. .	23.4	3	45 0.0	6.9	+40.8	1	Mer.
17.48613	Paris	18.13	.33	16.9	2	44 7.0	6.0	17.6	2	19
18.33128	Milan	41 45.18	.39	29.8	2	48 39.6	5.5	23.2	3	19
18.36765	Padua	50.33	.28	26.4	2	40.0	6.2	27.8	1	. .
18.37756	Vienna	49.33	.21	3.2	1	. . . .	. .	. .	.	. .
18.39704	Prague	57.96	.03	53.9	0	. . . .	. .	. .	.	19
18.41455	Seeberg	57.87	.13	30.5	2	55.0	6.5	20.4	2	. .
18.41762	Bremen	57.86	.14	26.0	3	52.8	6.5	16.7	2	17, 18
18.42277	Paris	58.56	.19	26.3	2	43.6	6.6	7.5	2	19
18.44602	Vienna	42 1.60	. .	19.2	2	. . . . .	. .	. .	.	Mer.
18.46171	Seeberg	5.00	. .	30.4	2	49 13.0	6.7	27.8	2	Mer.
18.46800	Mannheim	5.20	. .	23.5	3	40.0	6.7	53.4	0	Mer.
18.47500	Greenwich	10.50	+0.05	69.4	0	26.0	6.7	37.9	2	. .
18.48123	Palermo	7.83	−0.05	30.3	2	12.0	5.1	+22.0	2	. .
18.48498	Paris	7.90	. .	25.5	3	48 46.2	6.8	− 3.7	1	Mer.
19.33019	Milan	44 11.44	+0.37	31.6	2	51 48.8	5.4	+17.8	3	19
19.34273	Padua	13.33	.34	32.3	2	58.0	5.7	25.8	1	. .
19.34820	Munich	13.60	.31	27.0	3	55.0	5.8	+22.0	1	. .
19.36811	Kremsmünster	14.60	.24	9.2	1	. . . . .	. .	. .	.	. .
19.37510	Prague	19.79	.04	+46.5	1	. . .	. .	. .	.	19
19.38226	Vienna	13.33	.19	−22.1	0	. . . . .	. .	. .	.	. .
19.40191	Berlin	20.33	.14	+16.6	1	29.0	6.4	−11.6	0	. .
19.42255	Seeberg	25.00	.11	32.2	2	59.0	6.5	+15.3	2	.
19.42632	Bremen	27.03	+0.14	58.0	0	57.5	6.3	13.4	2	19
19.44475	Vienna	27.27	. .	22.6	2	. . . . .	. .	. .	.	Mer.
19.45903	Florence	29.33	. .	22.9	1	53.0	6.7	4.2	1	Mer.
19.46050	Seeberg	30.53	. .	32.0	2	52 20.0	6.6	30.8	2	Mer.
19.46267	Göttingen	29.73	. .	+21.8	3	12.0	6.5	22.4	2	Mer.
19.47173	Palermo	28.53	0.05	5.6	2	+51 52 30.0	+ 5.1	+ 8.8	2	. .
20.34510	Kremsmünster	46 32.20	+0.29	+ 8.3	1	. . . . .	. .	. .	.	. .
20.34779	Prague	7 46 33.36	+0.05	+13.0	1	. . . . .	. .	. .	.	19

Date July	Place	α apparent h m s	π s	O — C Δα cos δ ″	Wt.	δ apparent ° ′ ″	π ″	O — C Δδ ″	Wt.	*
20.37557	Vienna	7 46 36.87	+0.19	+11.6	1	. . . .	. .	. .	.	. .
20.41487	Dorpat	41.79	. .	33.0	3	+51 54 22.1	. .	+37.3	2	Mer.
20.41845	Wilna	41.96	. .	29.8	1	9.7	+ 6.3	+30.8	1	Mer.
20.42502	Berlin	43.13	0.07	5.1	1	53 34.0	5.4	— 6.2	1	. .
21.32761	Milan	48 48.56	.36	31.0	2	54 56.6	5.2	+24.2	3	19
21.33802	Milan	50.48	.36	+36.8	2	51.0	5.2	18.3	3	19
21.46007	Göttingen	49 2.27	. .	— 9.4	0	58.0	6.3	22.5	2	Mer.
21.46990	Greenwich	48 39.00	.05	—3′ 53″.4	0	55 0.0	6.3	21.5	2	. .
22.35418	Paris	51 4.08	.33	+31.6	2	51 52.1	5.3	11.8	2	19
22.39115	Seeberg	9.93	.17	39.6	2	13.0	5.9	3.9	2	. .
22.41152	Dorpat	12.08	. .	33.9	3	. . . .	. .	. .	.	Mer.
22.45657	Seeberg	17.53	. .	30.2	2	55 5.0	6.2	27.4	2	Mer.
22.45877	Göttingen	18.27	. .	34.4	3	54 56.0	6.1	18.3	2	Mer.
22.48634	Greenwich	21.00	. .	26.7	2	55 24.0	6.2	47.0	0	Mer.
23.33005	Palermo	53 8.60	+0.36	31.0	2	54 6.0	5.2	1.1	2	. .
23.35840	Paris	12.39	—0.12	+28.5	2	25.5	6.1	22.9	2	19
23.36400	Vienna	8.07	.21	—14.9	0	. . . .	. .	. .	.	. .
23.41081	Dorpat	11.78	. .	+37.3	3	45.8	. .	40.2	2	Mer.
23.41107	Padua	18.73	+0.12	27.9	2	46.0	6.1	46.5	0	. .
23.41532	Berlin	16.33	.19	1.9	1	21.0	5.7	+21.2	1	. .
23.47845	Paris	27.14	. .	26.1	3	53 46.5	6.1	— 9.1	1	Mer.
23.48192	Greenwich	27.00	. .	+18.0	2	54 19.0	6.1	+73.8	0	Mer.
24.33558	Milan	55 13.63	+0.33	+35.9	2	53 20.5	+5.1	+28.6	3	19
24.33565	Kremsmünster	11.80	.28	18.6	1	. . . .	. .	. .	.	. .
24.38507	Paris	19.77	.24	36.3	2	18.1	5.5	30.9	2	19
24.40430	Bremen	22.14	+0.13	34.5	3	20.0	5.8	34.0	2	19, 20
24.40935	Dorpat	22.23	. .	29.6	3	52 59.6	. .	+ 9.2	2	Mer.
24.41301	Wilna	21.47	. .	18.7	1	39.0	5.9	— 5.2	1	Mer.
24.47700	Paris	30.55	. .	30.4	3	51.5	6.1	+13.4	1	Mer.
24.48348	Greenwich	30.00	. .	18.0	2	53 3.0	6.0	25.3	2	Mer.
24.61872	Palermo	50.40	—0.46	50.9	2	52 32.0	4.1	5.4	2	. .
25.33224	Milan	57 13.55	+0.33	38.8	2	51 29.9	5.1	21.4	3	19
25.34947	Kremsmunster	15.80	.24	39.6	1	. . . .	. .	. .	.	. .
25.35848	Padua	16.00	.25	31.8	2	41.0	5.5	36.0	1	. .
25.37879	Vienna	16.73	.15	15.5	1	. . . .	. .	. .	.	. .
25.39152	Paris	20.05	.22	33.0	2	19.8	5.5	+18.7	2	19
25.39633	Munich	21.13	.14	37.1	3	50 52.0	5.8	— 8.2	1	. .
25.41161	Wilna	20.80	. .	15.9	1	51 38.4	5.8	+40.1	1	Mer.
25.41931	Bremen	23.29	+0.09	30.6	3	32.4	5.8	34.5	2	19, 20
25.43646	Vienna	25.40	. .	31.4	2	. . . .	. .	. .	.	Mer.
25.47553	Paris	29.98	. .	31.2	3	18.5	6.0	28.1	1	Mer.
25.48200	Greenwich	30.00	. .	25.2	2	23.0	5.9	+32.6	2	Mer.
25.61604	Palermo	46.80	—0.45	30.1	2	50 31.0	4.1	— 4.1	2	. .
26.36865	Prague	59 13.83	+0.04	31.6	1	. . . .	. .	. .	.	19
26.37755	Vienna	11.40	.22	2.8	1	. . . .	.	. .	.	. .
26.40523	Bremen	17.81	+0.12	+29.7	3	49 21.0	5.6	+32.3	2	20
26.40673	Dorpat	. .	. .	. .	.	12.9	. .	18.6	2	. .
26.41000	Wilna	13.28	. .	—17.5	0	0.3	5.6	13.1	1	Mer.
26.43546	Vienna	17.93	. .	+19.8	2	. . . .	. .	. .	.	Mer.
26.45292	Göttingen	24.67	. .	42.5	3	48 49.0	5.8	8.1	2	Mer.
26.47402	Paris	27.42	. .	45.4	3	49 10.0	5.9	32.3	1	Mer.
26.48048	Greenwich	26.00	. .	25.7	2	5.0	5.8	+28.1	2	Mer.
26.60812	Palermo	41.07	—0.43	26.3	2	48 13.0	4.0	— 7.0	1	. .
27.33118	Milan	8 1 8.09	+0.30	+1′ 24″.1	0	46 38.3	5.1	+10.7	3	19
27.34079	Padua	4.13	.27	36.9	2	47 9.0	5.1	42.9	1	. .
27.34603	Kremsmünster	0.87	.23	1.3	1	. . . .	. .	. .	.	. .
27.37395	Prague	8.37	.03	39.8	1	. . . .	. .	. .	.	19
27.38647	Vienna	5.60	.20	2.9	1	. . . .	. .	. .	.	. .
27.39849	Seeberg	9.80	+ .13	28.8	2	. . . .	. .	. .	.	. .
27.40482	Dorpat	11.70	. .	+38.7	3	46 38.5	. .	17.6	2	Mer.
27.40842	Wilna	8 1 5.67	. .	—21.0	0	+51 46 19.0	+ 5.6	+ 4.3	1	Mer.

Date	Place	α apparent	π	O — C Δα cos δ	Wt.	δ apparent	π	O — C Δδ	Wt.	*
July		h m s	s	″		° ′ ″	″	″		
27.41007	Berlin	8 1 9.87	+0.08	+17.0	1	+51 46 28.0	+ 5.7	+13.8	1	. .
27.43393	Vienna	11.60	. .	7.6	2	. . . .	. .	. .	.	Mer.
27.44916	Seeberg	15.07	. .	24.2	2	46 17.0	5.7	9.1	2	Mer.
27.45133	Göttingen	14.87	. .	20.0	3	. . . .	. .	. .	.	Mer.
28.33257	Kremsmünster	2 51.60	0.26	21.8	1	. . . .	. .	. .	.	. .
28.36018	Vienna	58.57	.18	58.1	0	. . . .	. .	. .	.	. .
28.36563	Prague	57.09	.03	37.2	1	. . . .	. .	. .	.	19
28.40323	Dorpat	3 1.04	. .	36.0	3	44 8.8	. .	37.9	2	Mer.
28.40691	Wilna	1.78	. .	39.0	1	13.0	5.6	+48.3	1	Mer.
28.42222	Paris	2 59.48	+0.14	3.7	2	43 20.5	5.5	− 1.4	2	19
28.43235	Vienna	3 1.73	. .	16.8	2	. . . .	. .	. .	.	Mer.
28.44267	Mannheim	4.07	. .	24.5	3	40.0	5.7	+20.1	1	Mer.
28.44513	Munich	4.20	. .	23.2	3	38.0	5.7	20.2	1	Mer.
28.44976	Göttingen	. .	. .	. .	.	46.0	5.6	29.3	3	. .
28.47383	Palermo	7.80	−0.06	+27.4	2	37.0	4.5	22.7	2	. .
29.34325	Kremsmünster	4 36.60	+0.23	− 7.8	0	. . . .	. .	. .	.	. .
29.36899	Vienna	41.27	.15	+ 9.5	1	. . . .	. .	. .	.	. .
29.37443	Prague	44.39	.03	31.8	1	. . . .	. .	. .	.	19
29.40526	Wilna	45.81	. .	14.7	1	41 3.4	5.4	42.5	1	Mer.
29.41804	Seeberg	49.53	+0.07	37.4	2	. . . .	. .	. .	.	. .
29.44352	Munich	51.40	. .	29.3	3	40 54.0	5.6	40.5	1	Mer.
29.45007	Mannheim	52.13	. .	29.7	3	38.0	5.6	25.8	1	Mer.
29.62546	Palermo	5 11.40	−0.43	33.8	2	39 57.0	3.8	+16.7	2	. .
30.33134	Padua	6 23.17	+0.27	29.5	2	37 17.0	4.9	− 3.0	1	. .
30.37014	Vienna	26.53	.14	19.3	1	. . .	. .	. .	.	. .
30.39927	Dorpat	31.72	. .	38.5	3	31.1	. .	+19.1	2	Mer.
30.40359	Wilna	29.60	. .	14.7	1	31.2	5.4	25.5	1	Mer.
30.40587	Paris	31.81	.15	34.6	2	16.7	5.3	11.5	2	19
30.44649	Göttingen	35.47	. .	28.4	3	23.0	5.4	25.9	2	Mer.
30.46760	Paris	37.90	. .	30.8	3	1.0	5.6	8.4	1	Mer.
31.35019	Padua	8 7.07	+0.22	+30.9	2	33 54.0	+5.0	+ 1.3	1	. .
31.37288	Vienna	8.13	.13	18.9	1	. . . .	. .	. .	.	. .
31.37832	Prague	9.97	.02	29.6	1	. . . .	. .	. .	.	19
31.38284	Seeberg	12.13	.14	47.1	2	. . . .	. .	. .	.	. .
31.39829	Dorpat	13.51	. .	43.9	3	. . . .	. .	. .	.	Mer.
31.40141	Paris	12.55	+0.15	+33.6	2	34 0.4	5.3	18.6	2	19
31.40187	Wilna	6.63	. .	−23.4	0	15.2	5.3	33.3	1	Mer.
31.42740	Vienna	14.53	. .	+33.8	2	. . . .	. .	. .	.	Mer.
31.46664	Paris	21.16	. .	51.8	3	33 32.7	5.5	+ 4.8	1	Mer.
31.49704	Kremsmünster	20.27	−0.16	+13.7	1	. . . .	. .	. .	.	. .
31.60460	Palermo	27.00	−0.41	−25.7	0	0.0	3.8	− 0.8	2	. .
August										
1.32376	Padua	9 44.00	+0.27	+37.6	2	30 57.0	4.7	+29.3	1	. .
1.33488	Milan	45.83	.26	47.8	2	31 1.6	5.0	36.6	3	19
1.37116	Vienna	48.00	.13	30.5	1	. . . .	. .	. .	.	. .
1.37659	Prague	54.09	.02	81.0	0	. . . .	. .	. .	.	19
1.39433	Seeberg	51.07	+0.11	38.1	2	30 22.0	5.3	7.8	2	. .
1.40020	Wilna	49.92	. .	+20.9	1	47.3	5.3	36.6	1	Mer.
1.42564	Vienna	48.73	. .	−13.2	0	. . . .	. .	. .	.	Mer.
1.46421	Paris	57.47	. .	+33.2	3	2.2	5.4	5.3	1	Mer.
1.49615	Kremsmünster	58.67	−0.16	13.9	1	. . . .	. .	. .	.	. .
2.33164	Padua	11 21.47	+0.25	40.5	2	28 58.0	4.8	+2′ 7″.7	0	. .
2.39841	Wilna	25.16	. .	13.3	1	27 0.1	5.2	21.6	1	Mer.
2.42395	Vienna	27.33	. .	11.0	2	. . . .	. .	. .	.	Mer.
3.33503	Milan	12 55.50	.25	37.6	2	23 30.1	5.0	17.9	3	19
3.35584	Milan	57.66	+0.21	39.4	2	30.3	5.0	22.6	3	19
3.39671	Wilna	13 1.44	. .	37.2	1	1.3	5.2	3.6	1	Mer.
3.43962	Göttingen	1.60	. .	29.6	3	25.0	5.2	37.5	2	Mer.
3.61719	Palermo	21.87	−0.41	38.6	2	22 29.0	3.4	17.5	2	. .
4.39130	Dorpat	14 33.07	. .	41.8	3	19 37.9	. .	15.1	2	Mer.
4.39492	Wilna	8 14 32.16	. .	+30.2	1	+51 19 48.5	+ 5.1	+31.6	1	Mer.

Date August	Place	α apparent h m s	π s	O − C Δα cos δ ″	Wt.	δ apparent ° ′ ″	π ″	O − C Δδ ″	Wt.	*
4.43137	Seeberg	8 14 37.73	+0.01	+51.2	2	+51 20 5.0	+5.2	+56.2	0	. .
4.43783	Göttingen	35.67	. .	26.8	3	19 46.0	5.2	38.6	2	Mer.
4.60860	Palermo	53.27	−0.40	13.4	2	18 51.0	3.5	49.7	2	. .
5.35822	Prague	15 56.87	+0.03	16.0	1	. . . .	. .	. .	.	19
5.38951	Dorpat	16 2.61	. .	43.8	3	16 6.7	. .	24.1	2	Mer.
5.39310	Wilna	15 59.90	. .	15.7	1	16 41.1	5.1	+1′ 4″.4	0	Mer.
5.60049	Palermo	16 21.73	−0.39	45.6	2	14 52.0	3.5	− 0.6	2	.
6.38698	Dorpat	17 28.70	. .	34.4	3	. . . .	. .	. .	.	Mer.
6.39134	Wilna	32.16	. .	+1′ 3″.4	0	12 10.3	5.0	+12.9	1	Mer.
7.38587	Dorpat	18 55.40	. .	47.4	3	. . . .	. .	. .	.	Mer.
7.39012	Wilna	52.65	. .	18.5	1	8 54.4	1.9	34.5	1	Mer.
7.40273	Paris	57.83	+0.06	57.8	2	51.0	5.0	35.2	2	15
7.45348	Paris	19 1.07	. .	47.4	3	. . . .	. .	. .	.	Mer.
8.33398	Padua	20 14.00	+0.15	43.8	2	5 32.0	4.8	35.4	1	. .
8.38399	Dorpat	18.56	. .	46.7	3	34.4	. .	+42.4	2	Mer.
8.45161	Paris	23.60	. .	41.6	3	3 58.5	5.0	−33.2	0	Mer.
8.59532	Palermo	35.27	−0.38	36.6	2	4 10.0	3.4	+ 7.3	2	. .
9.38210	Dorpat	21 39.26	. .	40.2	3	1 16.4	. .	25.1	2	Mer.
9.38573	Wilna	37.33	. .	19.1	1	2 15.2	4.9	+60.5	0	Mer.
9.59835	Palermo	58.40	−0.38	53.0	2	0 28.0	3.4	− 3.8	2	. .
10.34071	Padua	55.00	+0.18	+34.8	2	50 57 19.0	4.6	38.3	.	. .
10.38382	Wilna	50.41	. .	−45.0	0	36.1	4.8	12.2	.	Mer.
10.44782	Paris	23 4.20	. .	+40.6	3	56 56.2	5.0	−38.9	1	Mer.
11.44584	Paris	24 20.47	. .	+27.0	3	54 18.0	+4.9	+ 3.8	1	Mer.
11.60136	Palermo	34.20	−0.39	40.2	2	53 44.0	3.0	− 1.3	2	. .
12.35168	Munich	25 31.87	+0.13	48.1	3	51 39.0	4.7	+22.4	1	. .
12.36221	Vienna	34.60	.09	68.4	1	. . . .	. .	. .	.	. .
12.37635	Dorpat	33.44	. .	46.3	3	46.8	. .	+28.3	2	Mer.
12.41726	Seeberg	36.20	.01	43.3	2	. . . .	. .	. .	.	. .
13.36232	Seeberg	26 17.67	.11	54.8	2	47 51.0	4.7	−14.4	0	. .
13.37162	Munich	47.00	.10	41.0	3	48 31.0	4.7	+27.4	1	. .
13.37440	Dorpat	47.96	. .	46.7	3	. . . .	. .	. .	.	Mer.
13.44201	Paris	51.67	. .	34.9	3	47 47.0	4.8	− 3.4	1	Mer.
14.32676	Padua	27 54.20	.19	17.7	2	41 16.0	4.4	52.8	0	. .
14.38616	Vienna	59.70	+0.03	27.6	1	. . . .	. .	. .	.	22
14.44006	Seeberg	28 4.00	−0.05	+30.9	2	. . . .	. .	. .	.	. .
15.43751	Paris	29 10.80	. .	− 9.0	.	41 52.0	4.8	− 2.1	1	Mer.
16.33524	Vienna	30 19.35	+0.13	+48.6	1	. . . .	. .	. .	.	22
16.33798	Prague	23.32	.02	85.4	0	. . . .	. .	. .	.	23
16.36608	Seeberg	18.53	.09	19.8	2	. . . .	. .	. .	.	. .
16.41678	Paris	24.29	.04	41.3	2	. . . .	. .	. .	.	15
17.31690	Padua	31 25.20	+0.19	37.7	2	38 23.0	4.3	+1′ 37″.8	0	. .
17.45996	Paris	38.24	−0.06	67.7	2	36 56.2	4.7	34.9	2	15
18.35040	Prague	32 37.90	+0.02	73.4	0	. . . .	. .	. .	.	23
18.40606	Bremen	38.33	. .	40.4	3	34 37.0	4.6	37.5	2	Mer.
18.45496	Paris	41.91	−0.05	73.9	0	2.3	4.6	10.8	2	15
19.34770	Bremen	33 38.87	+0.11	33.8	3	32 14.0	4.4	28.8	2	. .
19.37317	Seeberg	40.67	.07	34.9	2	. . . .	. .	. .	.	. .
20.33003	Padua	34 48.93	.15	+1′ 45″.6	0	29 58.0	4.4	24.9	1	. .
21.35407	Bremen	35 46.40	.10	45.8	3	27 46.0	4.4	21.3	2	. .
21.37264	Paris	45.93	.10	31.4	2	27.5	4.4	6.0	2	28
22.32025	Padua	36 47.60	.14	69.9	2	26 16.0	4.3	+42.7	1	. .
22.35724	Bremen	45.27	.09	26.0	3	. . . .	. .	. .	.	. .
22.37126	Seeberg	47.20	.06	36.0	2	. . . .	. .	. .	.	. .
22.39139	Paris	47.32	.06	26.1	2	25 3.5	4.5	−17.2	1	28, 29
23.32029	Padua	37 46.07	+0.15	+61.5	2	23 17.0	+4.3	−30.1	1	. .
23.34583	Vienna	43.33	.07	20.3	1	. . . .	. .	. .	.	. .
23.35112	Seeberg	47.60	.09	58.3	2	. . . .	. .	. .	.	. .
23.35399	Dorpat	46.00	. .	42.0	3	. . . .	. .	. .	.	Mer.
23.35127	Prague	44.45	.01	33.7	1	. . . .	. .	. .	.	21
23.36675	Paris	8 37 48.03	+0.11	+54.3	2	+50 23 55.2	+4.3	+13.4	2	28, 29

Date	Place	α apparent h m s	π s	O – C Δα cos δ ″	Wt.	δ apparent ° ′ ″	π ″	O – C Δδ ″	Wt.	*
August 23.46692	Kremsmünster	8 37 55.60	–0.15	+67.7	1	. . . .	. .	. .	.	. .
24.32104	Padua	38 43.00	+0.15	51.5	2	+50 23 17.0	+4.3	+66.5	1	. .
24.32690	Prague	39.46	.02	15.4	0	. . . .	. .	. .	.	28, 21
24.33049	Vienna	41.07	.10	27.5	1	. . . .	. .	. .	.	. .
24.34749	Seeberg	44.87	.09	54.4	2	. . . .	. .	. .	.	. .
24.35189	Dorpat	45.25	. .	54.2	3	. . . .	. .	. .	.	Mer.
24.35727	Bremen	44.73	.08	47.6	3	21 37.0	4.4	–30.2	1	. .
25.35127	Seeberg	39 42.73	+0.09	65.9	2	. . . .	. .	. .	.	. .
25.50391	Paris	48.19	–0.17	34.9	2	21 14.0	4.1	+45.0	2	29
26.33037	Bremen	40 31.00	+0.12	+ 6.7	1	19 38.0	4.2	35.5	2	. .
26.34063	Prague	29.60	+0.01	– 0.6	0	. . . .	. .	. .	.	21
26.46677	Kremsmünster	40.60	–0.16	+31.6	1	. . . .	. .	. .	.	. .
27.34543	Dorpat	41 29.94	. .	55.0	3	. . . .	. .	. .	.	Mer.
28.33288	Prague	42 15.65	+0.01	6.4	.	. . . .	. .	. .	.	21
28.34324	Dorpat	21.75	. .	52.8	3	. . . .	. .	. .	.	Mer.
29.32808	Bremen	43 12.67	.11	63.1	3	17 24.0	4.3	41.8	2	. .
30.31838	Prague	44 57.69	+0.02	32.1	1	. . . .	. .	. .	.	21
30.44255	Kremsmünster	11.53	–0.15	99.2	0	. . . .	. .	. .	.	. .
30.49316	Paris	6.64	.15	29.2	2	16 21.2	4.1	+18.8	2	28, 29
September 1.54381	Seeberg	45 43.27	–0.15	32.1	2	14 57.0	3.3	–34.3	1	. .
9.32183	Bremen	50 55.53	+0.07	+48.0	3	19 42.0	+4.1	–62.9	0	. .
10.32563	Bremen	51 27.93	.06	35.2	3	21 54.0	4.1	22.6	2	. .
11.35010	Bremen	52 0.73	+0.02	34.4	3	23 41.0	4.1	21.7	2	. .
13.38365	Dorpat	53 2.10	–0.11	42.0	3	27 30.6	–3.8	–48.6	2	24, 26
14.30547	Prague	25.87	+0.01	30.8	1	. . . .	. .	. .	.	23
15.30429	Prague	52.89	.01	41.5	1	. . . .	. .	. .	.	23
15.31315	Bremen	51.40	+0.06	52.5	3	33 12.0	+4.1	+17.6	2	. .
15.33748	Dorpat	57.58	–0.10	75.3	0	32 28.4	–3.8	–37.9	2	26
16.29316	Prague	54 18.70	+0.01	59.3	1	. . . .	. .	. .	.	23
17.30713	Bremen	40.13	.07	42.2	3	38 59.0	4.0	+23.3	2	. .
18.30140	Prague	55 1.81	.01	51.7	1	. . . .	. .	. .	.	23
19.29931	Bremen	20.07	+0.07	50.6	3	46 0.0	4.0	+53.8	0	. .
23.36293	Dorpat	56 22.21	–0.12	+59.7	3	51 00 0.1	+ 3.7	–59.0	2	27
24.34153	Bremen	30.87	.02	48.8	3	6 18.0	4.1	+58.2	2	. .
24.35550	Dorpat	37.07	.11	105.2	0	5 0.1	3.7	–23.7	2	27
24.53037	Bremen	34.33	.26	63.1	3	6 51.0	2.7	+38.0	2	. .
25.36067	Dorpat	44.08	.12	93.4	0	10 31.9	3.7	23.5	2	27
October 12.29163	Bremen	53 23.60	–0.02	+ 6.5	3	53 0 52.0	+4.0	+31.2	2	. .
12.32982	Dorpat	25.00	.14	28.2	3	+53 0 22.0	+3.5	–23.9	2	. .
15.31612	Dorpat	8 51 42.08	–0.14	+86.2	3	. . . .	. .	. .	.	. .

The weights attached to the observations were determined by plotting the residuals, and computing the mean error of the various series with regard to this curve. In forming the Normal Places, the observations of each single day were united into a weighted mean. These means were then used as the basis of equations of the form $x + ty = n$, by whose solution the position of the comet was found. The perturbations were computed for all the planets except *Uranus* and *Neptune*. Naturally they are small, as the comet very rapidly receded from the plane of the solar system. The positions for the dates of the Normal Places and the perturbations are

	Δα cos δ ″	Per. ″	Wt.	Δδ ″	Per. ″	Wt.
July 5.0	+ 5.88	+0.01	120	+ 6.57	0.00	88
11.0	15.50	3	126	11.42	0.00	100
19.0	24.81	6	124	19.45	+0.02	87
27.0	29.05	8	124	20.36	4	77
Aug. 4.0	37.47	9	95	17.48	10	54
16.0	39.66	10	61	14.27	21	26
26.0	46.54	11	44	+26.14	34	13
Sept. 14.0	44.16	8	25	–11.47	69	12
24.0	57.20	+0.05	9	+ 7.4	86	10
Oct. 14.0	+40.39	–0.06	9	+ 3.6	+1.37	4

The differential equations are

$-8.9027\,\partial\kappa$	$+9.4232\,\kappa\sqrt{2}\,\partial T$	$+9.1018\,\partial q$	$+9.5013\,\partial\lambda$	$+9.5094\,\partial\nu$	$-8.1719\,\frac{\partial e}{2}$	-0.7701	$= 0$
−9.0860	+9.4278	+9.5985	+9.2028	+9.6362	−7.9716	−1.1912	
−9.2435	+9.4173	+9.7135	+7.9439	+9.6877	+8.0306	−1.3957	
−9.3119	+9.3955	+9.7699	−8.9773	+9.7039	+8.5216	−1.4643	
−9.4105	+9.3733	+9.8032	−9.2303	+9.7092	+8.7206	−1.5747	
−9.4863	+9.3431	+9.8353	−9.4064	+9.7118	+8.8617	−1.5991	
−9.5351	+9.3228	+9.8517	−9.4943	+9.7146	+8.9061	−1.6688	
−9.6057	+9.2917	+9.8803	−9.6099	+9.7275	+8.8542	−1.6458	
−9.6329	+9.2773	+9.8885	−9.6616	+9.7406	+8.7169	−1.7578	
−9.6590	+9.2432	+9.8851	−9.7588	+9.7810	−8.3200	−1.6053	
+8.8721	−9.9826	+9.7648	−7.3455	−7.3536	+9.1590	−0.8176	
−9.1773	−9.5698	+9.8988	−8.1949	−8.6283	+9.3722	−1.0577	
−9.4967	−9.0255	+9.9723	−7.2148	−8.9586	+9.5725	−1.2894	
−9.6126	−8.3753	+9.9963	+8.3769	−9.1035	+9.7118	−1.3096	
−9.6789	+7.7766	+0.0050	+8.7095	−9.1881	+9.8157	−1.2450	
−9.7456	+8.3252	+0.0103	+8.9663	−9.2714	+9.9325	−1.1608	
−9.7895	+8.4210	+0.0163	+9.1023	−9.3226	+0.0104	−1.4229	
−9.8619	+8.5260	+0.0352	+9.2803	−9.3979	+0.1279	+1.0326	
−9.8990	+8.5961	+0.0511	+9.3514	−9.4304	+0.1800	−0.9170	
−9.9706	+8.7586	+0.0930	+9.4516	−9.4738	+0.2605	−0.6964	

From these by the ordinary least-square methods:

$$\partial\kappa -9.5271\,\kappa\sqrt{2}\,\partial T -0.3446\,\partial q +9.0216\,\partial\lambda -9.7251\,\partial\nu -9.9915\,\frac{\partial e}{2} +1.8368 = 0$$

$$\kappa\sqrt{2}\,\partial T -9.7274\,\partial q +8.4489\,\partial\lambda +9.7204\,\partial\nu -9.6058\,\frac{\partial e}{2} -0.9173 = 0$$

$$\partial q +8.8517\,\partial\lambda +0.0171\,\partial\nu -9.6769\,\frac{\partial e}{2} -1.6827 = 0$$

$$\partial\lambda -9.7769\,\partial\nu -9.7855\,\frac{\partial e}{2} +1.7027 = 0$$

$$\partial\nu -9.1604\,\frac{\partial e}{2} -1.5297 = 0$$

$$\frac{\partial e}{2} +1.0926 = 0$$

And finally,

$$\partial\kappa = -11''.50 \qquad \partial\lambda = -31''.11$$
$$\kappa\sqrt{2}\,\partial T = +\ 6.78 \qquad \partial\nu = +19.57$$
$$\partial q = +24.14 \qquad \frac{\partial e}{2} = -12.38$$

When these values of the unknown quantities are substituted in the original equations the residuals are

$\Delta\alpha\cos\delta$	$\Delta\delta$	$\Delta\alpha\cos\delta$	$\Delta\delta$
″	″	″	″
+0.5	+1.7	+ 1.2	+ 0.5
−0.9	−3.7	+ 5.2	+14.5
−0.5	+0.3	− 2.5	−18.2
−1.7	+1.6	+ 7.7	+ 2.7
+3.1	+0.4	−14.9	+ 1.1

On account of the predominance of the positive sign in the declination residuals, and the excessive values for Aug. 26 and Sept. 14, these latter were entirely omitted, and a second solution was made. Also, Olbers's observation for Oct. 12 was removed from the last normal place in right ascension, as it differs by nearly a minute of arc from the ephemeris place that is in accord with the other normal places, and by its presence gives rise to the large negative residual in this last normal in right-ascension. With these modifications there now results:

$$\partial\kappa = -47''.4 \qquad \partial\lambda = -26''.1$$
$$\kappa\sqrt{2}\,\partial T = -\ 4.1 \qquad \partial\nu = +22.5$$
$$\partial q = +14.0 \qquad \frac{\partial e}{2} = -21.5$$

and the residuals are

$\Delta\alpha\cos\delta$	$\Delta\delta$	$\Delta\alpha\cos\delta$	$\Delta\delta$
″	″	″	″
+0.3	+1.1	0.0	−1.1
−0.4	−2.7	+3.2	. .
−0.2	+1.1	−6.1	. .
−1.7	+1.6	+3.5	−0.3
+2.7	−0.2	−3.3	−3.1

Corresponding to this new series of values of the unknown quantities are the following corrections to the elements:

$$\begin{array}{rcl|rcl} \partial T &=& -0.00083 & \partial\omega &=& -52''.4 \\ \partial\Omega &=& +30''.9 & \partial q &=& +0.000068 \\ \partial i &=& +15.8 & \partial e &=& -0.000208 \end{array}$$

Computing the probable errors the definitive elements are

$$\begin{aligned} T &= 1819 \text{ June } 27.71464 \pm 0.00037 \\ \left.\begin{aligned} \Omega &= 273^\circ\ 42'\ \ 2''.9 \pm \ 4''.7 \\ i &= \ \ 80\ 44\ 53.8 \pm \ 2.7 \\ \omega &= \ \ 13\ 25\ 21.6 \pm 21.4 \end{aligned}\right\} & 1819.0 \\ \log q &= 9.533319 \pm 0.000041 \\ e &= 0.999792 \pm 0.000079 \end{aligned}$$

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EQUATORIAL COORDINATES.

$$\begin{aligned} x &= [9.237881]\, r \sin (\ 35^\circ\ 20'\ 18''.1 + v) \\ y &= [9.996719]\, r \sin (260\ 41\ 39.4 + v) \\ z &= [9.996738]\, r \sin (349\ 49\ 28.2 + v) \end{aligned}$$

The coefficients in the differential equations were tested by the method of arbitrary variation of the elements at the beginning of the work. In order to test the accuracy of the definitive elements the position of the comet was computed for the date of the last normal place with the result:

$$\Delta\alpha \cos\delta = -3''.8 \quad , \quad \Delta\delta = -3''.6$$

In conclusion, I wish to express my thanks to the staff of astronomers at the Harvard Observatory for the many courtesies extended during the collection of the observations used as a basis for the computation.

MR. STOCKWELL ON THE THEORY OF TIDAL FRICTION.

BY SIR GEORGE DARWIN.

As Mr. STOCKWELL's name is well known for his contributions to Dynamical Astronomy, it seems advisable that his paper on this subject in the *Astronomical Journal* (No. 581, Jan. 15, 1906) should be noticed.

After some general remarks, and an extract from my book, "*The Tides*," he proves by numerical examples the well-known result, that in the variational curve, the moon's radius-vector in syzygies is less than in quadratures. This is taken to show that a body detached from the earth must necessarily collide with and be re-absorbed by the earth. It does prove it if the detachment occurs in quadratures, but not so if it occurs with a proper velocity in syzygies. However, the whole argument seems to me beside the point.

I prove that the moon may be traced back to a position near the earth, and that it then had no motion relatively to the earth. Such a position is one of dynamical instability, and mathematical analysis is powerless to trace the configuration of the system in retrospect behind such a stage. This primitive configuration of the moon seems, to me at least, to indicate a previous detachment from the earth of the matter now forming the moon; but the process of such a detachment escapes our powers of analysis. Mr. STOCKWELL's argument only shows that one particular mode of separation of the moon from the earth leads to a difficulty, unless the separation occurs at syzygy.

He next proceeds to discuss tidal friction, and gives a table of numerical values, which seems to me redundant. He then follows FERREL in placing the tidal apex, under the influence of friction, eastward of the moon. He omits to remark that this is the correct supposition for "direct" tides, when, save for tidal friction, the figure would be a prolate ellipsoid, with longest axis directed to the moon; but incorrect for shallow oceans when the tides are "inverted," and the figure approximates to an oblate ellipsoid with minor axis pointed to the moon. In the latter case under friction the minor axis is directed westward of the moon.

We now come to the fundamental error in his argument. Following FERREL's supposition, he correctly finds that the moon's radius-vector is augmented, and her mean angular motion retarded. He then says, "Now, since action and reaction are equal, and in opposite directions, it follows that a *negative* secular equation of the moon's motion from this cause must produce a *positive* secular equation of the tidal sphere, which is supposed to be attached to the earth's surface, and thereby *increase* the earth's velocity of rotation." This is an erroneous appeal to the equality of action and reaction, and that it must be incorrect may be shown by the following argument. He finds an increase in the moon's radius-vector, and therefore the sum of the potential and kinetic energies of the moon's orbital motion is increased. If now the earth's rotation increases, the kinetic energy of rotation increases, and thus the whole effect of tidal friction is to augment the energy of the system; which is absurd. In fact, the earth's rotation is retarded, and that to such an extent as to make the whole energy of the system decrease. Action and reaction are, of course, equal and opposite, but the argument is incorrectly applied by Mr. STOCKWELL.

I would refer the reader to Appendix G (*b*) in THOMSON and TAIT's Natural Philosophy, or to Art. "*Tides*," §46,

in the *Encyclopedia Britannica* for a correct treatment of the problem. In the latter place there will be found a clear statement, due to Sir GEORGE STOKES, of the *modus operandi* of tidal friction.

Mr. STOCKWELL then attacks my result that the minimum time required for the change of position of the moon from her initial to her present position is sixty million years. He describes it as a rough estimate, and so it is, but not in the sense that it is a mere shot at the truth. He has clearly not looked at the laborious integrations of which it is the outcome.

He has found as the result of his numerical table that with the lunar apex 45° in advance of the moon, the moon's mean distance is increasing at 3.0353 feet per year. To find the aggregate change he simply multiplies this by sixty millions, and omits to notice that the efficiency of tidal friction varies inversely as the sixth power of the moon's distance. His actual numerical result is of no importance, since I was discussing bodily tides in the earth, but his omission is vital.

Later we are informed that "the fundamental conception of the theory of tidal evolution is that the moon's motion should be accelerated." This is diametrically wrong, although it is true that a terrestrial observer, who takes the earth as his time-keeper, would regard the real retardation of the moon's angular motion as an acceleration.

Next, by a loose argument, he concludes that the position of stability of the actual moon must be with her longest equatorial diameter at right angles to her radius-vector. This is incorrect, for the supposed configuration is unstable as being one of greater energy than if the longest equatorial diameter coincided with the radius-vector. The views of NEWTON and of LAPLACE are therefore correct.

Finally, we come to this statement:

"There is, however, one feature of the theory of the tides that is so universally accepted, and at the same time so obviously erroneous, that I may incidentally speak of it in this place; and that is, that the necessary effect of the attraction of the sun or moon upon the waters of the ocean is to heap them up directly under the attracting body. In this case, if the bodies were fluid and without rotation, they would obviously assume the form of a prolate spheroid whose longer axis would be directed towards each other. But we have seen that such a position would be one of unstable equilibrium; and consequently could not exist in nature. The form which the surface of the water would assume under the given conditions would be that of an oblate spheroid, whose shorter axis would be directed towards the attracting body; and consequently it ought to be low water under the attracting body."

The alleged universal acceptance of the prolate form is astonishing in face of the fact that it was NEWTON who had the marvellous insight to perceive that in a shallow equatorial canal low water would occur under the moon, and that LAPLACE showed that on an ocean-covered planet the same would be true of the equatorial regions, whilst in the polar regions the tides would be "direct." In a deep canal, however, or with a deep ocean, the tides are everywhere "direct."

Mr. STOCKWELL is of course entitled to hold any opinions he likes on the subject of tidal friction, but I venture to prophesy that he will gain but few adherents.

MAXIMA OF LONG-PERIOD VARIABLES,

BY IDA WHITESIDE.

The maxima of the following twenty long-period variables were determined from the single light-curves deduced from observations made with a four-inch Dolland telescope.

103. *T Andromedae.*

Seventeen observations of this star, extending from Aug. 18, 1905, to Dec. 26, 1905, gave a maximum on Oct. 14, 1905, at which time the star had a magnitude of 8M.5. The date of maximum predicted in CHANDLER'S "Ephemerides of Long-Period Variables" was Sept. 24, 1905.

8230. *S Aquarii.*

This star reached a maximum of 8M.0 on Dec. 10, 1905. This maximum was deduced from eight observations covering the time between Nov. 8, 1905, and Jan. 24, 1906. The predicted time of maximum was Nov. 27, 1905.

7468. *T Aquarii.*

Eleven observations of this star were made, the first on Sept. 22, 1905, and the last on Dec. 14, 1905. A maximum of 7M.4 was reached on Oct. 5, 1905, thirty-nine days before the time predicted (Nov. 13, 1905).

782. *R Arietis.*

From thirteen observations of this star, the maximum was found to be 8M.7 on Jan. 8, 1906. The observations extended from Nov. 8, 1905, to March 1, 1906, and gave a well-defined curve. The maximum was predicted for Dec. 26, 1905. Though at the maximum of Dec. 19, 1904, the observed and predicted times agreed exactly, the form of the curve will not allow a variation of more than a day or so from the date given above for this maximum.

715. *S Arietis.*

The time of maximum for this star agrees exactly with that predicted Oct. 20, 1905. The star reached a brightness of $9^{M}.8$. Nine observations were made from Sept. 20, 1905, to Nov. 27, 1905.

976. *T Arietis.*

Fourteen observations of this star were made, covering the time from the fifth of October, 1905, to the fifth of February, 1906. The star varied but a little more than 1½ magnitudes during the entire period. The curve has a very gradual slope near maximum, and the time is, therefore, not very well defined. Apparently, the star reached a maximum of $8^{M}.9$ at about the predicted time of maximum, Oct. 24, 1905.

243. *U Cassiopeae.*

The time of maximum of this star (Dec. 27, 1905) was found from fourteen observations, which gave a well-defined curve and maximum. The star reached $7^{M}.8$. The observations extended from Oct. 28, 1905, to March 1, 1906. The maximum was predicted for Nov. 3, 1905.

8324. *V Cassiopeae.*

This star was observed nine times, from Oct. 30, 1905, to Jan. 18, 1906. Only two observations were made before maximum. Nevertheless, the form of the curve shows pretty clearly that the maximum occurred on Nov. 11, 1905, and not on Dec. 7, which was the date predicted. At maximum the star had a brightness of $8^{M}.7$.

7085. *RT Cygni.*

From eight observations, the maximum of this star was determined as $7^{M}.1$ on Nov. 20, 1905. The observations covered the time between Oct. 30, 1905, and Jan. 12, 1906. The predicted date of maximum was Dec. 1, 1905.

7242. *S Geminorum.*

The maximum of this star was predicted to occur on Jan. 19, 1906, but the observations from Dec. 7, 1905, to Feb. 21, 1906, showed a maximum of $8^{M}.9$ on Dec. 15, 1905. As the star was nearly at its brightest when first observed, the maximum may have occurred a little earlier than the date given, but the form of the curve precludes entirely its coming later.

6512. *T Herculis.*

Ten observations of this star were made between Aug. 17, 1905, and Oct. 28, 1905. They show a maximum of $8^{M}.1$ on Sept. 12, 1905. By a very slight and possible change in the curve, the date of maximum could be made to agree with that predicted (Sept. 8, 1905).

1761. *R Orionis.*

Nine observations of this star, made between Nov. 25, 1905, and Feb. 22, 1906, gave a maximum of $9^{M}.5$ on Dec. 27, 1905. Though the curve is not very well determined near maximum, the time was probably not far from that given, and not that predicted (Jan. 11, 1906).

8290. *R Pegasi.*

The last maximum of this star occurred on Sept. 30, 1905, when it reached $8^{M}.3$. The observations, thirteen in number, extended from Aug. 18 to Dec. 14, 1905. The maximum was predicted for Sept. 17, 1905, but the curve shows with a good deal of certainty that it occurred later than that date.

8373. *S Pegasi.*

This star was observed from Sept. 21, 1905, to Jan. 12, 1906, twelve times in all. The curve shows a well-determined maximum of $7^{M}.7$ on Nov. 6, 1905, instead of on Oct. 14, as was predicted.

8369. *W Pegasi.*

On account of its position in the sky, it was possible to obtain but six observations of this star. They extended from Jan. 12 to Feb. 22, 1906. Though nearly all the observations preceded maximum, they point to a maximum of $8^{M}.3$ on Feb. 14, 1906, instead of on Jan. 25, 1906, as predicted.

678. *U Persei.*

Fourteen observations made between July 27 and Oct. 28, 1905, show that this star had a maximum of $7^{M}.6$ on Aug. 6, 1905. The maximum was predicted for Aug. 19, 1905.

431. *S Piscium.*

This star reached a maximum of $9^{M}.7$ on Nov. 15, 1905. The first of the six observations was made on Oct. 16, and the last on Dec. 26, 1905. By a very possible change in the curve, the time of maximum could be made to agree with that predicted (Nov. 10, 1905).

5501. *S Serpentis.*

Seven observations of this star were made between Aug. 18 and Oct. 16, 1905. The magnitude decreased steadily from $8^{M}.2$ on the first to $10^{M}.1$ on the last date. Therefore, the maximum must have preceded Aug. 18, and could not have been on the predicted date (Sept. 30, 1905).

1577. *R Tauri.*

Seven observations of this star gave a maximum of $9^{M}.4$ on Nov. 14, 1905. The observations extended from Oct. 30 to Dec. 26, 1905. The predicted time of maximum, Dec. 7, 1905, cannot be made to fit the observations.

5601. *S Ursae Minoris.*

Observations of this star were made from Sept. 21, 1905, to Jan. 24, 1906. Fourteen were made in all, and they show a maximum of $8^{M}.3$ on Nov. 3, 1905, not on the predicted date (Oct. 29, 1905).

South Cambridge, N.Y.

THE PROPER MOTION OF B.D. +38°3095,

By GEORGE A. HILL.

[Communicated by Rear-Admiral ASA WALKER, Superintendent Naval Observatory.]

Recently, in collecting the individual places of the star B.D. +38°3095, whose position for 1875.0 is R.A. 18h 5m 29s, Declination +38° 27′ 4″, from observations made of it with the prime-vertical transit instrument, I find the star has a relatively large proper motion in declination.

It is somewhat surprising that this discovery has not been made before, as the object is in the sixth magnitude class, and well up in the northern heavens.

I have found it in the following catalogues:

			R.A.	Decl.	
			h m s	° ′ ″	
Lalande	33439	1800	18 3 0.77	+38 27 15.5	1797.5
Yarnall	7837	1860	4 59.16	26 55.5	1875.5
Paris	23526	1875	5 29.25	27 5.0	1872.3
Lund	7521	1875	18 5 29.03	+38 27 0.6	1881.4

I have secured the following places with the prime vertical:

		° ′ ″		
Prime vertical	1900	+38 27 4.2	1901.2	5 obs.
Prime vertical	1904	+38 27 4.9	1904.4	4 obs.

The minute in right-ascension for Lalande is wrong; it should be 3. See Bonn, Vol. 7, page 230.

Reducing the above to 1900, merely by precession, we have

		Weight	Epoch	No. Obs.
	° ′ ″			
Lalande	+38 27 56.1	0.1	1797.5	1
Paris	17.9	0.6	1872.3	4
Yarnall	15.3	0.4	1875.5	2
Lund	13.5	0.5	1881.4	2
P.V. 1900	4.2	0.9	1901.2	5
P.V. 1904	+38 27 2.7	0.9	1904.4	4

In the Paris Catalogue, Vol. 4, page 10, a comparison between Lalande and Paris, is given, and is P–L $-2^s.21$, and $+38''.3$, but no mention is made of suspected proper motion. The same remark also applies to the Lund Catalogue. This large difference, in declination, is real, and from the above data I find, by a least-square solution, the proper motion to be $-0''.42$.

Introducing the above value into the annual precession for the star, the following differences between the corrected assumed declination for 1875, and each catalogue place is found to be

	″		″
Lalande	−6.7	Lund	+0.8
Yarnall	+1.7	P.V. 1900	0.0
Paris	0.0	P.V. 1904	−0.4

This star I am now using in my latitude work, and I will have more data in the future with which to correct this proper motion.

ELEMENTS AND EPHEMERIS OF COMET a 1906 (BROOKS),*

BY HERBERT R. MORGAN.

The following elements were derived from three observations made at Glasgow, on Jan. 27, Feb. 2, and Feb. 8; the last being taken during the total eclipse of the moon.

ELEMENTS.

$$T = 1905 \text{ Dec. } 22^d.330855 \text{ Gr. M.T.}$$

$$\left.\begin{aligned} \pi &= 16°\ 20'\ 1''.5 \\ \Omega &= 286\ 25\ 14.3 \\ i &= 126\ 25\ 45.0 \end{aligned}\right\} 1906.0$$

$$q = 1.296509$$

$$\cos\beta\Delta\lambda = +2''.1$$
$$\Delta\beta = +0.9$$

HELIOCENTRIC EQUATORIAL COORDINATES.

$$x = r\,[9.803386]\sin(243°\ 31'\ 7''.0 + v)$$
$$y = r\,[9.999833]\sin(331\ 55\ 37.2 + v)$$
$$z = r\,[9.887770]\sin(\ 60\ 17\ 20.7 + v)$$

EPHEMERIS.

Gr. M.T.	True α	True δ	log Δ	Br.
1906	h m s	° ′ ″		
Feb. 24.5	6 17 1.1	+74 34 5	0.0088	0.78
28.5	5 55 46.4	68 9 30	0.0376	
Mar. 4.5	5 46 38.9	62 28 11	0.0693	
8.5	5 42 27.2	57 31 48	0.1022	0.45
12.5	5 40 43.8	53 16 25	0.1351	
16.5	5 40 26.1	49 36 36	0.1674	
20.5	5 41 2.8	46 26 57	0.1986	0.25
24.5	5 42 16.2	43 42 35	0.2284	
28.5	5 43 55.9	41 19 19	0.2567	
Apr. 1.5	5 45 54.8	+39 13 46	0.2835	0.15

Brightness Jan. 27, 1906 = 1.00.

* *From Supplement to No. 583.*

MEASURES OF DOUBLE STARS,

MADE WITH THE 12-INCH EQUATORIAL OF THE MORRISON OBSERVATORY,

BY HERBERT R. MORGAN.

The stars in the following list were observed nine years ago with the 26-inch McCormick equatorial (*A.J.*, No. 439). They have been reobserved, not so much for themselves, as to test the conditions and instrument of this observatory. Two nights out of twenty have been good, and two others fair. Often a 2″ star is only a blurr. Some of the distances are large. I have measured these, sometimes, by pure guess. The micrometer is good. The oil lamps are replaced by electric, though oil was used to give the red threads used for these measures. The electric current is brought out by private wire from Glasgow. Eye-pieces magnifying 275 and 450 have generally been used. The rule for a night's measure is six angles and four double distances. Excepting two nights each for Σ1291 and Σ1338, each star was measured on three nights. The largest variation from night to night is, in p, 4°.3 on Σ60; and in s, 0″.45 on OΣ38 A.B. The α and δ are for 1880.0. The value of one revolution of the micrometer screw 12″.910 has been used. It was checked Oct. 27 by a measurement of the arc, *Celaeno–Taygeta*. Only a provisional examination of the position of the instrument has been made.

I am indebted to Mr. KAVANAUGH, student assistant, for reading the telescope circles, and recording.

Star	α	δ	Epoch	p	s
	h m	° ′		°	″
Σ3062	0 0.1	+57 46	1905.866	351.7	1.73
Σ60	0 41.7	+57 11	.867	233.1	5.43
Σ73	0 48.3	+22 57	.870	24.3	1.16
OΣ38 A.B.	1 56.6	+41 45	.869	62.8	10.11
Σ333	2 52.3	+20 51	.870	209.9	1.53
Σ460	3 50.0	+80 22	.889	53.7	0.91
Σ1110	7 26.9	+32 9	.953	222.8	5.39
Σ1291	8 46.5	+31 4	.953	327.4	1.33
Σ1338	9 13.5	+38 42	.954	170.6	1.62
Σ2130	17 2.8	+54 38	.828	142.3	2.38
Σ2173	17 24.2	− 0 58	.830	320.1	0.97
Σ2262	17 56.5	− 8 11	.822	256.0	2.05
Σ2272	17 59.4	+ 2 33	.831	176.7	2.39
Σ2382	18 40.4	+39 33	.828	10.2	2.86
Σ2383	18 40.4	+39 29	.828	127.2	2.30
β441	20 12.6	+28 46	.839	67.1	5.94
β366 A.B	20 44.8	+50 3	.839	130.0	1.48
Σ2758	21 1.1	+38 7	.840	127.5	22.56
β75	21 49.7	+10 19	1905.888	41.7	1.11

Glasgow, Mo., 1906 *Jan.* 12.

OBSERVATIONS OF COMETS *b* AND *c* 1905,

MADE WITH THE 12-INCH EQUATORIAL OF THE MORRISON OBSERVATORY,

BY HERBERT R. MORGAN.

1905 Glasgow M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
				COMET *b*.						
h m s			m s	′ ″	h m s	° ′ ″			s	″
Nov. 19 13 52 21	1	6 , 4	+1 56.9	+3 20.1	0 2 15.2	+67 22 33.8	0.074	0.239	+5.05	+30.9
21 7 20 50	2	12 , 6	+0 48.8	−0 29.4	23 45 42.0	+52 7 30.4	n8.882	n0.293	+3.46	+30.8
21 10 19 12	3	d5 , 7	−0 42.0	−2 9.5	23 44 58.6	+51 3 59.7	9.661	n9.810	+3.46	+30.8
22 6 43 32	4	d6 , 6	+0 1.1	−2 40.6	23 40 29.4	+44 14 35.2	n9.155	n9.827	+3.20	+29.8
25 7 24 39	5	12 , 8	+1 17.8	−1 11.1	23 34 8.8	+24 18 40.8	8.221	0.353	+2.72	+25.6
28 7 9 28	6	14 , 8	+1 30.2	−6 56.4	23 31 27.8	+11 55 3.1	8.146	0.606	+2.60	+22.0
				COMET *c*.						
Dec. 7 16 58 37	7	d5 , 8	−0 23.1	−7 20.0	14 27 42.4	+20 25 32.2	n9.648	0.627	+0.71	− 9.3

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	0 0 13.26	+67 18 42.8	Christiania, A.G. Zones 166 No. 18, 170 No. 12	5	23 32 48.27	+24 19 26.3	Berlin B, A.G. 9047
2	23 41 49.71	+52 7 29.0	Camb., U.S., A.G. 8480	6	23 29 55.00	+12 1 37.5	Munich I, 32562
3	23 45 37.16	+51 5 38.4	Eastman, 5075	7	14 28 1.81	+20 33 1.5	Mic. Comp. with *8
4	23 40 25.05	+44 16 46.0	Bonn, A.G. 18121	8	14 28 29.71	+20 31 16.5	Berlin B, A.G. 5096

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THE DIFFERENTIAL EQUATIONS OF DISTURBED ELLIPTIC MOTION,

By A. HALL.

The general differential-equations of planetary motion are well known, and need not be repeated here. These equations can be solved only by approximation. The first step is to neglect the disturbing forces, and thus reduce the problem to one of two bodies. In this case the equations can be solved, and neglecting the mass of the planet we have six arbitrary constants of integration, which are the elements of the conic section described by the planet around the sun. The next step is to assume that the arbitrary constants are variable. LAGRANGE has given a general and beautiful solution of this question. The orbit remains a conic, but the elements are varied to fit the observations. From the three coordinates of the planet at any time, and the three velocities along the axes, we can compute the orbit. The action of the disturbing forces changes the velocities, and hence we shall find different orbits.

In the *Berliner Jahrbuch* for 1837 ENCKE published a form of the Lagrangian equations for the variations of the elements, which has been very useful for computing the special perturbations of the minor planets, and comets, and for determining the mass of *Jupiter*. These formulas were given to ENCKE by GAUSS. The proof of them is probably that of GAUSS, which is essentially that of LAGRANGE, with some geometrical illustrations. The disturbing force is resolved into three rectangular components: R along the radius-vector; S perpendicular to the radius, and in the plane of the orbit; and W perpendicular to the plane of the orbit.

In the *Berliner Jahrbuch* for 1855, ENCKE has given an elementary proof of these formulas. For this purpose he introduced another system of rectangular components, T along the tangent, N along the normal, and W as before. If we call c the velocity of the planet, and ψ the angle between the direction of the velocity and the prolongation of the radius, we shall have in the common notation,

$$c \cos\psi = \frac{dr}{dt} = \frac{e \sin v}{p} \cdot r^2 \frac{du}{dt}$$

$$c \sin\psi = r\frac{du}{dr} = \frac{k\sqrt{p}}{r}$$

The components of the disturbing force are connected by the equations,

$$R = T\cos\psi - N\sin\psi$$
$$S = T\sin\psi + N\cos\psi$$

and inversely

$$T = R\cos\psi + S\sin\psi$$
$$N = -R\sin\psi + S\cos\psi$$

Also, $$T = \frac{dc}{dt} \quad , \quad N = c\,.\frac{d\psi}{dt}$$

The motions of the undisturbed orbit being in the orbit plane the above equations follow from geometry. Hence

$$cT = Rc\cos\psi + Sc\sin\psi = \frac{k}{\sqrt{p}}.\left(e\sin v\,.\,R + \frac{p}{r}.\,S\right)$$

$$cN = -Rc\sin\psi + Sc\cos\psi = \frac{k}{\sqrt{p}}.\left(-\frac{p}{r}.\,R + e\sin v\,.\,S\right)$$

In these equations we omit the mass of the planet. This can be done in nearly every case, but if the mass is given we must multiply the Gaussian constant k by $\sqrt{1+m}$, m being the mass.

In his derivation of the formulas, ENCKE has introduced the second, or vacant focus, with an anomaly and radius counted from this focus. He uses the components T and N, but as these are not convenient for computing, he transforms to R and S. This transformation is long, "*weitläufig*," as ENCKE calls it, and it does not appear necessary. We begin with the simpler equations. The equation of living force is

$$c^2 = k^2.\left(\frac{2}{r} - \frac{1}{a}\right)$$

In the differentiations the polar coordinates, r and u, are considered constant in the plane of the orbit, while the elements vary. From

$$\frac{1}{a} = \frac{2}{r} - \frac{c^2}{k^2}$$

$$\frac{da}{dt} = \frac{2a^2 . e}{k^2} . \frac{de}{dt} = \frac{2a^2 e}{k^2} . T$$

or

$$\frac{da}{dt} = \frac{2a^2}{k\sqrt{p}} . \left(e \sin v . R + \frac{p}{r} . S\right)$$

Since the mean motion

$$\mu = \frac{k}{a^{\frac{3}{2}}};\ \text{and}\ \frac{d\mu}{\mu} = -\frac{3}{2} . \frac{da}{a}$$

$$\frac{d\mu}{dt} = -\frac{3a\mu}{k\sqrt{p}} . \left(e \sin v . R + \frac{p}{r} . S\right)$$

The next equation is for the eccentricity. Since the component S is the only force that changes the description of areas, and the element of double areas is

$$r^2 . \frac{du}{dt} = k\sqrt{p}$$

$$\frac{d . \sqrt{p}}{dt} = \frac{r . S}{k}$$

$$\frac{dp}{dt} = \frac{2r\sqrt{p}}{k} . S$$

If we differentiate the equation $p = a(1-e^2)$, we have

$$2ede . \frac{a}{p} = \frac{da}{a} - \frac{dp}{p}$$

or

$$ede = \frac{1}{k\sqrt{p}} . \left(pe \sin v . R + \frac{p^2}{r} . S - \frac{r}{a} p . S\right)$$

But

$$\frac{p}{r} - \frac{r}{a} = e \cos v + e \cos E$$

and thus

$$\frac{de}{dt} = \frac{1}{k\sqrt{p}} . \{p \sin v . R + p(\cos v + \cos E) . S\}$$

To find $ed\omega$ we differentiate the polar equation of the ellipse, and since $v = u - \omega$, we have

$$ed\omega = \frac{p}{ar \sin v} . da - \frac{a(\cos E + e)}{r \sin v} . de$$

Substitute for da and de, take out the common factor $\frac{1}{k\sqrt{p}} . \frac{a}{\sin v}$, and the coefficient of R is

$$\frac{2p}{r} e \sin v - \frac{p}{r}(\cos E + e) \sin v = \sin v . \frac{p}{r} . (e - \cos E)$$

As $e - \cos E = -\frac{r}{a} . \cos v$, by restoring the factor the coefficient of R is $-\frac{1}{k\sqrt{p}} . p \cos v$. For the coefficient of S, taking out the same factors, we have

$$\frac{2p^2}{r^2} - \frac{p}{r}(\cos E + e) . (\cos v + \cos E)$$

Eliminate $\cos E$, and multiply and divide by a, then we have

$$\frac{2a(1 + e\cos v)^3 - a(\cos v + 2e + e^2 \cos v)(2 \cos v + e + e \cos v^2)}{a(1 + e \cos v)}$$

$$= \frac{2a + ae\cos v + 2ae^2 \cos v^2 + ae^3 \cos v^3 - 2a \cos v^2 - ae \cos v^3 - ae^3 \cos v - 2ae^2}{a(1 + e \cos v)}$$

The first and last terms give $2p$. The second and penultimate terms give $pe \cos v$, and

$$\frac{2p + pe\cos v}{1 + e \cos v} = p + r$$

The remaining terms give $a \cos v^2 . \{e^2(2 + e \cos v) - (2 + e \cos v)\}$. Now, $2 + e \cos v = \frac{p + r}{r}$; and since $\frac{p}{r}$ is divided out by $1 + \cos v$, by restoring the factors the coefficient of S is, $(p + r) \sin v$; so that

$$\frac{e . d\omega}{dt} = \frac{1}{k\sqrt{p}} . \{-p \cos v . R + (p + r) \sin v . S\}$$

To this we must add the change of the node. The remaining element in the plane of the orbit is the mean anomaly, for which we have

$$M = E - e \sin E$$

$$dM = dE - e \cos E . dE - \sin E . de$$

From the equation

$$r = a(1 - e \cos E)$$

$$e \cos E . dE = \frac{\cos E^2}{\sin E} . de - \frac{r \cos E}{a^2 \sin E} . da$$

If we differentiate

$$r \sin(u - \omega) = a \cos E - ae$$

$$dE = -\sqrt{1-e^2} . d\omega + \frac{\cos E - e}{a \sin E} . da - \frac{1}{\sin E} . de$$

From these values we find

$$dM = -\sqrt{1-e^2} . d\omega - \frac{2}{\sin E} . de + \frac{r}{a^2 \sin E}(\cos v + \cos E) . da$$

Substituting the values of de and da,

$$dM = -\sqrt{1-e^2} . d\omega - \frac{2}{\sin E} . \frac{1}{k\sqrt{p}} . \{p \sin v . R + p(\cos v + \cos E) . S\}$$

$$+ \frac{r}{a^2 \sin E} . (\cos v + \cos E) . \frac{2a^2}{k\sqrt{p}} . \left(e \sin v . R + \frac{p}{r} . S\right)$$

The factor of S is zero, and the factor of R reduces to

$$-\frac{1}{k\sqrt{p}} \cdot 2r\sqrt{1-e^2}.$$

Hence

$$dM = -\sqrt{1-e^2} \cdot d\omega - \frac{1}{k\sqrt{p}} \cdot 2r \cdot \sqrt{1-e^2} \cdot R$$

This value requires two corrections; one for the motion of the orbit plane, and one for the variation of the mean motion. When we pass from one value of M to another we must use the actual, or disturbed value of μ; and hence must add to $dM \int \frac{d\mu}{dt} \cdot dt$. We have, therefore, a double integral in the value of M.

The component W is the disturbing force that alters the node and inclination. This force makes the orbit plane revolve about the radius-vector (Hansen, *Auseinandersetzung*, p. 67), through the small angle $d\xi$ in the time dt. Consider at any moment the spherical triangle, equinox, node, and planet, or EAP. The side EA is Ω; the side AP is u; and the side EP is constant. The angle at E is constant; the angle at A is $180° - i$; and the variation of the angle at P is

$$d\xi = \frac{W \cdot dt}{c \sin\psi} = \frac{r \cdot W \cdot dt}{k\sqrt{p}}$$

The differential formulas of a spherical triangle give

$$\begin{aligned} di &= \cos u \cdot d\xi \\ du &= -\cos i \cdot d\Omega \\ \sin i \cdot d\Omega &= \sin u \cdot d\xi \end{aligned}$$

Hence we have

$$\frac{di}{dt} = \frac{W}{k\sqrt{p}} \cdot r\cos u$$

$$\frac{d\Omega}{dt} = \frac{W}{k\sqrt{p}} \cdot \frac{r\sin u}{\sin i}$$

The variation du, arising from the rotation of the orbit plane, must be applied to arcs measured on the orbit; such as ω, the distance from the node to the perihelion, the longitude of the perihelion, $\pi = \Omega + \omega$, and the longitude in the orbit, $L = \Omega + u$.

Collecting results we have

$$\begin{aligned}
\frac{da}{dt} &= \frac{2a^2}{k\sqrt{p}} \cdot \left(e\sin v \cdot R + \frac{p}{r} \cdot S\right) \\
\frac{d\mu}{dt} &= -\frac{3a\mu}{k\sqrt{p}} \cdot \left(e\sin v \cdot R + \frac{p}{r} \cdot S\right) \\
\frac{de}{dt} &= \frac{1}{k\sqrt{p}} \cdot \{p\sin v \cdot R + p(\cos v + \cos E) \cdot S\} \\
e \cdot \frac{d\omega}{dt} &= \frac{1}{k\sqrt{p}} \cdot \{-p\cos v \cdot R + (p+r)\sin v \cdot S\} - e\cos i \cdot \frac{d\Omega}{dt} \\
\frac{di}{dt} &= \frac{r\cos u}{k\sqrt{p}} \cdot W \\
\frac{d\Omega}{dt} &= \frac{r\sin u}{k\sqrt{p}} \cdot \frac{1}{\sin i} \cdot W \\
\frac{dM}{dt} &= -\sqrt{1-e^2}\left\{\frac{d\omega}{dt} + \cos i\,\frac{d\Omega}{dt}\right\} - \frac{2r\sqrt{1-e^2}}{k\sqrt{p}} \cdot R + \int\frac{d\mu}{dt} \cdot dt
\end{aligned}$$

This method of resolving the disturbing force into the components R, S, and W, is old-fashioned, as well as that of T, N, and W, but the former can be easily computed, while the latter cannot. The above formulas give an idea of the action of the disturbing force on the orbit of the planet. The results which Airy has pointed out in his book on Elementary Celestial Mechanics can be deduced immediately from these formulas. The formula for $e \cdot d\omega$ explains the retrograde motion of the line of apsides of the orbit of *Hyperion* through the action of *Titan*.

1906 *March* 29.

NOTES ON SOME LONG-PERIOD VARIABLE STARS.

By A. STANLEY WILLIAMS.

The following notes are in continuation of those already published in the *Astronomical Journal*. The introductory remarks in No. 559, p. 62, will apply generally to the present series. The observations were all made with a $6\frac{1}{2}$-in. reflector, a power of 73 being usually employed. The maxima and minima have all been derived from single curves.

RW Andromedae (189.1904).

R.A. $= 0^h\,41^m\,56^s$. Decl. $= +32°\,8'.4$ (1900).

Hartwig suggested a period of 435 days for this variable, with a maximum on 1904 Dec. 10 (*A.N.*, No. 4009). According to observations obtained here on 13 nights, between 1904 Dec. 7 and 1905 Feb. 2, the maximum occurred in 1904 on Dec. 23 (about $8^M.0$). From 1905 Aug. 5 to Nov. 23 the variable remained below 12^M, though on several nights a faint star, about 12^M, was seen at or near its place. On Dec. 27 of last year it was still invisible in a $6\frac{1}{2}$-in. reflector (fainter than $11\frac{1}{2}^M$); but on 1906 Jan. 22, it was nearly equal in brightness to DM. +31°114 ($9^M.0$), and further observations on 5 nights, between this date and Feb. 25, indicate a well-defined maximum for 1906 Feb. 5 (about $8^M.6$). The interval between the two maxima is 409 days, nearly the same as that suggested by Hartwig, so that the resulting elements of variation will be,

Maximum = 1904 Dec. 23 (J.D. 2416838) + 409^d E.

RU Andromedae (11.1903).

R.A. = $1^h\ 32^m\ 47^s$, Decl. = $+38°\ 9'.5$ (1900).

Observations made on 21 nights, between 1905 Sept. 27 and 1906 Jan. 13, show that a well-defined maximum occurred on 1905 Dec. 9 ($9^M.5$). The following maxima and minima have now been observed:

E	Date	J.D.	Mag.
	MAXIMA.		
5	1903 Dec. 11	2416460	9.9
8	1905 Dec. 9	7189	9.5
	MINIMA.		
4	1902 Dec. 18	2416102	12.7
5	1903 Aug. 25	6352	12.2

From which the following elements have been derived:

Maximum = 1900 Aug. 2 (J.D. 2415234) + $244^d.7$ E.

the interval $M-m$ being 108 days.

RV Andromedae.

R.A. = $2^h\ 4^m\ 34^s$, Decl. = $+48°\ 27'.6$ (1900).

According to 13 observations, between 1904 Nov. 12 and 1905 Mar. 19, a well-defined minimum ($10^M.4$) occurred on 1905 Jan. 29, two days later than the expected date. Six observations in the early part of 1905, by Miss MARY WHITNEY, at Vassar College Observatory, have been published in the *A.N.*, No. 4050. Making allowance for a systematic difference of scale,* these observations are in very good agreement with those made here, excepting that the first estimate makes the star about one-fifth magnitude brighter than mine.

In 1905 the star rose rapidly from $10^M.1$ on Aug. 5 to a sharply-defined maximum on Sept. 14 ($8^M.0$), and then declined, at first nearly as quickly, to a minimum ($10^M.3$) on Nov. 27. The variable then rose to $9^M.9$ on Dec. 19, but then remained nearly stationary until 1906 Jan. 22, when a slow rise set in again. This minimum occurred nearly two months earlier than the expected date, and the maximum preceding it nearly a month earlier. There are, perhaps, traces of a secondary minimum about 1906 Jan. 14. Observations were made on 35 nights, between 1905 Aug. 5 and 1906 Feb. 20, so that the light-changes during the intervening period are well ascertained. Owing to the irregularities referred to above, it is doubtful at present what modification of the published elements is indicated. It would seem as though the recent prolonged minimum, or standstill, will have the effect of making the next maximum occur nearly on the computed date.

1205. *Y Persei.*

Observations made on 9 nights, between 1904 Dec. 13 and 1905 Mar. 19, indicate a minimum for 1905 Feb. 11 ($9^M.8$), but the results are insufficient to give a good determination of the date. Observations on 22 nights, between 1905 Aug. 5 and 1906 Jan. 13, give a good determination of another minimum for 1905 Oct. 29 ($9^M.8$). The above dates of minimum are 11 and 12 days respectively later than those given by the elements of GRAFF.*

RU Persei (187.1904).

R.A. = $3^h\ 23^m\ 57^s$, Decl. = $+39°\ 18'.9$ (1900).

Observations on 36 nights, between 1905 Aug. 5 and 1906 Feb. 25, show that the star was at minimum brightness ($10^M.4$) on 1905 Sept. 17; and that a well-defined maximum ($9^M.4$) occurred on 1905 Nov. 26. At the end of February of this year the star was apparently just about at minimum again.

RY Lyrae.

R.A. = $18^h\ 41^m\ 15^s$, Decl. = $+34°\ 34'.3$ (1900).

Observations were made on 9 nights, between 1905 May 5 and Oct. 19. On the former date the variable was $9^M.2$, and it diminished steadily in brightness to $12^M.4$ on July 27. On Oct. 19 it was invisible in a 6½-in. reflector, and must have been fainter than 12^M. Maximum brightness was probably attained about 1905 May 2. As four maxima have now been observed, fresh elements of variation have been derived as under.

Maximum = 1900 Nov. 13 (J.D. 2415337) + 327^d E.

The observed maxima are:

E	Date	J.D. 241+	Mag.	O—C
0	1900 Oct. 23	5316	. .	-21^d
3	1903 July 25	6321	9.5	+ 3
4	1904 June 10	6642	10.6	− 3
5	1905 May 2±	6968±	9.2	− 4

The first maximum is a photographic one, but it appears to be quite well determined, and it can hardly be more than a few days in error. The maximum of 1905 cannot well be later than May 2, whilst it might be somewhat earlier. It appears probable, therefore, that the period is not of uniform length. According to M. and G. WOLF the variable is not visible, and fainter than 13^M, on a Heidelberg plate, dated 1896 Nov. 4 (see *A.N.*, No. 4046). The computed date of maximum in that year is May 22, so that the star should have been faint at the date of the photograph, which thus confirms the general correctness of the elements.

SU Lyrae (59.1905).

R.A. = $18^h\ 50^m\ 7^s$, Decl. = $+36°\ 23'.1$ (1900).

This star, the variability of which was discovered at Heidelberg by M. and G. WOLF, was about 11^M in the early part of May, 1905, and from this time it decreased in brightness at a nearly uniform rate until July 30, after

* Miss WHITNEY gives $9^M.4$ for the magnitude of DM. +48°614. I had assumed it to be $8^M.7$, after the DM.

* *Mitteilungen der Hamburger Sternwarte*, Nr. 8, p. 24.

which it could no longer be seen with a 6½-in. reflector. The last negative observation was made on Oct. 4.

6816. *Z Lyrae.*

A well-defined maximum ($9^M.9$) occurred 1905 June 17. Nineteen observations between May 5 and Sept. 3.

6783. *RX Lyrae.*

A maximum ($10^M.9$) occurred 1905 May 19, but this is not a good determination, the curve being somewhat flat, and the observations not well distributed. Twelve observations between the limiting dates May 5 and June 24.

6895. *RU Lyrae.*

Nineteen observations, between 1905 June 22 and Oct. 27, give a good determination of the time of maximum, namely, 1905 Sept. 2 ($10^M.2$). The maximum was a well-defined one.

7019. *T Cygni.*

A well-defined maximum occurred 1905 Sept. 19 ($8^M.6$). Twenty observations between June 22 and Nov. 16.

7571*a*. *TW Cygni.*

A well-defined maximum occurred 1905 May 26 ($9^M.0$). Fourteen observations between May 5 and Aug. 5.

RV Pegasi (159.1904).

R.A. = $22^h\ 21^m\ 2^s$, Decl. = $+29°\ 57'.9$ (1900).

This star was invisible ($<12\frac{1}{2}^M$) in a 6½-in. reflector on 1905 June 21, but on July 30 it was observed about 11^M, and a pretty definite maximum ($10\frac{1}{2}^M$) was reached on Aug. 5. From this date there was a slow decline to below 12^M by Nov. 23, with a distinct secondary maximum about Aug. 27. The date of the previous maximum is somewhat uncertain, though, having regard to the photographic observations of Görz at Heidelberg, it probably occurred somewhere about 1904 Aug. 5, so that the period is evidently nearly equal in length to a year. It is noteworthy that the recent maximum was about two whole magnitudes fainter than that of 1904, since in the latter year the star must have attained to a visual magnitude of 8½ or brighter. The decline after the maximum of 1904 was also very much more rapid than that after the recent maximum. The comparatively rapid decline in 1904 is shown by the observations of Mr. WICKHAM at the Radcliffe Observatory, Oxford, published in the *Monthly Notices*, Vol. 65, p. 161.

20 Hove Park Villas, Hove, 1906 March 9.

MICROMETRICAL OBSERVATIONS OF THE FIFTH SATELLITE OF *JUPITER*,

BY E. E. BARNARD.

The present season has been very bad for observations of the Fifth Satellite of *Jupiter*.

Though the planet has been splendidly placed for observation, only a few measures of the satellite have been obtained, and those with very great difficulty, with the exception of the latter set of measures on 1905 Dec. 5, at which time the satellite was very easy to measure. Unfortunately the measures were interrupted at midnight on that date, and the complete elongation could not be observed.

I have devoted most of the time to measures from the polar limbs of the planet, because such measures have been very few in the previous observations, and are much needed to determine the inclination of the satellite's orbit. The results are not so satisfactory as I could wish. In bad seeing the polar limbs are not so well seen as the equatorial, because of the duskiness of the polar regions of *Jupiter*, and hence the settings are not so accurate.

Following are all the observations I have been able to secure. The times are $6^h\ 0^m$ slow of Greenwich Mean Time.

OBSERVATIONS OF THE SATELLITE.

1905	Central Stand. Time		Appt. lat.	Comp.	
		From N. limb			
Oct. 28	10^h 3^m 54^s	23″.48	−0″.78	5	Satel. preceding. Seen for only a few minutes
		From S. limb			
Oct. 28	10 10 53	22.45	−0.24	1	
		From S. limb			
Nov. 25	12 55 56	25.05	+1.84	4	Satel. following. Seeing very poor.
		From N. limb			
Nov. 25	13 8 21	22.18	+1.03	4	
		From S. limb			
Nov. 25	13^h 12^m 31^s	24″.28	+1″.07	4	Very difficult.
		From N. limb			
Nov. 25	13 15 43	22.65	+0.56	4	
		From S. limb			
Nov. 25	13 19 1	24.18	+0.97	4	
		From f. limb	From center		
Nov. 25	13 24 18	34.21	58.96	4	
	13 26 52	34.28	59.04	3	

1905	Central Stand. Time		From center	Comp.	
		From p. limb			
	h m s	″	″		
Nov. 25	13 29 6	83.33	58.58	3	
	13 31 16	83.06	58.31	3	
		From f. limb			
Nov. 25	13 33 43	33.30	58.05	3	
	13 35 56	32.79	57.55	3	
	13 38 26	32.38	57.13	3	
	13 40 58	31.70	56.46	4	
		From p. limb			
Nov. 25	13 43 49	80.67	55.91	3	
	13 46 12	79.96	55.79	3	
		From N. limb	Appt. lat.		
Nov. 25	13 50 37	23.49	−0.28	3	
	13 53 23	23.68	−0.47	4	
		From S. limb			
Nov. 25	13 56 31	22.92	−0.29	3	
	13 58 36	23.08	−0.13	3	
	14 1 9	22.54	−0.67	3	
		From N. limb			
Nov. 25	14 3 51	24.51	−1.30	3	
	14 6 43	24.57	−1.36	4	
		From S. limb			
Nov. 25	14 10 9	22.58	−0.63	3	
	14 12 49	22.56	−0.65	3	
	14 15 6	21.85	−1.35	3	
		From N. limb			
Nov. 25	14 18 12	24.73	−1.53	3	
	14 21 32	24.83	−1.62	4	
		From p. limb	From center		
Dec. 5	6 59 45	29.58	54.18	2	Satel. preceding
	7 1 45	28.99	53.59	2	
		From f. limb			
Dec. 5	7 4 7	77.17	52.57	2	Seeing very good;
	7 5 50	77.27	52.67	2	satellite. v. faint;
					difficult
		From N. limb	Appt. lat.		
Dec. 5	7 10 53	22.47	+0.58	3	
	7 13 23	22.73	+0.32	3	
		From S. limb			
Dec. 5	7 16 10	23,37	+0.32	3	
	7 18 26	23.87	+0.82	3	
		From N. limb			
Dec. 5	7 21 33	22.24	+0.81	3	
	7 23 52	22.27	+0.79	3	

1905	Central Stand. Time		Appt. lat.	Comp.	
		From S. limb			
	h m s	″	″		
Dec. 5	7 26 55	24.04	+0.98	3	
	7 29 37	24.21	+1.16	3	
		From f. limb	From center		
Dec. 5	11 36 12	34.97	59.59	3	Satellite follow-
	11 38 58	35.43	60.05	3	ing. Seeing very
	11 40 40	35.86	60.48	2	g'd. Satel. bright
		From p. limb			
Dec. 5	11 42 56	84.73	60.13	3	
	11 45 45	85.20	60.60	3	
	11 48 45	85.19	60.59	3	
		From f. limb			
Dec. 5	11 53 0	36.48	61.10	3	
	11 55 40	36.58	61.20	3	
	11 58 17	36.34	60 96	2	
		From S. limb	Appt. lat.		
Dec. 5	12 2 46	23.72	+0.66	3	
		From N. limb			
Dec. 5	12 5 0	22.45	+0.60	3	
		From S. limb			
Dec. 12	11 48 57	23.89	+1.04	3	Satel. fol. Seeing
	11 51 25	23.84	+0.99	2	excessively bad.
		From N. limb			
Dec. 12	11 54 25	22.57	+0.28	3	Satellite very
	11 57 57	22.55	+0.30	2	difficult
		From S. limb			
Dec. 12	11 59 59	23.66	+0.81	3	
		From N. limb			
Dec. 12	12 3 2	23.26	−0.41	4	
		From S. limb			Satellite follow-
Dec. 16	12 11 8	22.08	−0.63	4	ing. Seeing very
		From N. limb			bad, and satellite
Dec. 16	12 13 58	24.26	−1.56	5	very difficult.
		From f. limb	From center		
Dec. 26	9 39 44	33.25	56.97	3	Satel. fol. Obs. v.
	9 46 29	34.51	58.23	4	diffi't and uncert.
		From p. limb			
Dec. 26	9 53 51	82.56	58.84	3	
	9 59 1	82.68	58.96	2	
		From f. limb			
Dec. 26	10 3 10	33.48	57.20	1	

Following are the computed apparent diameters of *Jupiter* used in the reductions:

1905	App. eq. diam.	App. polar diam.
	″	″
Oct. 28	. .	45.392
Nov. 25	49.513	46.416
Dec. 5	49.200	46.412
12	. .	45.701
16	. .	45.409
26	47.441	. .

Following are the position-angles of the micrometer wires at the times of the latitude measures.

1905 Oct.	28	80.°31
Nov.	25	79.46
Dec.	5	79.24
	12	79.10
	16	78.56
	26	78.20

A number of position-angles of the belts of *Jupiter* were accidentally omitted in the previous papers on the Fifth Satellite. They are collected and given here as a continuation of those previously printed.

1899 May 22	P.A. of belts $113^\circ.08$ (5)	at	$9^h\ 20^m$
1900 Apr. 18	100.20 (4)		14 50
24	102.77 (4)		13 35
June 1	101.59 (4)		10 50
July 10	102.73 (4)		7 50
11	105.04 (4)		8 0
25	102.02 (4)		7 55
29	101.55 (4)		7 50
30	102.63 (3)		8 15
Aug. 6	101.92 (4)		7 30
1900 Aug. 7	P.A. of belts $101^\circ.84$ (4)	at	$7^h\ 40^m$
14	101.95 (4)		7 20
1901 July 29	88.60 (5)		9 30
1903 July 20	66.57 (5)		13 20
1904 Dec. 31	67.34 (5)		6 10
1905 Nov. 25	78.97 (6)		14 30
Dec. 16	76.94 (4)		12 30
1906 Feb. 6	77.00 (6)		8 20

In these last observations there was no north equatorial belt. There was, however, a strong south equatorial belt, and the wires were made parallel to it. The measures may not be so good as previous ones because of a want of symmetry of the belts with respect to the equator. The Great Red Spot was observed in transit 1905 Dec. 9, at $6^h\ 4^m$ Central Standard Time. The Spot was very pale.

Yerkes Observatory, 1906 *March* 14.

MAXIMA OF LONG-PERIOD VARIABLES,

By MARY W. WHITNEY.

The following variable stars have been observed at Vassar College during periods including their predicted maxima for 1905:

6449. *T Draconis*. Predicted maximum June 10th.

Observations of this star extended from June 3d to Oct. 28th, 1905. During that interval it did not differ to a noticeable degree from B.D. 78°1771, magnitude 9.5, by B.D. standard. *T Draconis* is not given in the original B.D. catalogue, but it is included in the revised charts of 1898. If the star is correctly located on the chart, the right star was observed.

5955. *R Draconis*. Predicted maximum July 4th.

Eleven observations, May 24th to Oct. 6th, indicate a maximum of $7^M.7$ on July 9th. An observation on Jan. 26th, 1906, gives a magnitude 9.5. A minimum probably occurred early in November.

5237. *R Bootis*. Predicted maximum July 18th.

From ten observations, May 24th to Sept. 21st, a maximum of $6^M.8$ on July 16th was derived.

6044. *S Herculis*. Predicted maximum July 19th.

Nine observations, May 24th to Sept. 11th, place maximum at $7^M.7$, on July 20th.

4948. *R Can. Ven.* Predicted maximum Aug. 11th.

Observations extended from June 25th to Aug. 17th, and magnitudes ranged from 8.9 to 10.2, all on the downward slope of the curve. Maximum may have occurred about June 15th.

678. *U Persei*. Predicted maximum Aug. 19th.

Nine observations, June 25th to Oct. 28th, give maximum $7^M.8$, on July 30th.

5950. *W Herculis*. Predicted maximum Aug. 31.

Eleven observations, May 25th to Oct. 7th, indicate a maximum of $8^M.2$ on Aug. 22d.

6512. *T Herculis*. Predicted maximum Sept. 7th.

Curve based on nine observations, July 3d to Oct. 28th, gives a maximum of $7^M.8$ on Sept. 3d.

5194. *V Bootis*. Predicted maximum Sept. 10th.

Eight comparisons, July 9th to Oct. 30th, show that maximum occurred about Oct. 20th, with magnitude 7.8. Two later observations, Feb. 23, 1906, magnitude 10.4, and April 16th, magnitude 10.2, indicate a minimum about the middle of March.

Vassar College Observatory, 1906 *April* 21.

SECONDARY NUCLEUS TO COMET *b* 1906 (KOPFF'S, *March* 3),*

By E. E. BARNARD.

On March 17—the first observation of this comet here—I noticed with the 40-inch telescope a faint star-like condensation close preceding the nucleus. The unusually slow motion of the comet made it uncertain whether the object was a faint star or a secondary nucleus. It was seen again, however, on March 24, at a somewhat greater distance from the nucleus, and was (as before) involved in the tail, and rather difficult to see. By hiding the brighter

part of the comet the object appeared as a small star-like nucleus. The following position is referred to the nucleus of the comet:

1906 March 24^d 18^h 40^m Gr. M.T.

Position-angle, 300°.0 (5 obs.) Distance, 4″.68 (4 obs.)

The second nucleus was about 14^M–15^M.

If this object were free from the nebulosity of the main body, it would doubtless appear as a distinct comet — like one of the companions of comet V 1889. It is a similar case to that of SWIFT's comet (a 1899), which was observed here and at the Lick Observatory to have a second nucleus or small comet receding from it (see *A.J.* 464).

Yerkes Observatory, 1906 *March* 26.

* *From Supplement to Nos.* 584–5.

Looking back over the past twenty-five years at the various cases of double or multiple comets, it would appear that BIELA's double comet was not such a rare object after all.

The following additional observation of the double head of KOPFF's comet has been made:

1906 March 31^d 13^h 0^m Central Standard Time.

Position-angle 302°.7 Distance 5″.67.

There seems to be a slight change in the angle and distance, but it is quite difficult to measure satisfactorily. The small nucleus is 3 or 4 magnitudes fainter than the principal one.

COMET c 1906 (*ROSS, March* 17),*

A comet was discovered by Ross, at Melbourne, Australia, on March 17, in the position first given below. It is described as circular, 3′ diameter, brightness 8^M, with some central condensation.

The other positions below were observed by Prof. H. R. MORGAN, of the Morrison Observatory, Glasgow, Missouri, and received by telegraph *via* Harvard College Observatory.

1906 Gr. M.T.	α	δ	Observer
	h m s	° ′ ″	
Mar. 17.914	2 3 52	−7 41 –	Ross
19.5779	2 9 31.4	−5 47 25	Morgan
21.5782	2 16 3.8	−3 35 21	Morgan
22.5773	2 19 3.7	−2 32 29	Morgan

The following elements and ephemeris, telegraphed by Admiral ASA WALKER, Superintendent Naval Observatory, Washington, also *via* H.C.O., were computed by Miss ELEANOR LAMSON, from observations of Mar. 19, 20, 21.

ELEMENTS.

$$T = 1906 \text{ Feb. } 21.47 \text{ Gr. M.T.}$$

$$\left.\begin{aligned} \omega &= 278^\circ\ 43' \\ \Omega &= 72\ \ 51 \\ i &= 81\ \ 27 \end{aligned}\right\} 1906$$

$$q = 0.743$$

EPHEMERIS.

1906	α	δ	Br.
	h m s	° ′	
March 25	2 27 47	+ 0 26	0.67
29	38 45	4 11	
April 2	48 52	7 36	
6	2 58 17	+10 42	0.39

* *From Supplement to Nos.* 584–5.

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A COMPARISON OF TWO METHODS OF OBTAINING LEVEL CORRECTIONS IN PRIME-VERTICAL TRANSIT OBSERVATIONS.

By GEORGE A. HILL.

[Communicated by Rear-Admiral ASA WALKER, Superintendent Naval Observatory.]

In ascertaining small displacements that occur in star positions, due to a number of causes, astronomers depend for a part of the accuracy of their results, especially if the measurements are made with the zenith-telescope, or the prime-vertical transit, upon the change of the air bubble in a level vial indicating a corresponding change in position of the instrument.

In modern practice, those who use the zenith-telescope have equipped their instruments with two level vials, so that one will check the other. It is somewhat cumbersome to attach two vials to the striding frame, used in connection with the prime-vertical transit.

As is well known, each time the lack of horizontality of the pivots is to be measured, the level box must be reversed end for end, and to obtain accordant values of the level, a perpendicular plane must pass through the points of contact of vial in its box, and the axis of the instrument. I have hesitated as to the wisdom of attaching two vials to the striding frame of our instrument, fearing that each of them would not continue to conform to that plane.

The error due to lack of level in the axis of the prime-vertical transit instrument enters an observed position of a star with practically its full amount, because objects selected for the transits are relatively near the zenith when they cross the prime-vertical. It is, therefore, of the utmost importance, that the level correction should be determined with the greatest care.

In what follows, it will be understood that STRUVE's method of observing, that is, a reversal of the instrument on both verticals, to eliminate the collimation, and the inequality of pivots, is the only one considered.

It is now in order to consider a mode by which the deviation of the axis of the prime-vertical transit from a horizontal plane can be obtained, without the aid of a level vial. It is beyond question that an independent measurement of the error, or its elimination by the process of observing, will greatly increase the value of a declination, or latitude secured with the form of instrument here being considered.

To fix our ideas upon a proper theory, suppose we revert to the general equation of a transit instrument, assuming the following notation. These equations, which are introduced to illustrate what follows, are by BESSEL, but are here given, with one or two modifications, as they appear in the second volume of CHAUVENET.

Let φ = the latitude of the place in which the instrument is mounted, δ the declination of a star, τ the hour-angle of the same when on the prime-vertical, a the azimuth of the rotation axis of the instrument, positive when the north end is east of north; c the collimation constant of a thread, positive if the thread is north of the collimation axis; b the inclination of the rotation axis, positive when the north end is highest; λ the longitude of the meridian of the instrument, positive when west of the true meridian; and D the declination of the north end of the axis, then we have by proper substitutions, these equations:

$$\left.\begin{aligned} \cos D\cos\lambda &= -\sin b\cos\varphi + \cos b\cos a\sin\varphi \\ \cos D\sin\lambda &= \quad\cos b\sin a \\ \sin D &= \quad\sin b\sin\varphi + \cos b\cos a\cos\varphi \end{aligned}\right\} \quad (1)$$

Now assume h and β to satisfy

$$\left.\begin{aligned} h\sin\beta &= \sin b \\ h\cos\beta &= \cos b\cos a \end{aligned}\right\} \quad (2)$$

and from equation (1) we have

$$\left.\begin{aligned} \cos D\cos\lambda &= h\sin(\varphi-\beta) \\ \cos D\sin\lambda &= \cos b\sin a \\ \sin D &= h\cos(\varphi-\beta) \end{aligned}\right\} \quad (3)$$

From the general equation of the transit instrument we have,

$$(4)\quad \begin{cases} -\sin c = \sin D \sin \delta - \cos D \cos \delta \cos (T-\lambda) \\ \sin c = \sin D \sin \delta - \cos D \cos \delta \cos (T'-\lambda) \end{cases}$$

Let us designate $(T-\lambda)$ and $(T'-\lambda)$ by t and t' respectively, in which t represents one-half the elapsed sidereal time it takes a star to pass from a certain thread in the instrument, on the east vertical, to the same thread on the west, in one position of the axis, and t' similarly for a reversed position.

The sum of the equations in (4) gives

$$\cot D = \tan \delta \sec \tfrac{1}{2}(t+t') \sec \tfrac{1}{2}(t-t'),$$

and equation (3) gives

$$\cot D \cos \lambda = \tan (q-\beta).$$

In the last equation it will be seen that β represents the correction for level, corresponding to the time of the passage of the star over the east and west verticals, respectively.

Now let $q' = q-\beta$, and we have

$$(5)\quad \begin{cases} \tan q' = \tan \delta \,.\, \sec \tfrac{1}{2}(t+t') \sec \tfrac{1}{2}(t-t') \cos \lambda \\ q = q' + \beta \end{cases}$$

Equation (5) is the equation developed by STRUVE for reducing a prime-vertical transit made in accordance with his method of observing.

We now wish to devise a method by which the value of β, whatever it may be, will have opposite effects upon the hour-angle during the process of noting the transits of the star as it crosses the several threads in the instrument.

It may be seen from a purely mechanical analysis that, as the instrument points toward the heavens, at a certain angular distance from the zenith on the east vertical, should the south end of the axis be too high, for a position in our latitude, although it had been accurately adjusted in collimation and azimuth, a star would transit the line of collimation before it did the great circle of the prime-vertical. Likewise, if the instrument is turned down, and pointed toward a basin filled with mercury, and at a similar angular distance from the nadir, the star would appear to transit, by reflection, the line of collimation after it did the great circle of the prime-vertical. From this it follows, the effect of the inclination of the axis, on the transit time, would have opposite signs from the two methods, and by a proper combination can be eliminated.

We may prove the same thing by an algebraic analysis. The general equation of the transit instrument in the prime-vertical can be so reduced that the term containing the correction for level error would be

$$\frac{\cos z \sin b}{\cos a \cos b}$$

in which z is the zenith-distance of the star in the prime-vertical, a its right-ascension, and b the error of level. The sum of the cosines of zenith-distance, for all the threads, both by direct and reflected transits, that are to enter the numerator of the above expression, would be practically zero.

The writer, anxious that the observations he has carried on with the prime-vertical transit at the Naval Observatory should be put to the most exhaustive test, to prove beyond doubt whether there are defects in securing levelings of the instrument that are in any manner causing erroneous results, which are not manifest by one method of securing that correction, has introduced the mode of securing direct and reflected transits into his observing with the instrument. So far as I am aware no one has up to this time published results with the prime-vertical transit from which the level error has been eliminated by the method of observing.

In the Fall of 1903 the instrument was dismounted for a general cleaning, after it had been in constant use for ten years, and advantage was taken of that opportunity to place in the reticule of the micrometer a system of threads that would permit of making these double observations on the same night. Experience has shown the writer, that in general, it is practicable to obtain the transits by the two methods, and at the same time carry out the STRUVE mode of using the instrument.

The system of threads is made up of four groups, each containing seven threads. Two groups are placed on each side of the thread that marks the middle of the field, with sufficient time allowance to permit the observer, after the star has crossed one set, to change from the couch to a step ladder, or the opposite.

In the following table are printed results of the declination of *a Lyrae* obtained in accordance with what has been outlined above. A comparison is made of the correction for level that has been secured from the transits, with that obtained by the use of the level vial.

It is self-evident, if the method of direct and reflected transits eliminates the level error completely, when the results of a direct and reflected observation are combined into one quantity, then one-half of the difference of these results should be equal to the level error derived from the vial. If it is not so, it is fair to presume that erroneous levelings are being obtained.

The observations printed include both day and night work. It should be stated that at times, on account of high winds, it becomes impracticable to secure a quiet surface of mercury, and then only direct observations are obtained.

The data in the table are made up as follows: In the second and third columns will be found the declination of the star for the date, reduced to 1904.0, the seconds only

being printed. Each of the quantities in the two columns is that resulting from the mean of seven threads, without the application of the level correction secured with the leveling apparatus, the first being the reflected, and the second the direct transit, marked R' and D', respectively. The fourth column contains the mean of the two quantities in the second and third, or the declination free from level and collimation error, and inequality of pivots.

The fifth column contains declinations resulting from the reflected transit, with the level correction, as obtained by aid of the spirit vial applied to it. The sixth column contains declinations resulting from the direct transits, with the level correction as obtained with the vial applied to it.

The seventh column contains one-half the difference between each reflected and direct transit. Along side this, in the eighth column, is the level correction obtained by the aid of the leveling apparatus, expressed as a correction to a direct transit. The last column contains the difference between the levels as obtained by the two methods.

A study of these figures clearly indicates that the level error of the prime-vertical transit, as determined by means of the spirit-vial, corresponds very satisfactorily with that secured by an entirely independent method.

A Table of Comparison of Direct and Reflected Declinations, and the Correction for Level as Obtained from the Transits, and the Leveling Apparatus of the Instrument.

Date	R'	D'	$\frac{R'+D'}{2}$	R	D	$\frac{R'-D'}{2}$	Level	$\frac{(R'-D')}{2}$ −Level
1904	″	″	″	″	″	″	″	″
Apr. 5	38.95	38.12	38.54	38.53	38.54	+0.42	+0.42	0.00
14	38.90	38.73	38.82	38.79	38.84	+0.08	+0.11	−0.03
17	38.76	38.67	38.72	38.70	38.73	+0.04	+0.06	−0.02
Aug. 15	39.95	37.53	38.74	38.60	38.88	+1.21	+1.35	−0.14
28	38.89	38.45	38.67	38.70	38.64	+0.22	+0.19	+0.03
Sept. 17	38.23	38.32	38.28	38.21	38.34	−0.04	+0.02	−0.06
22	39.02	38.14	38.58	38.57	38.59	+0.44	+0.45	−0.01
Oct. 1	38.27	39.31	38.79	38.72	38.86	−0.52	−0.45	−0.07
3	39.64	37.65	38.64	38.57	38.72	+1.00	+1.07	−0.07
4	39.58	37.35	38.46	38.42	38.51	+1.12	+1.16	−0.04
7	39.33	37.61	38.47	38.51	38.43	+0.86	+0.82	+0.04
16	39.48	38.02	38.75	38.86	38.64	+0.73	+0.62	+0.11
18	40.08	35.74	37.91	38.10	37.72	+2.17	+1.98	+0.19
19	38.72	38.62	38.67	38.65	38.69	+0.05	+0.07	−0.02
31	40.10	36.39	38.24	38.17	38.32	+1.86	+1.93	−0.07
Nov. 1	39.30	37.72	38.51	38.32	38.70	+0.79	+0.98	−0.19
16	39.09	37.75	38.42	38.57	38.27	+0.67	+0.52	+0.15
17	38.50	37.99	38.24	38.37	38.12	+0.26	+0.13	+0.13
19	39.20	37.55	38.38	38.15	38.60	+0.82	+1.05	−0.23
21	39.23	37.07	38.15	38.16	38.14	+1.08	+1.07	+0.01
Dec. 23	38.96	37.94	38.45	38.52	38.38	+0.51	+0.44	+0.07
1905								
Jan. 8	38.26	38.30	38.28	38.28	38.28	−0.02	−0.02	0.00
14	37.62	39.13	38.38	38.39	38.36	−0.76	−0.77	+0.01
Feb. 9	40.56	36.64	38.60	38.55	38.65	+1.96	+2.01	−0.05
Mar. 12	38.90	38.18	38.54	38.45	38.63	+0.36	+0.45	−0.09
13	38.16	39.09	38.62	38.55	38.70	−0.46	−0.39	−0.07

Taking means of the three values of the declinations given above, we have

	Reflected	$\frac{1}{2}(R+D)$	Direct
1904.0	$+38° 41' 38''.48$	$38''.50$	$38''.51$

Not considering the variation of latitude, the probable error of a single observation derived from the column headed $\frac{R'+D'}{2}$ is $\pm 0''.15$.

OBSERVATIONS OF THE CRATER *LINNÉ* DURING THE LUNAR ECLIPSE OF FEBRUARY 8, 1906,

By JOEL STEBBINS.

In accordance with the suggestion of Prof. W. H. Pickering, in *Popular Astronomy*, the following measures of the diameter of *Linné* were made at the time of the total lunar eclipse of February 8, 1906. The instrument was the twelve-inch equatorial of this observatory, with filar micrometer, and eyepiece giving a power of 170. Pre-

liminary measures were made on the nights of February 4 and 5, and the method adopted was that used by Professor PICKERING, where both micrometer wires are placed on the same side of the object, and the width of the space between the wires is changed until it appears equal to the diameter of the spot.

As a check upon the work, I also measured the small crater, which is about 49″ northwest of *Linné*, the approximate selenographic latitude and longitude of which are respectively N. 30°.03, and W. 13°.75. This little crater appeared as a well-defined round spot, and will be referred to as *Crater* 2.

Both spots were measured in a north and south direction, or at right-angles to the diurnal motion. Each observed diameter is the result of six settings, that is, three double distances. The results are given as measured, except that the width of a micrometer wire, 0″.48, has been subtracted from the observed diameters. No correction for change of *Linné*, due to variation in the moon's age, has been applied, and the reduction to a common distance has been neglected. This latter correction amounts to 0″.1 to be added to the measures of *Linné* on February 5, but the distances at other dates were so near that at the time of the eclipse, that the correction is less than 0″.1. On account of the large systematic errors possible in measuring so hazy an object, the diameters are rounded off to one decimal place. The seeing is given on a scale from 0 to 5, 5 being the best.

The last measure of *Linné* before the eclipse, was made when the spot was rapidly being lost in the umbra; and again at reappearance the first setting was obtained while *Linné* was still very dark.

For some time preceding totality the seeing was called 2, or fair; but when *Linné* had emerged, it was much worse and was called 1. The measures were made as well as possible, and without prejudice until $21^h\ 40^m$, when the settings were compared with those taken before the eclipse, and much to my surprise I found that I was measuring *Linné* larger by about thirty per cent. I then took another set of *Linné* and of *Crater* 2, and could not help noticing an immediate decrease. Rather than go on after I had become prejudiced, and with such bad seeing, the observations were discontinued. Measures on subsequent dates were made after I had forgotten the data of the eclipse night.

It will be noticed that both *Linné* and *Crater* 2 were measured larger in poor than in good seeing, and in my judgement the large apparent increase in the diameters after the eclipse was due almost wholly to this cause. The fact that the change was greater for *Linné* than for *Crater* 2, is easily explained by the hazy outline of the former. If there was any increase in the size of *Linné*, as has been reported at the time of previous eclipses, it was too small to be detected here under the conditions when the following measures were made.

DIAMETERS OF *Linné* AND OF *Crater* 2.

1906		G.M.T. h m	*Linné* ″	*Crater* 2 ″	Seeing
February	5	15 ±	3.0	..	
"	"	.. ..	3.0	..	
"	"	.. ..	3.2	..	
"	"	.. ..	3.4	..	
"	"	Mean	3.2	..	0
February	8	18 52	2.4	..	
"	"	18 57	2.5	..	
"	"	19 7	..	2.1	
"	"	19 11	..	2.0	
"	"	19 14	2.5	..	
"	"	19 17	2.4	..	
"	"	19 20	..	2.1	
"	"	19 22	..	2.1	
"	"	19 26	2.3	..	
"	"	19 29	2.4	..	
"	"	Mean	2.4	2.1	2
Linné Eclipsed.					
February	8	21 19	3.4	..	
"	"	21 22	..	2.4	
"	"	21 24	3.0	..	
"	"	21 28	3.2	..	
"	"	21 31	..	2.2	
"	"	21 34	3.2	..	
"	"	21 36	3.0	..	
"	"	21 40	..	2.5	
"	"	Mean	3.2	2.4	1

See remarks.

1906		G.M.T. h m	*Linné* ″	*Crater* 2 ″	Seeing
February	8	21 49	2.6	..	1
"	"	21 51	..	2.0	1
February	10	16 35	2.6	..	
"	"	16 37	2.7	..	
"	"	16 41	..	1.9	
"	"	16 43	..	1.8	
"	"	16 45	2.5	..	
"	"	16 47	2.6	..	
"	"	16 49	..	2.0	
"	"	16 51	..	2.0	
"	"	Mean	2.6	1.9	2
March	9	14 41	2.2	..	
"	"	14 44	2.1	..	
"	"	14 51	..	1.8	
"	"	14 53	..	1.8	
"	"	14 57	2.2	..	
"	"	15 0	2.1	..	
"	"	15 3	..	1.8	
"	"	15 5	..	1.8	
"	"	Mean	2.2	1.8	2

University of Illinois Observatory, Urbana, Ill., 1906 March.

SUNSPOT OBSERVATIONS,

MADE AT THE AMHERST COLLEGE OBSERVATORY.

By ROBERT H. BAKER.

1906 d h	New Gr.	New Spots	Disapp. Gr.	Disapp. Spots	Reapp. Gr.	Reapp. Spots	Total Gr.	Total Spots	Def.
Jan. 1 22	1	1	–	–	–	–	2	21	3
2 21	1	1	1	1	1	1	2	12	3
* 6 1	–	1	1	11	–	–	1	2	2
* 7 2	–	–	–	–	–	–	1	1	2
7 23	–	–	–	–	–	–	1	1	1
8 22	1	7	–	–	1	7	1	7	3
10 3	–	2	–	–	–	–	1	9	3
10 21	–	–	–	–	–	–	1	8	3
12 2	–	–	–	–	–	–	1	7	4
16 3	–	–	–	–	–	–	1	3	4
17 2	–	–	–	–	–	–	1	2	5
18 3	–	–	–	–	–	–	1	1	4
18 21	–	–	–	–	–	–	1	1	3
21 3	3	30	1	1	3	30	3	30	5
23 21	2	9	–	–	2	7	5	36	3
24 21	–	12	–	–	–	–	5	42	4
25 21	1	10	–	–	1	8	6	45	4
26 21	–	–	–	–	–	–	6	28	3
27 22	–	3	–	–	–	–	6	25	3
28 21	–	3	1	1	–	–	4	25	3
29 21	–	–	–	–	–	–	4	18	3
31 2	1	3	1	2	–	–	4	15	2
31 21	–	4	1	1	–	–	3	22	4
Feb. 1 22	–	–	1	1	–	–	2	14	3
3 2	–	–	–	–	–	–	2	12	4
4 0	–	–	1	1	–	–	1	3	1
6 0	1	1	–	–	1	1	2	4	4
7 0	–	–	1	3	–	–	1	1	3
8 0	–	2	–	–	–	–	1	3	5
9 22	–	5	–	–	–	–	1	8	5
11 3	–	–	–	–	–	–	1	4	4
15 3	1	1	–	–	1	1	2	2	4
16 0	1	2	1	1	–	–	2	3	4
17 0	1	1	–	–	1	4	3	6	4
18 22	–	7	1	1		–	2	12	4
19 21	–	4	–	–	–	–	2	16	3
20 21	1	2	–	–	–	–	3	12	2
22 3	–	6	–	–	–	–	3	22	4
23 0	–	2	–	–	–	–	3	25	5
23 22	–	–	–	–	–	–	3	18	5
24 21	–	–	–	–	–	–	3	25	1
25 23	–	–	–	–	–	–	3	12	4
27 22	1	5	2	9	–	–	2	8	3
28 23	–	–	–	–	–	–	1	4	2
Mar. 4 4	1	3	1	4	1	3	1	3	5
5 0	–	–	–	–	–	–	1	3	3
5 21	2	9	–	–	1	1	3	12	3
7 22	1	8	–	–	–	–	4	18	5
9 22	–	17	1	10	–	–	2	21	4
11 21	–	–	–	–	–	–	2	14	3
13 3	–	–	–	–	–	–	2	8	2
14 5	1	11	–	–	–	–	3	21	4
15 23	1	1	1	14	1	1	3	5	5
16 23	1	9	–	–	1	6	4	14	4
Mar. 18 4	1	9	1	1	1	4	4	22	3
20 22	2	60	–	–	2	12	6	78	5
21 22	–	1	–	–	–	–	6	48	4
22 23	–	–	–	–	–	–	6	31	3
23 23	–	1	1	1	–	–	5	24	4
25 2	1	4	–	–	1	4	6	29	5
27 22	1	13	3	12	1	5	4	21	5
* 29 0	–	–	–	–	–	–	4	20	3
* 31 5	4	20	2	5	1	3	6	35	4
31 22	1	8	–	–	1	1	7	43	3
Apr. 1 21	–	3	2	2	–	–	4	41	5
2 19	–	–	–	–	–	–	4	24	3
3 6	–	3	–	–	–	–	4	21	1
3 20	1	2	1	2	1	2	4	15	4
4 20	1	10	–	–	1	2	5	22	5
5 22	–	8	–	–	–	–	5	22	3
6 19	–	4	2	5	–	–	3	21	1
8 4	2	24	–	–	2	14	5	47	4
10 23	–	–	1	1	–	–	4	24	4
12 20	–	5	1	1	–	–	3	16	4
13 21	–	–	–	–	–	–	3	12	3
15 21	2	13	–	–	1	7	5	22	5
16 21	–	3	1	3	–	–	4	15	5
17 21	–	–	2	3	–	–	2	10	4
18 6	–	4	–	–	–	–	2	15	5
18 21	–	–	1	1	–	–	1	12	3
20 1	–	2	–	–	–	–	1	16	5
20 21	–	–	–	–	–	–	1	8	4
21 21	–	–	–	–	–	–	1	3	4
23 21	1	2	–	–	1	2	2	5	3
24 21	–	17	–	–	–	–	2	21	3
25 21	–	8	–	–	–	–	2	29	3
26 21	–	–	1	2	–	–	1	15	2
27 22	–	6	–	–	–	–	1	33	4
29 0	–	–	–	–	–	–	1	25	3
30 2	–	–	–	–	–	–	1	20	4
30 21	–	–	–	–	–	–	1	19	4
May 2 21	–	–	–	–	–	–	1	6	5
3 21	1	1	–	–	1	1	2	4	4
6 21	2	22	1	3	2	17	3	23	5
7 19	–	1	–	–	–	–	3	22	3
10 21	1	2	1	1	1	2	3	13	1
11 21	–	–	–	–	–	–	2	10	3
12 21	2	31	–	–	–	–	4	40	4
13 22	–	23	–	–	–	–	4	63	4
14 21	–	–	–	–	–	–	4	41	3
15 19	–	–	1	2	–	–	3	33	4
17 19	–	–	2	9	–	–	1	20	5
18 21	1	9	–	–	1	9	2	23	4
19 21	–	3	–	–	–	–	2	23	3
20 21	–	8	–	–	–	–	2	24	3
21 22	–	3	1	1	–	–	1	23	5
22 21	1	6	–	–	1	2	2	29	3
23 21	1	16	–	–	–	–	3	38	3

1906	New Gr.	New Spots	Disapp. Gr.	Disapp. Spots	Reapp. Gr.	Reapp. Spots	Total Gr.	Total Spots	Def.
May 25^d 3^h	2	22	–	–	1	6	5	52	5
25 21	–	1	–	–	–	–	5	37	3
28 21	–	–	–	–	–	–	5	26	2
29 21	–	–	–	–	–	–	5	22	4
31 6	1	20	1	1	–	–	5	46	5
31 22	–	–	–	–	–	–	4	24	3
June 1 21	1	12	1	3	–	–	3	30	4
3 6	1	12	–	–	1	10	4	31	5
4 5	–	9	1	1	–	–	2	31	3
5 23	2	2	–	–	2	2	4	22	3
8 23	–	21	1	4	–	–	3	33	5
9 22	–	2	–	–	–	–	3	24	5
10 19	–	–	1	5	–	–	2	19	4
June 11^d 19^h	–	1	–	–	–	–	2	19	4
12 21	–	–	–	–	–	–	2	7	4
13 21	1	4	–	–	–	–	2	7	4
14 23	–	–	–	–	–	–	1	1	2
19 22	2	10	–	–	–	–	3	14	5
20 22	1	2	–	–	1	2	3	11	3
22 4	–	10	–	–	–	–	3	23	5
24 3	2	12	1	15	2	10	4	20	4
25 22	2	16	–	–	2	11	6	28	3
27 3	2	38	1	3	1	1	7	64	5
27 22	–	2	–	–	–	–	7	58	4
29 2	–	32	–	–	–	–	7	95	5
30 23	–	–	1	2	–	–	4	44	3

Observed with 6-inch Reflector, except Feb. 1, 3, 4, 6, 7, 9, 11, with 3-inch Refractor.
*Observed by Professor David Todd.

OBSERVATIONS OF MINOR PLANETS AND COMET (1906 *b*).

MADE WITH THE 18-INCH EQUATORIAL OF THE FLOWER OBSERVATORY,

By SAMUEL G. BARTON.

1905–6 Wash'n M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
					(372) *Palma.*					
Dec. 26 8^h 58^m 34^s	1	30 , 5	−1^m 51.87^s	−2′ 19.1″	6^h 36^m 1.32^s	+55° 46′ 56.6″	*n*9.772	*n*9.777	+6.22^s	−12.4″
26 10 11 50	2	24 , 5	−4 26.19	−1 45.9	6 35 56.64	+55 46 35.5	*n*9.596	*n*0.211	+6.20	−12.9
27 17 19 49	3	21 , 5	−3 11.00	−2 20.0	6 33 56.01	+55 38 0.5	9.891	0.331	+6.30	−12.0
Jan. 5 9 48 22	4	35 , 5	+0 22.79	+3 8.9	6 21 9.73	+54 22 9.6	*n*9.310	*n*0.262	+1.04	− 0.9
5 9 48 22	5	35 , 5	−0 26.45	−0 41.9	6 21 9.57	+54 22 5.7	*n*9.310	*n*0.262	+1.01	− 0.9
					(9) *Metis.*					
Dec. 27 9 57 53	6	30 , 5	−2 25.42	+2 13.8	5 32 29.16	+26 59 6.8	*n*9.142	0.315	+1.49	− 5.1
27 12 50 16	7	27 , 5	−2 56.94	−0 17.7	5 32 20.46	+26 59 20.2	9.365	0.359	+1.48	− 5.1
Jan. 5 12 59 45	8	21 , 4	−4 17.87	+0 45.2	5 23 41.35	+27 16 37.6	9.519	0.423	+0.47	− 4.0
					(3) *Juno.*					
Feb. 9 14 40 9	9	25 , 6	−4 0.49	+5 7.1	8 46 2.79	+ 4 35 14.3	9.526	0.720	+1.01	−10.2
9 14 43 18	10	29 , 6	−5 38.78	−0 30.3	8 46 3.38	+ 4 35 9.1	9.532	0.720	+1.01	−10.2
16 12 58 54	11	29 , 5	+0 20.36	−2 35.0	8 49 54.35	+ 5 46 32.5	9.359	0.557	+1.00	−10.6
16 12 58 54	12	29 , 5	−0 43.21	−0 45.8	8 49 54.37	+ 5 46 33.1	9.359	0.557	+1.00	−10.6
					(17) *Thetis.*					
Feb. 16 10 1 49	13	30 , 5	+0 39.37	+4 41.2	8 32 53.33	+20 8 13.8	*n*8.911	0.475	+0.96	− 9.1
16 10 1 49	14	30 , 5	−1 26.51	+1 46.3	8 32 53.23	+20 8 13.2	*n*8.911	0.475	+0.96	− 9.1
23 11 26 38	15	30 , 5	+0 1.96	−5 33.6	8 27 19.61	+20 39 58.9	9.211	0.480	+1.05	− 8.1
23 11 26 38	16	30 , 5	−0 38.48	−2 29.8	8 27 19.51	+20 39 59.2	9.211	0.480	+1.05	− 8.1
					(118) *Peitho.*					
Feb. 23 15 27 29	17	21 , 5	+4 58.43	+3 48.9	9 32 47.74	+28 50 45.0	9.661	0.512	+0.94	− 8.7
23 15 27 29	18	21 , 5	+3 5.11	+5 37.6	9 32 47.71	+28 50 45.0	9.661	0.512	+0.91	− 8.7
					(67) *Asia.*					
Apr. 13 13 59 25	19	30 , 6	+0 35.26	+2 27.5	13 36 32.43	− 9 43 19.3	9.316	0.815	+1.45	+ 4.6
					(65) *Cybele.*					
Apr. 27 11 35 28	20	30 , 6	+2 28.30	+1 56.8	15 3 54.18	−12 30 39.2	*n*9.079	0.839	+1.58	− 2.0
27 11 35 28	21	30 , 5	−0 50.95	+6 34.8	15 3 54.25	−12 30 12.2	*n*9.079	0.839	+1.58	− 1.9

1906 Wash'n M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(148) *Gallia.*					
h m s			m s	′ ″	h m s	° ′ ″			s	″
Apr. 27 14 43 58	22	30 , 5	+0 43.49	−4 35.4	15 22 18.95	+17 25 19.3	9.339	0.553	+1.40	− 2.9
27 14 43 58	23	30 , 5	+0 37.98	−1 5.1	15 22 18.80	+17 25 17.7	9.339	0.553	+1.40	− 2.9
May 11 11 2 21	24	24 , 5	−4 30.25	+2 46.4	15 12 27.17	+18 29 57.6	*n*8.984	0.510	+1.54	− 0.6
11 11 2 21	25	24 , 5	+3 36.67	+0 36.2	15 12 27.10	+18 29 58.5	*n*8.984	0.510	+1.54	− 0.6
11 11 4 4	26	23 , 5	+3 52.71	+0 40.4	15 12 27.31	+18 29 59.2	*n*8.968	0.509	+1.54	− 0.6
18 10 58 6	27	30 , 5	+3 49.85	−3 45.3	15 6 53.12	+18 44 33.5	*n*8.570	0.500	+1.57	+ 0.6
21 9 42 55	27	30 , 5	−1 33.55	−1 29.6	15 4 36.82	+18 46 49.8	*n*9.207	0.515	+1.57	+ 1.2
25 9 32 2	27	30 , 6	−1 24.29	−1 56.9	15 1 39.00	+18 46 23.3	*n*9.164	0.512	+1.59	+ 2.0
					(92) *Undina.*					
May 4 13 18 59	28	10 , 10	−0 2.27	+1 16.7	16 7 18.26	−10 13 8.4	8.162	0.828	+1.56	+ 0.3
4 14 7 23	29	23 , 5	−1 51.35	−2 38.3	16 7 16.98	−10 13 10.9	9.061	0.825	+1.56	+ 0.3
11 12 59 47	30	10 , 10	+0 9.03	+0 45.4	16 2 15.45	−10 3 30.9	8.624	0.827	+1.68	+ 0.4
18 14 14 24	31	24 , 5	+6 36.18	−5 22.4	16 7 40.98	− 9 56 19.8	9.353	0.815	+1.76	+ 0.5
18 14 14 24	32	24 , 5	+4 20.59	−5 23.0	16 7 40.95	− 9 56 18.7	9.353	0.815	+1.76	+ 0.6
25 13 46 10	33	11 , 3	+5 57.10	−3 28.6	15 51 7.01	− 9 52 30.6	9.402	0.813	+1.83	+ 0.2
25 13 46 10	34	11 , 0	−0 31.80	. . . .	15 51 6.99	. . . . . .	9.402	. .	+1.83	. .
					Comet (1906 *b*)					
Apr. 16 12 36 43	35	9 , 10	+0 41.65	−7 11.3	11 20 38.22	+ 2 29 25.9	9.505	0.733	+1.11	− 8.7
16 13 38 30	36	10 , 10	−0 23.21	+6 12.8	11 20 37.34	+ 2 29 27.6	9.595	0.733	+1.11	− 8.7
18 8 56 58	35	33 , 7	+0 35.90	−6 18.9	11 20 32.46	+ 2 30 18.4	*n*7.614	0.726	+1.10	− 8.6
18 9 47 16	36	32 , 7	−0 28.05	+7 4.4	11 20 32.49	+ 2 30 19.3	7.399	0.726	+1.10	− 8.6
23 11 51 18	37	26 , 6	+0 24.50	+2 27.6	11 20 1.69	+ 2 28 24.2	9.470	0.733	+1.03	− 8.3
23 12 29 2	36	32 , 5	−0 58.81	+5 28.1	11 20 1.66	+ 2 28 23.3	9.540	0.735	+1.03	− 8.3
25 8 25 12	37	24 , 5	+0 22.03	+1 30.6	11 19 59.20	+ 2 27 27.2	*n*8.837	0.817	+1.01	− 8.3
25 8 25 44	36	29 , 5	−1 1.30	+4 12.2	11 19 59.16	+ 2 27 27.5	*n*8.842	0.817	+1.02	− 8.2
27 8 38 46	36	31 , 6	−1 0.66	+2 52.0	11 19 59.79	+ 2 26 7.3	*n*8.426	0.727	+1.01	− 8.2
27 8 39 15	37	28 , 6	+0 22.60	+0 9.7	11 19 59.77	+ 2 26 6.4	*n*8.410	0.727	+1.01	− 8.2

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	6 37 46.97	+55 49 28.1	Hels.-Goth. A.G. 4696	20	15 1 24.60	−12 32 34.0	Radcliffe 1890, 3903
2	6 40 16.63	+55 48 34.3	Hels.-Goth. A.G. 4714	21	15 4 43.62	−12 37 15.1	Bruxelles 6065
3	6 37 0.71	+55 40 32.5	Hels.-Goth. A.G. 4691	22	15 21 34.06	+17 29 57.6	Berlin A, A.G. 5536
4	6 20 45.90	+54 19 1.6	Camb., U.S., A.G. 2504	23	15 21 39.42	+17 26 25.7	Berlin A. A.G. 5538
5	6 21 34.98	+54 22 48.5	Camb., U.S., A.G. 2514	24	15 7 55.38	+18 27 11.8	Berlin A, A.G. 5474
6	5 34 50.09	+26 56 58.1	Camb., Eng., A G. 2586	25	15 8 48.89	+18 29 22.9	Berlin A. A.G. 5483
7	5 35 12.92	+26 59 43.0	Camb., Eng., A.G. 2592	26	15 8 33.06	+18 29 19.4	Berlin A, A.G. 5478
8	5 28 28.75	+27 15 56.4	Camb., Eng., A.G. 2517	27	15 3 1.70	+18 48 18.2	Berlin A, A.G. 5444
9	8 50 2.27	+ 4 30 17.4	Albany, A.G. 3590	28	16 7 18.97	−10 14 25.4	Wien-Ott., A.G. 5630
10	8 51 41.15	+ 4 35 49.6	Albany, A.G. 3603	29	16 9 6.77	−10 10 32.9	Wien-Ott., A.G. 5643
11	8 49 32.99	+ 5 49 18.1	Leipzig II, A.G. 4874	30	16 2 4.74	−10 4 16.7	Wien-Ott., A.G. 5601
12	8 50 36.58	+ 5 47 29.5	Leipzig II, A.G. 4868	31	16 1 3.04	− 9 50 57.9	Wien-Ott., A.G. 5593
13	8 32 13.00	+20 3 38.7	Berlin, B, A.G. 3453	32	16 3 18.60	− 9 50 56.3	Wien-Ott., A.G. 5610
14	8 34 18.81	+20 6 36.0	Berlin, B, A.G. 3475	33	15 45 8.08	− 9 49 2.2	Wien-Ott., A.G. 5518
15	8 27 16.60	+20 45 40.6	Fund. Catal.	34	15 51 36.96	− 9 47 19.2	Wien-Ott., A.G. 5545
16	8 27 56.94	+20 42 37.1	Berlin, B, A G. 5422	35	11 19 55.46	+ 2 36 45.9	Albany, A.G. 4265
17	9 27 48.37	+28 47 5.1	Camb., Eng., A.G. 5005	36	11 20 59.44	+ 2 23 23.5	Albany, A.G. 4271
18	9 29 41.36	+28 45 16.1	Camb., Eng., A.G. 5017	37	11 19 36.16	+ 2 26 4.9	Albany, A.G. 4264
19	13 35 55.72	− 9 45 51.4	Wien-Ott., A.G. 4869				

1906 *Ivyland, Pa.*

OBSERVATIONS OF COMETS AND MINOR PLANETS,

MADE AT THE VASSAR COLLEGE OBSERVATORY,

BY MARY W. WHITNEY AND CAROLINE E. FURNESS.

1905–6 Greenw. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Comet 1904 *e*.										
1905 d h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 23 14 36 37	1	* 6, 6	+1 9.28	−6 43.1	3 5 41.05	+29 11 29.3	9.682	0.615	−0.04	− 2.9[1]
23 14 52 33	1	* 4, 4	+1 11.56	−6 25.8	3 5 43.33	+29 11 46.6	9.691	0.631	−0.04	− 2.9[2]
24 14 15 10	2	* 8, 8	−0 20.83	+5 16.4	3 8 23.02	+29 40 45.7	9.670	0.610	−0.05	− 2.9[2]
25 13 44 28	3	* 4, 4	+1 31.71	−2 9.7	3 11 4.87	+30 10 0.3	9.644	0.529	−0.05	− 2.8[2]
(79) *Eurynome*.										
Oct. 7 13 19 13	4	†10, 10	−0 9.19	+7 30.8	0 0 24.14	+ 3 12 28.2	n9.436	0.745	+2.99	+19.6[2]
9 14 13 15	5	10, 10	+0 49.16	+3 48.3	23 58 58.42	+ 2 55 17.9	n9.224	0.743	+2.98	+19.6[2]
10 14 11 11	5	†10, 10	+0 8.97	−4 26.0	23 58 18.22	+ 2 47 3.6	n9.212	0.744	+2.97	+19.6[2]
12 13 31 59	6	10, 8	−2 46.85	+6 5.9	23 57 1.36	+ 2 30 55.9	n9.333	0.748	+2.96	+19.4[2]
14 13 26 58	7	†10, 10	+0 14.41	+5 57.6	23 55 48.99	+ 2 15 3.7	n9.319	0.750	+2.93	+19.5[2]
(455) *Bruchsalea*.										
Oct. 28 14 59 3	8	†10, 8	−0 16.47	−6 21.8	0 32 50.23	−17 29 16.6	n7.660	0.879	+2.84	+14.4[2]
30 12 14 47	10	10, 8	+0 56.46	−4 36.2	0 32 4.22	−17 9 52.4	9.466	0.851	+2.83	+14.3[2]
31 12 21 31	10	10, 8	+0 35.03	+6 4.3	0 31 42.79	−16 59 12.0	9.442	0.854	+2.83	+14.2[2]
Nov. 1 14 28 58	11	† 5, 13	−0 22.56	−0 47.0	0 31 21.12	−16 47 16.1	n8.483	0.876	+2.83	+14.1[1]
2 13 4 6	12	†10, 8	−0 3.36	−7 15.0	0 31 5.39	−16 36 44.6	n9.273	0.866	+2.83	+14.1[2]
(511) *Davida*.										
Oct. 28 16 22 46	13	10, 8	+2 1.09	+1 4.9	1 52 4.37	−14 21 34.1	7.690	0.864	+2.95	+12.8[2]
30 14 37 31	13	†10, 8	+0 30.35	−1 13.3	1 50 33.64	−14 23 52.5	n9.258	0.856	+2.96	+12.6[2]
31 15 0 44	13	†10, 8	−0 16.98	−2 0.2	1 49 46.35	−14 24 39.5	n9.110	0.860	+3.00	+12.5[2]
Nov. 2 15 58 38	14	10, 9	+0 47.28	+7 8.6	1 48 20.18	−14 25 28.1	7.537	0.864	+2.98	+12.3[1]
Comet 1905 *c*.										
Jan. 2 23 23 55	15	6, 6	+0 33.94	−7 43.4	17 5 35.14	+ 1 5 26.9	n9.569	0.762	−1.82	+ 6.6[1]
(372) *Palma*.										
1906 Jan. 4 14 12 24	16	10, 10	+1 10.58	+5 40.0	6 22 36.10	+54 32 39.4	n9.613	n0.058	+1.32	− 6.4[2]
(3) *Juno*.										
Feb. 5 13 29 0	17	† 8, 8	−0 26.87	−2 3.3	8 59 19.54	+ 3 52 40.5	n9.537	0.745	+0.99	− 9.7[2]
6 13 26 48	18	–, 4	. . .	−5 27.2	. . . .	+ 4 2 31.2	. .	0.744	+1.00	− 9.8[2]
6 13 34 3	18	† 8, –	−0 10.91	. . .	8 58 27.79	. . . . . .	n9.522	. .	+1.00	− 9.8[2]
Comet 1906 *a*.										
Jan. 30 18 2 56	19	† 8, 8	−0 29.64	−2 40.4	16 16 5.93	+53 42 39.9	n9.868	0.620	−1.42	− 9.9[1]
Feb. 13 14 18 10	20	10, 8	+1 40.77	−2 16.0	14 35 43.63	+80 56 9.6	n0.404	0.567	−0.14	−14.1[2]
15 14 8 42	21	6, 6	−3 50.33	−9 22.6	13 0 48.05	+83 58 33.8	n0.624	9.910	+7.16	−12.4[2]
16 13 37 13	22	10, 12	+0 25.95	+8 38.8	11 38 28.11	+84 50 37.8	n0.682	n9.924	+11.49	− 9.2[2]

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	3 4 31.81	+29 18 15.3	A.G. Camb.(Eng.) 1593	12	0 31 5.92	−16 29 43.7	Wash. Zones, 2 obs.
2	3 8 43.90	+29 35 32.2	Micr. Comp. with	13	1 50 0.33	−14 22 51.8	" " "
2	3 6 12.43	+29 28 22.4	A.G. Camb.(Eng.) 1598	14	1 47 29.92	−14 32 49.0	" " 1 obs.
3	3 9 33.21	+30 12 12.8	A.G. Camb. (Eng.) 1608 / A.G. Leiden 1223	15	17 5 3.02	+ 1 13 3.7	A.G. Albany 5672
4	0 0 30.34	+ 3 4 37.8	A.G. Albany 8243	16	6 21 24.20	+54 27 5.8	A.G. Camb. (U.S.) 2511
5	23 58 6.28	+ 2 51 10.0	" " 8234	17	8 59 45.42	+ 3 54 53.5	A.G. Albany 3655
6	23 59 45.25	+ 2 24 30.6	" " 8240	18	8 58 37.70	+ 4 8 8.2	A.G. Albany 3647
7	23 55 31.65	+ 2 8 46.6	" " 8223	19	16 16 36.99	+53 45 30.2	A.G. Camb. (U.S.) 4962
8	0 33 3.86	−17 23 9.2	B.D. −17°94 Micr. Comp. with Star 9	20	14 34 3.00	+80 58 39.7	B.D. 81°483 Greenwich Observations '99–'02
9	0 34 5.80	−17 16 6.1	Wash. Zones, 2 obs.	21	13 4 31.22	+84 8 8.8	B.D. +84°296 Gr. Obs.
10	0 31 4.93	−17 5 30.5	" " "	22	11 37 50.67	+84 42 8.2	Carr. 1743
11	0 31 40.85	−16 46 43.2	" " "				

* Observations made with square-bar occulting micrometer. † Δα measured direct.

[1] Observer, MARY W. WHITNEY. [2] Observer, CAROLINE E. FURNESS.

OBSERVATIONS OF COMETS,

MADE WITH THE 12-INCH EQUATORIAL OF THE MORRISON OBSERVATORY,

By HERBERT R. MORGAN.

1906 Glasgow M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
Comet *c* 1905 (Giacobini).										
h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 21 7 40 44	1	*d*6 , 6	+0 22.15	+3 53.6	. . . .	. . . .	9.639	0.774	−1.14	−12.8
24 8 0 46	2	12 , 5	+3 57.68	−1 28.7	1 25 9.59	− 9 1 14.9	9.646	0.765	−1.10	−12.7
27 7 44 50	3	*d*8 , 6	+0 12.17	+6 57.5	1 40 12.67	− 7 1 45.2	9.631	0.767	−1.05	−13.0
Comet *a* 1906 (Brooks).										
Jan. 27 16 13 34	4	18 , 8	+0 48.33	+1 4.4	16 18 47.14	+48 55 26.8	*n*9.753	9.821	−1.40	− 8.3
28 16 17 41	5	12 , 8	−2 36.62	+1 17.4	16 17 57.91	+50 34 51.4	*n*9.756	9.478	−1.42	− 9.1
30 16 31 44	6	15 , 6	−1 36.73	+3 51.3	16 15 50.65	+54 3 30.2	*n*9.751	*n*9.751	−1.40	−10.0
Feb. 2 16 52 23	7	18 , 8	+1 12.73	−2 22.9	16 10 54.76	+59 38 36.7	*n*9.736	*n*0.282	−1.44	−11.2
8 13 51 2	8	18 , 7	+2 54.98	+1 6.0	15 47 47.21	+71 28 21.3	*n*0.133	*n*9.575	−1.33	−13.5
27 9 35 6	9	15 , 8	+3 4.54	−0 24.2	5 58 50.16	+69 28 31.2	9.838	*n*0.545	+1.04	+ 7.2
Mar. 16 9 22 54	10	12 , 10	+0 33.58	−3 8.8	5 40 26.27	+49 30 24.4	9.730	9.354	−0.05	+ 2.5
17 8 26 41	11	15 , 8	+0 57.01	+4 36.0	5 40 31.66	+48 42 34.3	9.613	*n*9.683	−0.07	+ 2.2
20 8 43 11	12	15 , 8	+1 17.35	+0 13.5	5 41 3.53	+46 23 1.1	9.657	9.355	−0.13	+ 1.5
Comet *b* 1906 (Schaer).										
Mar. 17 10 35 55	13	*d*6 , 7	−0 5.43	−0 18.5	11 29 51.67	+2 0 30.5	*n*9.166	0.727	+1.19	− 8.7
20 10 4 13	13	12 , 6	−1 20.07	+4 7.7	11 28 37.04	+2 4 56.6	*n*9.258	0.727	+1.20	− 8.8
21 10 16 25	13	13 , 6	−1 44.92	+5 36.8	11 28 12.20	+2 6 25.7	*n*9.179	0.727	+1.21	− 8.8
Apr. 16 8 18 48	14	*d*6 , 6	−0 5.00	+0 39.0	11 20 39.56	+2 29 36.5	*n*9.216	0.723	+1.11	− 8.7
17 8 36 16	14	*d*5 , 4	−0 12.92	+0 41.9	11 20 31.63	+2 29 39.4	*n*9.220	0.723	+1.10	− 8.7
18 8 9 3	14	*d*5 , 4	−0 19.85	+0 43.3	11 20 24.69	+2 29 40.9	*n*9.225	0.723	+1.09	− 8.6
20 10 50 43	14	12 , 6	−0 32.13	+0 28.3	11 20 12.39	+2 29 26.0	9.214	0.723	+1.07	− 8.5
21 8 12 9	14	12 , 7	−0 35.76	+0 14.2	11 20 8.75	+2 29 11.9	*n*9.218	0.723	+1.06	− 8.5
24 9 19 2	14	12 , 6	−0 44.36	−0 55.0	11 20 0.12	+2 28 2.9	8.185	0.723	+1.03	− 8.3
Comet *c* 1906 (Ross).										
Mar. 19 7 40 55	16	18 , 7	−0 7.21	−4 43.0	2 9 31.37	−5 47 23.5	9.655	0.754	−1.10	−12.7
21 7 41 17	17	12 , 6	−3 33.31	−3 34.2	2 16 3.86	−3 35 21.5	9.654	0.751	−1.07	−12.5
22 7 40 0	18	15 , 8	−0 45.70	−6 12.6	2 19 3.66	−2 32 28.8	9.654	0.750	−1.07	−12.2

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	. . . .	. . . .	B.D. −11°228	10	5 39 52.74	+49 33 30.7	Bonn, A.G. Cat.
2	1 21 13.01	− 8 59 33.5	Vienna, A.G. Cat.	11	5 39 34.72	+48 37 56.1	Bonn, A.G. Cat.
3	1 40 1.55	− 7 8 29.7	Vienna, A.G. Cat.	12	5 39 46.31	+46 22 46.1	Bonn, A.G. Cat.
4	16 18 0.21	+48 54 30.7	Bonn, A.G. Cat.	13	11 29 55.91	+ 2 0 57.7	Albany, A.G. Cat.
5	16 20 35.95	+50 33 43.1	Harvard, A.G. Cat.	14	11 20 43.45	+ 2 29 6.2	c.f. with *15
6	16 17 28.78	+53 59 48.9	Harvard, A.G. Cat.	15	11 20 59.44	+ 2 23 23.5	Albany, A.G. Cat.
7	16 9 43.47	+59 41 10.8	Hels.&Gotha A.G.Cat.	16	2 9 39.68	− 5 42 27.8	Warsaw Cat.
8	15 44 53.56	+71 27 28.8	Green. Astro. Photographic Cat. / Dorpat, A.G. Cat.	17	2 19 38.24	− 3 31 34.8	Warsaw Cat.
9	5 55 44.58	+69 28 48.2	Christiania, A.G. Cat.	18	2 19 50.43	− 2 26 4.0	Warsaw Cat.

OBSERVATIONS OF THE SATELLITE OF *NEPTUNE* AT THE OPPOSITION OF 1905–1906,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by Rear-Admiral Asa Walker, U.S.N., Superintendent.]

In the following observations, the settings in position-angle were made, half before and half after, the measurements in distance, unless clouds prevented. The position-angle of the micrometer in measuring the distance was the mean of the first set of position-angles. This never differs by as much as one degree from the final mean,

and the correction to the distance for an error of this amount in the position-angle is inappreciable.

On certain nights, when the seeing was good, observations were made both east and west of the meridian to eliminate as far as possible errors due to the position of the observer.

A magnifying power of 600 was used for the first four measures, and 400 for all the remaining ones. Corrections have been applied for differential refraction and for the effect of the instrumental constants.

The computed positions, with which the comparisons are made, were derived from data given in the *Connaissance des Temps*.

Date	Wash. M.T.	Position-Angle p_o	Position-Angle p_c	Wash. M.T.	Distance s_o	Distance s_c	Comp.	O – C δp	O – C δs	Seeing
1905	h m s	°	°	h m s	″	″		°	″	
Oct. 29	13 34 5	343.64	342.85	13 34 31	11.36	11.34	6 , 6	+0.79	+0.02	Good
30	13 23 44	283.48	284.11	13 23 15	16.52	16.30	6 , 6	–0.63	+0.22	Excellent
Nov. 1	12 40 35	162.36	164.34	12 41 38	11.46	11.30	6 , 6	–1.98	+0.16	Poor
1906										
Jan. 5	11 49 31	136.46	136.94	11 49 18	12.77	13.17	7 , 6	–0.48	–0.40	Fair
6	10 15 7	91.10	91.77	10 15 33	16.68	16.90	6 , 8	–0.67	–0.22	Good
16	9 30 54	208.03	209.35	9 31 50	11.57	11.82	6 , 6	–1.32	–0.25	Fair
18	11 35 46	78.17	79.21	11 36 12	16.01	16.36	6 , 6	–1.04	–0.35	Good
24	8 39 47	78.17	79.13	8 31 53	16.13	16.36	6 , 6	–0.96	–0.23	Fair
24	10 40 3	74.06	75.49	10 40 20	15.85	16.04	6 , 6	–1.43	–0.19	Fair
28	10 41 8	184.01	183.32	10 42 0	10.93	11.06	8 , 8	+0.69	–0.13	Fair
29	8 22 25	115.24	116.93	8 22 55	15.16	15.10	8 , 8	–1.69	+0.06	Excellent
29	11 30 46	109.93	110.68	11 30 45	15.64	15.75	8 , 8	–0.75	–0.11	Excellent
31	8 30 13	5.73	6.11	8 30 59	11.43	11.06	8 , 8	–0.38	+0.37	Fair
31	10 46 38	355.88	357.29	10 44 38	11.34	11.06	8 , 8	–1.41	+0.28	Good
Feb. 11	7 10 7	63.51	64.87	7 11 27	14.73	14.92	8 , 9	–1.36	–0.19	Good
13	7 10 53	284.31	284.95	7 10 48	16.43	16.15	8 , 9	–0.64	+0.28	Fair
13	10 21 10	278.23	279.38	10 27 4	16.56	16.52	5 , 8	–1.15	+0.04	Excellent
16	7 38 40	100.01	101.50	7 39 56	16.00	16.37	8 , 8	–1.49	–0.37	Excellent
16	9 28 18	96.87	98.34	9 28 43	16.61	16.54	8 , 8	–1.47	+0.07	Excellent
17	7 34 31	56.02	57.38	7 34 20	13.96	14.08	8 , 8	–1.36	–0.12	Excellent
19	7 39 2	277.30	278.92	7 39 44	16.49	16.49	8 , 8	–1.62	0.00	Poor
23	7 23 58	48.45	50.53	7 24 49	13.36	13.35	8 , 10	–2.08	+0.01	Fair
23	9 53 54	42.83	43.68	9 54 10	12.92	12.72	8 , 8	–0.85	+0.20	Fair
24	9 49 46	318.51	319.79	9 49 34	12.46	12.59	8 , 8	–1.28	–0.13	Fair
25	7 9 39	273.28	274.72	7 20 13	16.88	16.60	4 , 9	–1.44	+0.28	Fair
25	9 34 32	269.68	270.67	9 33 3	16.21	16.64	8 , 8	–0.99	–0.43	Fair
Mar. 6	8 10 52	84.43	85.55	8 11 59	16.47	16.47	9 , 9	–1.12	0.00	Good
10	8 13 52	205.20	205.58	8 14 10	11.41	11.43	8 , 8	–0.38	–0.02	Good
20	8 0 55	294.46	295.40	8 1 38	15.02	14.83	8 , 10	–0.94	+0.19	Fair
23	8 25 48	111.08	111.00	8 25 58	14.86	15.19	10 , 9	+0.08	–0.33	Fair

CONTENTS.

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OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL OF THE U.S. NAVAL OBSERVATORY,

BY HERBERT L. RICE.

[Communicated by Rear-Admiral ASA WALKER, U.S.N., Superintendent.]

1905 Wash'n M.T.	*	Comp	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(30) *Urania*.					
Aug. 3 12^h 40^m 1^s	1	25 , 5	+1^m 54.80^s	+ 0′ 39.7″	21^h 43^m 7.06^s	−13° 1′ 12.8″	n8.475	0.839	+2.63^s	+18.4″
3 13 2 14	2	25 , 5	+2 25.23	− 3 32.6	21 43 6.31	−13 1 17.6	8.197	0.839	+2.63	+18.4
18 12 7 8	3	20 , 4	+1 13.11	− 2 15.9	21 28 44.02	−13 53 23.3	8.726	0.843	+2.79	+18.7
18 12 23 34	4	20 , 4	+1 4.19	− 0 58.1	21 28 43.38	−13 53 24.9	8.938	0.842	+2.79	+18.7
18 12 41 33	5	25 , 5	−0 26.04	+ 0 35.5	21 28 42.61	−13 53 27.3	9.089	0.840	+2.79	+18.8
21 11 17 3	6	25 , 5	+2 59.35	+ 2 35.3	21 25 51.10	−14 3 40.2	n8.293	0.845	+2.80	+18.7
					(1) *Ceres*.					
Aug. 21 12 34 39	7	25 , 5	+0 54.89	+ 0 26.5	23 28 58.46	−20 13 15.5	n9.062	0.873	+2.56	+20.0
26 13 43 53	8	21 , 7	+0 28.03	+ 2 3.0	23 25 18.49	−20 47 8.5	8.906	0.877	+2.65	+20.1
26 14 11 26	9	30 , 6	+1 46.58	− 7 1.6	23 25 17.55	−20 47 15.7	9.140	0.874	+2.65	+20.1
29 11 46 23	10	30 , 6	+2 6.53	+ 1 49.2	23 23 2.59	−21 5 50.0	n9.142	0.875	+2.70	+20.1
29 12 15 4	11	30 , 6	+1 55.54	+ 3 25.9	23 23 1.65	−21 5 58.0	n8.896	0.879	+2.70	+20.1
29 12 42 1	12	30 , 6	−1 57.38	− 0 27.5	23 23 0.74	−21 6 6.3	n8.329	0.880	+2.69	+20.1
					(84) *Klio*.					
Aug. 23 12 7 27	13	30 , 6	+1 31.59	+ 3 50.2	22 54 5.79	− 3 35 32.1	n8.894	0.772	+2.72	+18.9
26 11 42 58	14	25 , 5	+1 43.59	+ 0 23.1	22 51 9.89	− 3 20 49.5	n8.989	0.769	+2.75	+19.3
26 12 0 21	15	30 , 6	+0 21.56	− 0 14.1	22 51 9.00	− 3 20 47.5	n8.801	0.770	+2.75	+19.3
26 12 30 23	16	29 , 6	−1 4.71	− 1 39.7	22 51 7.62	− 3 20 41.6	n7.500	0.770	+2.75	+19.2
31 11 16 57	17	25 , 5	+1 1.01	− 0 12.4	22 46 2.29	− 2 58 24.4	n8.999	0.766	+2.80	+19.6
31 11 33 27	18	25 , 5	+0 35.80	− 2 59.9	22 46 1.48	− 2 58 20.4	n8.829	0.767	+2.80	+19.6
					(444) *Gyptis*.					
Sept. 7 12 41 21	19	29 , 6	+1 40.08	− 1 40.8	23 1 31.29	+ 3 33 56.8	8.964	0.705	+2.86	+19.8
7 13 0 17	20	25 , 5	+0 48.70	− 3 13.5	23 1 30.79	+ 3 33 50.6	9.110	0.706	+2.87	+19.8
9 14 17 10	21	25 , 5	+1 27.72	+ 0 29.6	23 0 7.55	+ 3 13 55.8	9.449	0.715	+2.87	+20.1
9 14 51 4	22	30 , 6	−2 45.77	− 0 38.8	23 0 6.39	+ 3 13 41.3	9.521	0.718	+2.87	+20.0
14 11 22 54	23	25 , 5	−2 12.92	+ 5 58.7	22 56 54.03	+ 2 24 56.9	n6.397	0.716	+2.88	+20.2
14 11 43 54	24	25 , 5	−0 49.12	+ 7 21.0	22 56 53.27	+ 2 24 47.6	8.621	0.716	+2.88	+20.3
					(21) *Lutetia*.					
Sept. 21 11 44 10	25	30 , 6	−2 36.52	+ 4 13.3	0 45 57.75	− 1 12 56.8	n9.075	0.751	+2.86	+17.2
21 12 15 2	26	30 , 6	−0 31.14	−10 12.8	0 45 56.68	− 1 13 2.8	n8.765	0.751	+2.86	+17.3
23 11 15 39	27	30 , 6	+2 31.90	− 2 25.1	0 44 17.09	− 1 23 21.3	n9.190	0.752	+2.88	+17.6
26 11 19 0	28	30 , 6	+0 43.58	+ 2 56.2	0 41 39.58	− 1 39 4.4	n9.084	0.755	+2.91	+17.7
26 11 37 31	29	30 , 6	−0 44.10	+ 1 6.5	0 41 38.93	− 1 39 4.1	n8.930	0.755	+2.90	+17.7
27 12 7 12	30	30 , 6	−1 28.01	+ 1 34.0	0 40 44.22	− 1 44 19.4	n8.218	0.756	+2.91	+17.7
					(455) *Bruchsalia*.					
Oct. 3 12 0 16	31	23 , 5	−1 50.96	− 5 10.7	0 50 39.39	−19 55 21.6	n7.500	0.875	+2.82	+17.0
3 12 21 52	32	20 , 4	−1 13.07	+ 1 52.4	0 50 38.36	−19 55 21.3	8.631	0.875	+2.82	+17.0
6 11 28 35	33	21 , 7	+0 39.10	+ 6 2.9	0 48 2.86	−19 49 55.1	n8.601	0.874	+2.84	+16.8

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	21 41 9.63	−13 2 10.9	Camb., U.S., A.G. Zones	18	22 45 22.88	− 2 55 40.1	Strassburg, A.G. Zones
2	21 40 38.45	−12 58 3.4	" " " "	19	22 59 48.35	+ 3 35 17.8	Albany, A.G. 7963
3	21 27 28.12	−13 51 26.1	1 Wash'n, U.S., A.G. Zones; 2 Camb., U.S., A.G. Zones	20	23 0 39.22	+ 3 36 44.3	" " 7969
4	21 27 36.40	−13 52 45.5	" " " "	21	22 58 36.96	+ 3 13 6.1	" " 7955
5	21 29 5.86	−13 54 24.6	" " " "	22	23 2 49.29	+ 3 14 0.1	" " 7979
6	21 22 48.95	−14 6 34.2	Wash., U.S., A.G. Zones	23	22 59 4.07	+ 2 18 38.0	" " 7959
7	23 28 1.01	−20 14 2.0	Algiers, A.G. Zones	24	22 57 39.51	+ 2 17 6.3	" " 7949
8	23 24 47.81	−20 49 31.6	" " "	25	0 48 31.41	− 1 17 27.3	Nicolajew, A.G. 164
9	23 23 28.32	−20 40 34.2	" " "	26	0 46 24.96	− 1 3 7.3	" " 154
10	23 20 53.36	−21 7 59.3	" " "	27	0 41 42.31	− 1 21 13.8	" " 136
11	23 21 3.41	−21 9 44.0	" " "	28	0 40 53.09	− 1 42 18.3	" " 132
12	23 24 55.43	−21 5 58.9	" " "	29	0 42 20.13	− 1 40 28.3	" " 140
13	22 52 31.48	− 3 39 41.2	Strassburg, A.G. Zones	30	0 42 9.32	− 1 46 11.1	" " 139
14	22 49 23.55	− 3 21 31.9	" " "	31	0 52 27.53	−19 50 27.9	Algiers, A.G. Zones
15	22 50 44.69	− 3 20 52.7	" " "	32	0 51 48.61	−19 57 30.7	" " "
16	22 52 9.58	− 3 19 21.1	" " "	33	0 47 20.92	−19 56 14.8	" " "
17	22 44 58.48	− 2 58 31.6	" " "				

The star places from the Algiers, Strassburg, and Cambridge (U.S.) Zones, were furnished through the courtesy of the Directors of the Observatories at these places.

OBSERVATIONS OF THE SIXTH SATELLITE OF *JUPITER*,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By HERBERT L. RICE.

[Communicated by Rear-Admiral ASA WALKER, U.S.N., Superintendent.]

1905 Wash'n M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
h m s			s	′ ″	h m s	° ′ ″			s	″
Oct. 21 13 17 6	2	7 , 6	+17.69	+3 5.7	4 11 0.44	+19 55 7.8	n9.053	0.462	+3.30	+2.1
29 12 39 0	3	9 , 11	+13.09	+5 19.1	4 8 18.23	+19 42 19.2	n9.083	0.468	+3.46	+2.6
30 12 22 7	3	9 , 9	− 9.24	+3 38.8	4 7 55.92	+19 40 38.9	n9.167	0.474	+3.48	+2.6
31 12 6 21	4	9 , 8	+15.09	−0 20.9	4 7 33.46	+19 38 58.8	n9.230	0.480	+3.51	+2.7
Nov. 1 11 58 32	4	9 , 7	− 8.16	−2 3.5	4 7 10.23	+19 37 16.3	n9.247	0.482	+3.53	+2.8
22 15 20 18	5	10 , 7	− 1.26	−5 14.2	3 57 54.21	+19 0 35.7	9.582	0.583	+3.83	+4.2
Dec. 4 13 59 15	7	9 , 10	+ 5.66	+2 7.7	3 52 30.13	+18 41 50.2	9.534	0.563	+3.92	+4.9
26 8 31 49	9	10 , 6	− 8.97	+5 19.1	3 44 36.06	+18 19 2.5	n9.045	0.495	+3.92	+5.4

The quantities Δα were all measured with the micrometer.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	4 11 18.02	+19 42 42.0	Three observations (Dec. 1905) on Wash. 9-inch Transit Circle	6	3 51 33.06	+18 33 53.2	Berlin A, A.G. 1052
2	4 10 39.45	+19 52 0.0	Fm. 1, $-0^m 38^s.57$ $+9' 18''.0$	7	3 52 20.55	+18 39 37.6	Fm. 6, $+0^m 47^s.49$ $+5' 44''.4$
3	4 8 1.68	+19 36 57.5	Fm. 1, $-3^m 16^s.34$ $-5' 44''.5$	8	3 46 55.67	+18 18 49.9	Berlin A, A.G. 1027
4	4 7 14.86	+19 39 17.0	Fm. 1, $-4^m 3^s.16$ $-3' 25''.0$	9	3 44 41.11	+18 13 38.0	Fm. 8, $-2^m 14^s.56$ $-5' 11''.9$
5	3 57 51.64	+19 5 45.7	Three observations (Dec. 1905) on Wash. 9-inch Transit Circle				

Star No. 1 is B.D. +19°687 ($8^M.5$) = Berlin A, A.G. 1119: its catalogue position was not used, however, owing to an indication of proper motion. No. 4 is the north, following component of the double star, B.D. +19°674 ($9^M.1$). No. 5 is B.D. +18°575 ($9^M.5$). No. 7 is the north, following component of B.D. +18°565 ($9^M.5$).

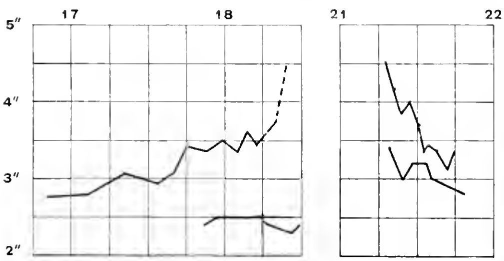
The values of Δα and Δδ in the columns "Authority" are reduced to 1905.0.

OBSERVATIONS OF THE CRATER *LINNÉ* DURING THE LUNAR ECLIPSE OF FEBRUARY 8, 1906.

By WILLIAM H. PICKERING.

In the *Astronomical Journal*, No. 587, p. 87, there are given some interesting measurements of the diameter of the white spot surrounding *Linné*, by Mr. JOEL STEBBINS, showing its apparent change in size as affected by the passage of the *Earth's* shadow. Unfortunately the observations were neither begun early enough, nor continued long enough, to show the phenomenon completely, but as far as they go it may be of interest to compare them with the longer series made at the same time by Mr. FROST, at Arequipa, Harvard Circular, No. 113.

The two observers used telescopes of practically the same aperture, but Mr. FROST had apparently more trouble from clouds. It was at first a little puzzling to make the comparison, since most of Mr. STEBBINS's earlier observations, on the night in question, were made according to his own figures, at a time when the *Moon* was totally eclipsed. This is impossible, since *Linné* would then have been invisible. Totality began according to the *American Ephemeris* at $18^h 58^m$, G.M.T., and Mr. STEBBINS's observations extended from $18^h 52^m$ to $19^h 29^m$. In what follows, I have assumed that he made a mistake of one hour in his earlier chronometer readings. It is to be regretted that he did not record the hour and minute when the shadow reached and left *Linné*. This would have served as a check, and also have been useful in the reductions, for comparison with the results of other years.



The curves on the left of the figure represent the observations before totality; those on the right the observations after *Linné* had reappeared from the shadow. The abscissas represent Greenwich Mean Time, and the ordinates the observed diameter of the spot in seconds of arc. The upper lines represent Mr. FROST's observations; the lower, Mr. STEBBINS's. Mr. FROST's measures were made in the usual way, by placing the spot between the micrometer threads. Mr. STEBBINS's measures, on the other hand, were obtained by the method that I have recommended in such cases, of placing the spot to one side of the two threads. This method has been found in my own case to give results about 0″.5 smaller than the other, when applied to small bright disks on a dark background. Measures of artificial disks have shown that the true value lay between the results obtained by the two methods. Mr. FROST expressed considerable doubt of the value of his last measure made before totality, on account of the density of the intervening cloud. It has therefore been joined to the others by a dotted line.

Mr. STEBBINS thinks that his later measures were larger than his earlier ones because the seeing was inferior. This at first sight might seem plausible, but a series of eighty-five measures, divided into seventeen groups, and taken on five different nights, under varying atmospheric conditions, shows that for a small bright artificial disk seen against a dark background, the poorer the seeing the smaller will the disk appear (*Annals Harvard Observatory*, XXXII, 136). If, therefore, Mr. STEBBINS is certain that the seeing after totality was inferior to that before it, then the difference in size of the spot before and after totality must really have been greater even than his measures would indicate. I do not think, therefore, that his explanation of his results will hold, and if he is not satisfied with the hoar-frost explanation, I should be very glad if he would suggest a more satisfactory one.

He explains the smallness of his last measure as due to prejudice on his part, because he found his later results were larger than he expected. Here, I think, he does himself an injustice. Judging by Mr. FROST's measures, the diminution in size is just about the amount that would naturally have been reached at that time. It is a pity he did not continue his observations a few minutes longer, as it would have added materially to their value.

The enlargement of the spot surrounding *Linné* after the passage of totality has now been recorded by six different observers, each working at a different observatory, several of them being originally prejudiced against it (*Popular Astronomy*, 1906, XIV, 4), and it would seem might now be accepted as fairly well confirmed. The variation in size of the spot in the ordinary course of a lunation, has been noted by several observers, and is indeed so large that it hardly requires further confirmation (*Annals Harvard Observatory*, XXXII, 207).

SUNSPOT OBSERVATIONS,

MADE AT BERWYN PENN., WITH A 4½-INCH REFRACTOR,

By A. W. QUIMBY.

1906	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Jan. 1	8	..	3	17	2	fair
2	8	..	3	13	2	fair
4	8	..	2	3	2	poor
5	4	..	2	2	1	poor
6	4	..	2	2	1	poor
7	8	1	2	2	2	fair
9	8	2	2	5	2	fair
10	8	..	2	8	3	fair
11	8	..	1	5	1	poor
12	3	1	2	8	2	poor
13	8	1	3	10	3	fair
14	1	..	2	8	..	poor
16	1	0	2	4	..	poor
17	11	..	1	1	2	poor
18	2	..	1	1	2	poor
19	2	..	1	1	2	fair
21	2	3	5	15	4	fair
22	1	..	5	16	2	poor
23	11	..	3	16	..	v.poor
24	8	..	5	34	1	fair
25	8	1	6	25	..	poor
26	11	1	4	16	..	poor
27	8	1	6	25	1	fair
28	8	..	5	22	2	poor
29	8	..	5	22	3	poor
30	8	..	4	17	3	poor
31	8	..	3	14	2	poor
Feb. 1	8	..	4	33	2	fair
2	8	..	3	16	3	fair
3	8	..	2	11	3	fair
4	8	..	2	9	3	fair
5	8	..	2	2	1	poor
6	8	..	2	2	2	poor
7	8	..	1	1	2	fair
8	8	..	1	2	2	poor
9	2	..	1	10	2	fair
10	8	..	1	15	2	fair
11	9	..	1	5	..	poor
12	9	..	1	3	..	poor
13	3	1	2	6	2	poor
14	2	1	2	5	2	fair
15	8	..	2	2	2	fair
16	2	0	2	2	2	fair
17	2	1	2	6	3	fair
18	12	0	2	11	1	fair
19	8	1	3	8	1	poor
20	8	..	2	14	3	fair
21	8	..	2	10	..	poor
22	2	1	3	14	2	fair
23	8	..	3	14	3	fair
24	9	1	4	26	1	good
25	8	0	3	10	..	poor
26	8	..	2	5	..	poor
28	8	..	1	4	3	fair
Mar. 1	8	2	3	6	2	fair
2	3	..	2	6	3	fair
4	8	..	1	4	2	fair
Mar. 5	3	..	1	3	2	fair
6	8	1	2	3	2	fair
7	8	1	2	2	..	poor
8	9	..	2	3	1	poor
9	8	2	4	14	1	fair
10	8	0	3	14	2	fair
11	8	..	3	16	2	fair
12	8	..	3	12	..	poor
13	8	..	2	5	..	poor
14	10	1	4	11	..	poor
16	8	1	3	6	3	fair
17	8	1	4	12	3	fair
18	8	1	5	16	3	fair
20	8	2	5	36	1	fair
21	8	1	6	54	2	good
22	8	..	6	25	2	poor
23	8	..	5	41	2	fair
24	8	..	5	26	2	poor
25	8	1	6	37	3	fair
26	9	..	2	22	..	v.poor
27	9	..	3	10	..	poor
28	12	1	4	14	1	fair
29	8	0	4	14	3	fair
Apr. 1	4	1	5	29	2	fair
2	8	..	4	23	3	fair
3	8	..	4	14	2	fair
4	8	..	4	8	2	fair
5	8	1	5	7	3	fair
6	8	1	5	18	3	fair
7	8	..	3	22	2	fair
8	7	1	4	21	1	fair
10	7	1	4	15	1	poor
11	7	..	4	15	2	fair
12	4	..	3	7	3	fair
13	8	..	3	7	2	fair
14	4	1	3	8	2	fair
15	2	..	3	5	..	poor
16	5	1	4	13	2	fair
17	7	..	4	10	2	fair
18	7	..	2	9	2	fair
19	7	1	2	17	2	fair
20	7	..	1	10	2	fair
21	7	..	1	7	3	fair
22	5	1	2	4	3	fair
23	7	..	1	2	..	poor
24	7	1	2	6	3	poor
25	7	..	2	8	1	poor
26	7	..	2	20	3	fair
27	7	..	2	20	3	fair
28	7	..	1	22	2	fair
29	7	1	2	20	..	poor
30	3	..	2	16	2	fair
May 1	7	1	2	20	1	fair
2	7	..	3	12	1	fair
3	7	..	3	7	3	poor
4	7	..	2	3	3	poor
May 5	7	2	2	7	3	poor
6	7	..	2	7	2	poor
7	9	..	2	5	..	poor
8	6	2	4	20	1	fair
9	6	..	3	17	..	poor
10	6	..	3	12	3	fair
11	6	..	3	9	2	fair
12	6	..	3	10	2	fair
13	1	1	4	32	2	fair
14	1	..	4	30	2	fair
15	10	..	4	30	0	poor
16	6	..	4	44	3	good
17	6	..	2	22	3	fair
18	6	1	2	17	2	good
19	6	..	2	11	2	fair
20	6	..	2	12	3	fair
21	6	..	2	16	2	fair
22	6	1	2	27	2	good
23	6	..	2	28	4	good
24	6	1	3	41	1	good
25	6	2	5	42	1	good
26	6	..	5	39	1	fair
27	5	1	6	36	2	fair
29	6	..	5	33	1	fair
30	6	..	5	28	1	fair
31	4	2	6	38	2	good
June 1	2	..	4	23	2	poor
2	6	..	4	28	3	fair
3	6	..	4	20	3	fair
4	6	..	4	25	1	fair
5	6	..	3	55	2	good
6	6	4	7	43	3	fair
7	6	..	5	18	3	fair
8	5	..	5	28	2	fair
9	11	..	5	13	2	poor
10	6	..	5	33	2	fair
11	6	..	2	20	2	poor
12	6	..	2	20	2	fair
13	6	..	2	11	3	fair
14	6	1	2	6	2	fair
15	4	1	2	11	2	fair
16	12	..	1	1	1	poor
17	7	..	1	1	1	poor
18	6	1	2	11	1	poor
19	6	..	2	12	2	poor
20	6	..	2	12	2	poor
*21	10	..	2	6	..	poor
*22	9	1	3	6	..	poor
23	6	..	3	9	..	poor
24	6	2	5	12	2	poor
25	6	2	7	32	3	fair
26	6	..	7	32	4	fair
27	6	..	7	54	3	good
28	6	1	8	57	3	good
29	6	0	8	54	3	good
30	6	..	8	65	5	good

*2½-inch Refractor.

OBSERVATIONS OF SUNSPOTS,

MADE AT BOSTON UNIVERSITY WITH A 5-INCH REFRACTOR,

By G. G. BULFINCH and H. O. COLE.

1905	Hour	Total Gps.	Total Spots	Def.	Obs'r	1905–6	Hour	Total Gps.	Total Spots	Def.	Obs'r	1906	Hour	Total Gps.	Total Spots	Def.	Obs'r
Oct. 10	3	3	19	F	B	Dec. 26	11	4	19	F	C	Mar. 2	3	3	8	F	B
12	2	3	15	F	B	27	11	3	19	F	C	5	4	1	1	P	B
17	3	2	49	G	C	28	11	3	22	G	C	6	3	2	3	F	B
18	3	2	12	P	C	1906 Jan. 1	11	3	37	G	C	8	1	2	3	G	B
19	11	2	71	F	B	2	11	3	19	F	C	12	4	3	12	F	B
21	11	2	56	F	C	3	12	2	8	P	B	14	3	4	10	G	C
23	11	2	41	P	C	8	12	2	2	P	C	16	1	3	4	G	B
25	11	2	25	P	C	9	10	1	8	F	C	17	1	4	10	G	B
26	11	1	18	F	C	10	1	1	14	F	B	21	12	7	66	E	C
27	2	1	12	F	C	11	11	2	11	F	C	22	4	6	44	G	B
30	11	2	23	F	C	12	11	1	9	P	C	23	1	7	38	G	B
31	3	3	16	F	C	17	3	2	5	F	C	24	1	6	37	G	B
Nov. 1	3	3	26	F	C	19	3	1	2	P	C	28	4	4	20	F	C
2	3	5	23	F	C	23	3	5	16	F	B	Apr. 2	4	4	28	P	B
3	11	5	38	G	B	24	4	5	21	P	B	3	4	4	16	F	B
4	3	7	44	P	C	26	1	6	41	F	B	4	3	3	7	F	C
9	3	8	35	P	C	29	11	4	18	F	C	5	3	4	10	P	B
11	11	10	63	F	C	29	3	4	23	F	B	10	4	3	5	P	B
13	11	10	49	P	C	30	12	4	24	G	C	11	3	3	20	F	C
14	3	11	45	P	C	Feb. 1	12	3	24	G	B	17	12	2	11	F	B
15	10	10	46	F	C	2	11	3	21	F	C	20	2	1	11	F	B
17	3	6	28	F	B	3	11	3	13	F	C	26	1	3	37	G	B
18	11	8	34	G	C	6	10	3	4	P	C	27	3	3	33	F	B
20	11	8	34	F	C	7	3	1	1	F	B	May 1	3	2	21	G	B
21	3	7	28	P	C	8	10	1	1	P	C	3	12	2	7	F	B
22	11	9	30	F	C	10	10	2	12	F	C	4	12	3	4	F	B
23	3	9	55	G	C	16	3	2	3	G	B	8	2	4	20	G	B
24	11	8	63	F	B	17	2	2	5	P	C	11	10	4	16	G	B
28	10	6	46	P	C	20	11	2	16	G	C	14	3	4	38	F	B
Dec. 1	11	5	47	F	C	20	2	2	17	G	B	15	4	4	32	F	B
5	10	5	15	P	C	21	11	3	14	P	C	16	10	4	28	G	C
6	10	6	14	F	C	22	1	3	21	G	B	17	1	3	21	G	B
7	3	4	11	F	C	23	11	3	14	F	C	18	10	1	20	G	B
8	3	5	16	F	B	23	4	3	15	F	B	21	4	2	17	F	B
13	11	3	14	F	C	24	3	4	19	F	B	22	2	2	19	G	B
14	10	4	14	F	C	27	10	2	4	F	C	23	1	2	32	G	C
18	11	3	23	G	C	28	11	2	5	F	C	24	11	3	44	G	C
20	10	5	33	G	C	Mar. 1	2	2	5	F	C	31	4	6	28	P	C
22	11	5	31	P	C	2	11	3	7	G	C						

Note — Definition: Poor, fair, good, excellent.

EPHEMERIS OF HOLMES'S COMET (*f* 1906).

(Abbreviated from Zwiers's ephemeris, *A.N.* 4085.)

1906	App. α	App. δ	log Δ	1906	App. α	App. δ	log Δ
	h m s	° ′			h m s	° ′	
Sept. 30	4 33 16	+48 19.2	0.3117	Oct. 28	4 26 10	+51 59.9	0.2803
Oct. 4	34 38	48 57.5	0.3062	Nov. 1	22 33	52 18.4	0.2778
8	34 53	49 31.2	0.3009	5	18 24	52 32.1	0.2760
12	34 28	50 8.9	0.2960	9	13 49	52 41.5	0.2748
16	33 23	50 41.2	0.2913	13	8 54	52 45.3	0.2745
20	31 38	51 10.8	0.2871	17	4 3 45	52 43.7	0.2750
24	4 29 13	+51 37.2	0.2834	21	3 58 30	+52 36.6	0.2764

OBSERVATIONS OF THE SATELLITE OF *NEPTUNE* IN THE YEARS 1905–06,

MADE WITH THE 40-INCH REFRACTOR OF THE YERKES OBSERVATORY,

By E. E. BARNARD.

When I first began these observations of the Satellite of *Neptune* in 1893, this object had been much neglected. It was my endeavor to remedy this defect as much as possible by frequent observations each year throughout the visibility of *Neptune*. The measures have been kept up until the end of the season of 1905–6 as faithfully as conditions and opportunity would permit.

Unless it is thought desirable by those interested in the planet, I shall not be able to continue the work systematically any longer.

The present observations are a continuation of those referred to above.

The previous measures have all been printed in the *Astronomical Journal*. The following references will indicate where they can be found.

Observations of 1893–1894,	*A.J.*,	Vol. 14,	p. 10
1894–1895,	"	" 15,	p. 41
1897–1898,	"	" 19,	p. 25
1898–1899,	"	" 20,	p. 41
1899–1900,	"	" 22,	p. 27
1901–1902 and 1902–1903,	"	" 23,	p. 105
1903–1904,	"	" 25,	p. 41

On 1904 December 10, *Neptune* was compared in the 4-inch finder with the star BD. +22°1408, which is given as 7.8 magnitude. *Neptune* was ¼ magnitude less than the star. This would make the planet 8.0 magnitude.

The observations of the satellite follow:

Date	h m s	°	″		
1905 Dec. 9	16 44 35	341.68	. .	6	Excessively bad seeing
	16 50 3	. .	11.65	4	
	16 53 57	. .	11.57	4	
16	14 53 42	277.84	. .	7	Clouded over before distances could be measured
19	11 9 28	102.60	. .	6	
	11 18 11	. .	16.90	5	Telescope swaying badly
	11 27 2	. .	17.24	5	
23	15 21 15	223.97	. .	6	
	15 26 42	. .	13.44	5	
	15 30 24	. .	13.26	5	
26	11 1 18	52.45	. .	6	
	11 7 4	. .	14.06	4	Seeing very bad
	11 10 36	. .	13.92	4	The second set of position-angles best
	11 18 33	53.12	. .	6	
30	13 46 21	137.18	. .	6	Seeing poor
	13 51 24	. .	12.97	4	
	13 54 37	. .	13.09	4	
1906 Jan. 23	11 26 28	115.45	. .	7	Seeing excessively bad
	11 32 19	. .	15.90	4	
	11 36 55	. .	16.09	5	
Feb. 6	12 10 15	337.87	. .	6	
	12 14 5	. .	11.18	4	
	12 16 37	. .	11.67	4	
27	9 49 21	132.00	. .	7	
	9 54 50	. .	13.30	4	
	9 57 46	. .	13.50	4	
Mar. 20	9 12 8	290.16	. .	6	
	9 17 20	. .	15.19	4	
	9 20 45	. .	14.91	5	
Apr. 17	7 27 40	11.19	. .	6	
	7 31 38	. .	12.01	5	
	7 34 20	. .	12.04	5	

Yerkes Observatory, 1906 *July* 18.

OBSERVATIONS OF THE SATELLITES OF *JUPITER* IN 1905–06,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By HERBERT L. RICE.

[Communicated by Rear-Admiral ASA WALKER, U.S.N., Superintendent.]

Position-angles were always taken about the inner satellite of each pair. In general, the observations in p consist of eight settings, half before and half after the measurements in distance. The latter also comprise eight measurements, four on each side of coincidence. Bright wire illumination was employed throughout the entire series.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	No.	Date	Wash. M.T.	p	Wash. M.T.	s
		h m s	°	h m s	″			h m s	°	h m s	″
					I–II.						
1	1905 Oct. 11	11 39 2	251.57	11 38 46	74.85	11	1905 Dec. 11	9 20 56	240.07	9 20 58	19.12
2	Nov. 2	10 42 46	255.27	10 42 34	46.40	12	12	8 48 31	259.51	8 48 27	278.12
3	4	10 17 25	79.34	10 17 27	347.39	13	19	8 17 55	258.17	8 17 51	303.94
4	8	10 39 53	85.59	10 39 56	129.20	14	30	8 0 32	265.13	8 0 33	116.45
5	10	14 44 44	262.73	14 44 35	215.69	15	1906 Jan. 5	8 3 13	247.47	8 2 57	52.07
6	11	10 10 59	78.32	10 10 54	342.64	16	6	7 41 24	261.06	7 41 14	187.01
7	14	10 16 36	327.31	10 16 36	21.26	17	9	7 25 48	255.53	7 26 33	261.17
8	Dec. 4	9 31 12	249.12	9 31 18	41.93	18	29	9 7 32	86.31	9 7 21	24.85
9	5	8 46 28	260.90	8 46 5	242.07	19	Feb. 9	9 56 9	81.05	9 55 41	62.63
10	6	12 43 8	79.60	12 43 43	258.70						
					I–III.						
1	1905 Oct. 11	12 8 6	81.13	12 9 9	195.42	12	Dec. 12	8 30 42	262.33	8 30 51	226.04
2	Nov. 4	9 59 47	253.40	9 59 57	151.25	13	18	8 50 38	259.80	8 50 22	216.69
3	8	11 27 40	79.47	11 28 2	272.68	14	19	8 34 48	260.68	8 34 55	260.25
4	10	15 6 32	252.60	15 6 29	169.05	15	30	8 13 10	219.08	8 13 20	20.45
5	11	10 54 54	251.77	10 54 44	127.61	16	1906 Jan. 5	7 30 33	77.69	7 30 43	438.60
6	14	9 32 17	50.27	9 32 23	58.63	17	9	7 42 53	257.71	7 42 38	290.72
7	21	9 36 41	311.51	9 36 18	34.67	18	10	5 58 25	61.44	5 58 27	93.85
8	25	11 17 55	245.57	11 17 37	69.18	19	18	5 39 1	73.46	5 38 59	149.02
9	Dec. 4	8 55 50	261.87	8 55 59	158.02	20	29	9 50 18	256.04	9 50 17	346.28
10	5	9 5 20	265.72	9 5 44	167.50	21	30	8 41 3	256.41	8 44 10	258.11
11	11	8 44 5	260.77	8 43 55	187.43	22	Feb. 9	9 11 26	75.30	9 11 20	384.60
					II–III.						
1	1905 Oct. 30	10 8 40	70.89	10 9 11	103.06	13	Dec. 18	8 26 47	261.03	8 27 6	223.36
2	31	10 32 16	74.21	10 32 49	89.20	14	19	8 1 0	63.43	8 1 0	42.69
3	Nov. 8	11 2 30	71.51	11 2 12	160.93	15	30	7 47 54	92.77	7 48 38	107.70
4	10	15 27 18	128.69	15 27 18	43.55	16	1906 Jan. 5	7 12 41	76.57	7 12 33	479.41
5	11	9 49 44	256.43	9 50 19	155.43	17	6	7 24 11	84.86	7 24 4	192.03
6	14	10 35 31	73.06	10 35 34	83.16	18	9	8 1 55	279.49	8 2 5	25.42
7	21	9 13 42	72.42	9 13 47	84.05	19	10	6 34 19	269.90	6 34 37	35.89
8	Dec. 4	9 13 42	266.25	9 13 57	118.20	20	16	6 35 40	265.46	6 35 47	62.06
9	5	9 24 29	69.95	9 21 1	68.61	21	18	5 57 43	49.99	5 57 32	37.28
10	6	13 3 35	61.54	13 2 28	66.01	22	25	9 46 19	52.48	9 46 56	42.95
11	11	8 26 0	262.77	8 26 20	178.73	23	30	9 0 51	261.36	9 0 44	96.76
12	12	9 6 42	67.82	9 7 29	57.53	24	Feb. 9	9 38 39	74.37	9 38 50	327.96
					III–IV.						
1	1905 Oct. 13	11 19 28	72.52	11 20 25	375.08	11	Dec. 12	8 13 31	252.38	8 13 43	456.02
2	30	10 29 31	77.92	10 29 33	414.19	12	18	8 5 58	73.72	8 6 0	445.19
3	31	10 55 35	78.20	10 55 49	292.75	13	19	9 1 55	75.74	9 1 46	450.37
4	Nov. 2	11 2 32	76.89	11 2 43	373.76	14	1906 Jan. 5	7 48 5	50.79	7 47 18	74.40
5	11	10 33 36	262.65	10 33 14	336.66	15	10	6 16 10	83.32	6 16 15	325.85
6	14	9 52 1	262.86	9 51 47	175.50	16	16	6 16 59	254.29	6 17 6	305.84
7	21	10 2 32	86.79	10 2 40	311.37	17	25	9 22 28	79.86	9 22 35	303.30
8	25	10 54 26	257.14	10 57 9	306.65	18	29	9 26 21	85.01	9 27 27	120.70
9	Dec. 6	13 28 43	81.73	13 29 4	294.29	19	30	9 19 42	233.18	9 20 6	73.68
10	11	9 2 20	230.09	9 1 58	79.05	20	Feb. 9	8 43 34	74.94	8 43 28	220.54

NOTE ON μ' *HERCULIS*,

By A. HALL.

I am indebted to Mr. AITKEN and Professor BARNARD for recent observations of this faint star, and a comparison with my orbit, *A.J.*, No. 324, gives the following results; (C−O):

Date	Obs. p	Δp	Obs. s	Δs	No. Obs'ns	Observer
	°	°	″	″		
1897.57	50.77	+3.60	1.35	+0.11	3	Aitken
1899.28	53.13	+5.20	1.67	−0.11	3	"
1901.41	59.77	+4.39	1.47	+0.11	3	"
1903.65	61.10	+7.10	1.41	+0.11	1	"
1905.46	69.00	+7.28	1.30	+0.10	1	"
1906.45	72.33	+7.57	1.34	−0.02	3	"
1906.48	74.00	+6.02	1.42	−0.10	4	Barnard

The mean residual is $+5°.92 = +0.15$, in angle. This residual would be removed by an increase of about a year in the time of periastron, or by a small increase of the periodic time.

If we assume the mass of this system equal to the mass of our sun, the hypothetical parallax is $+0''.11$. It would be interesting to have some measures for the parallax of this curious system.

COMET e 1906 (*KOPFF, August 22*).

Elements computed by CRAWFORD from observations of August 24, 25 and 26:

$$T = 1907 \text{ April } 12.26 \text{ Gr. M.T.}$$

$$\left.\begin{aligned}\omega &= 221°\ 38'\\ \Omega &= 230\ \ \ 2\\ i &= 12\ 45\end{aligned}\right\}1906.0$$

$$q = 1.118$$

Elements computed by MORGAN from observations of August 24, 25 and 26.

$$T = 1906 \text{ Dec. } 7.29 \text{ Gr. M.T.}$$

$$\left.\begin{aligned}\omega &= 243°\ 13'\\ \Omega &= 179\ 19\\ i &= 15\ 18\end{aligned}\right\}1906.0$$

$$q = 0.821$$

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OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH AND 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

BY JOHN C. HAMMOND AND MATT FREDERICKSON.

[Communicated by Rear-Admiral ASA WALKER, U.S.N., Superintendent.]

1905 Wash'n M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				(28) *Bellona.*						
			m s	′ ″	h m s	° ′ ″			s	″
Mar. 25 12 57 27*	1	30 , 6	+1 28.22	+ 6 56.7	12 18 8.69	+ 8 27 36.9	9.012	0.649	+1.48	− 9.3
25 13 22 32*	2	25 , 5	−3 37.24	+ 2 45.6	12 18 7.61	+ 8 27 47.7	9.180	0.652	+1.48	− 9.2
27 11 47 56*	3	30 , 6	−1 1.44	+ 8 36.7	12 16 40.30	+ 8 43 6.0	n8.258	0.643	+1.49	− 9.3
27 12 25 17*	4	20 , 4	+3 0.29	+ 7 48.7	12 16 39.02	+ 8 43 12.3	8.759	0.644	+1.49	− 9.4
27 12 49 41*	5	18 , 4	−1 18.00	− 4 21.4	12 16 38.16	+ 8 43 23.2	9.025	0.646	+1.49	− 9.3
28 11 26 35*	6	25 , 5	−3 6.93	− 1 30.6	12 15 55.59	+ 8 50 35.3	n8.714	0.642	+1.50	− 9.2
30 12 47 20*	7	26 , 6	+0 43.76	− 6 50.5	12 14 24.34	+ 9 5 47.8	9.110	0.642	+1.50	− 9.3
30 13 16 19*	8	26 , 6	−0 49.54	+ 0 22.8	12 14 23.13	+ 9 6 0.5	9.265	0.647	+1.50	− 9.2
Observed by FREDERICKSON.										
				(64) *Angelina.*						
Apr. 1 10 40 51*	9	19 , 5	−4 9.86	+ 2 29.6	12 22 37.39	− 4 29 23.5	n9.091	0.778	+1.63	− 9.4
2 11 28 23*	10	30 , 6	+0 46.86	− 3 48.4	12 21 45.52	− 4 24 3.1	n8.301	0.779	+1.63	− 9.6
2 12 16 6*	11	18 . 4	+3 19.78	+ 3 9.4	12 21 43.61	− 4 23 49.2	8.877	0.778	+1.63	− 9.8
7 14 20 16*	12	19 , 4	+2 9.92	+ 1 14.6	12 17 35.67	− 3 57 53.0	9.522	0.764	+1.62	− 9.8
8 14 45 32*	13	21 , 6	−0 11.94	− 1 9.6	12 16 48.08	− 3 52 49.2	9.568	0.760	+1.63	− 9.8
18 12 20 54†	14	20 , 4	+2 41.85	− 0 53.5	12 9 58.73	− 3 8 5.0	9.351	0.765	+1.58	−10.1
23 12 47 11*	15	25 , 5	−4 10.36	− 6 50.3	12 7 15.42	− 2 49 19.4	9.482	0.759	+1.57	− 9.9
24 10 10 8*	16	30 , 6	+0 47.98	− 9 33.7	12 6 50.30	− 2 46 28.7	8.420	0.765	+1.55	−10.1
24 10 43 24*	17	29 , 6	−1 37.32	+ 4 59.5	12 6 49.54	− 2 46 22.6	8.965	0.765	+1.56	−10.1
Observed by FREDERICKSON.										
				(124) *Alkeste.*						
Apr. 2 10 36 18*	18	27 . 6	+1 51.38	+ 0 49.2	12 26 45.38	− 2 46 46.5	n9.122	0.764	+1.61	− 9.4
8 13 22 14*	19	18 . 5	−1 43.51	+ 7 19.5	12 21 51.65	− 2 6 52.9	9.384	0.757	+1.62	− 9.4
14 13 12 17*	20	32 . 7	+3 10.05	+ 0 42.9	12 17 27.06	− 1 30 11.7	9.434	0.751	+1.59	− 9.8
22 12 53 44*	21	26 , 6	+2 20.00	+ 0 13.1	12 12 29.83	− 0 47 51.4	9.476	0.746	+1.55	− 9.6
22 13 23 57*	22	17 . 4	+3 42.43	− 5 4.1	12 12 28.83	− 0 47 43.3	9.534	0.745	+1.54	− 9.6
Observed by FREDERICKSON.										
				(26) *Proserpina.*						
May 2 11 27 19*	23	28 , 6	+1 5.99	− 3 39.7	14 34 7.16	−14 36 16.3	n8.714	0.848	+2.01	− 3.8
2 12 25 43*	24	19 , 4	+5 46.45	− 0 58.0	14 34 4.87	−14 36 11.6	8.837	0.847	+2.01	− 4.1
7 11 29 6*	25	18 , 6	+0 16.90	− 0 24.9	14 29 32.37	−14 23 30.3	7.281	0.847	+2.04	− 4.1
7 11 46 21*	26	18 , 6	−0 10.97	0 9.3	14 29 31.71	−14 23 26.4	8.574	0.847	+2.04	− 4.1
7 12 38 14*	27	18 , 4	−4 37.39	− 7 33.4	14 29 29.91	−14 23 20.4	9.154	0.842	+2.04	− 3.8
21 12 28 45*	28	20 , 4	−4 16.09	+ 5 4.1	14 18 4.48	−13 52 23.4	9.395	0.828	+2.06	− 4.6
21 13 3 48*	29	10 , 2	+3 56.31	+ 5 56.0	14 18 3.38	−13 52 20.2	9.487	0.818	+2.04	− 5.1
23 13 16 22*	30	24 , 5	−2 46.58	− 9 49.7	14 16 41.76	−13 48 57.2	9.530	0.811	+2.05	− 4.8
Observed by FREDERICKSON.										

(14) *Irene.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
May 8 11 25 27*	31	25 , 5	+0 39.36	+ 5 26.3	14 39 13.22	− 3 27 13.1	n8.228	0.771	+1.90	− 3.0
8 11 52 13*	32	29 , 6	−0 29.06	−13 23.2	14 39 12.06	− 3 27 17.3	8.565	0.771	+1.90	− 3.0
8 12 38 43*	33	35 , 7	+3 32.67	+ 7 20.5	14 39 10.17	− 3 27 17.7	9.108	0.770	+1.90	− 2.9
19 13 55 34*	34	25 , 5	−2 11.31	+ 6 37.1	14 29 49.24	− 3 45 23.3	9.537	0.762	+1.93	− 2.8
23 10 55 8*	35	25 , 5	+1 55.56	− 7 23.2	14 27 0.95	− 3 56 51.5	8.811	0.775	+1.92	− 3.0
23 12 12 48*	36	19 , 4	+3 52.14	+ 0 43.8	14 26 58.66	− 3 57 1.5	9.327	0.771	+1.92	− 3.1
27 12 53 42*	37	20 , 4	+2 19.09	−11 39.0	14 24 27.95	− 4 11 30.2	9.490	0.767	+1.91	− 2.9
28 10 38 35*	38	23 , 5	+2 46.20	−13 31.6	14 23 57.39	− 4 15 7.9	8.886	0.777	+1.91	− 2.8
June 3 12 38 12*	39	23 , 5	−2 45.73	+ 5 1.4	14 21 5.88	− 4 42 42.9	9.520	0.768	+1.92	− 2.4
3 12 58 30*	40	25 , 5	−0 47.11	+ 2 2.5	14 21 5.74	− 4 42 44.7	9.554	0.766	+1.91	− 2.5

Observed by Frederickson.

(19) *Fortuna.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
June 14 13 13 18*	41	24 , 5	+1 21.63	+ 8 4.2	18 34 37.49	−20 59 35.1	8.335	0.880	+2.43	+10.5
14 13 32 13*	42	24 , 5	+0 11.67	+ 3 16.6	18 34 36.88	−20 59 36.3	8.800	0.879	+2.43	+10.6
25 12 50 51*	43	15 , 5	−0 29.82	− 1 11.4	18 23 43.64	−21 1 50.8	8.944	0.878	+2.61	+10.2
25 13 15 6*	44	23 , 5	+1 11.53	−10 26.8	18 23 42.77	−21 1 54.5	9.145	0.874	+2.61	+10.1
July 7 12 5 10*	45	22 , 5	+3 15.17	+ 0 18.9	18 11 22.80	−21 4 34.5	9.072	0.876	+2.73	+ 9.3
7 12 51 6*	46	17 , 4	+3 9.94	+ 0 39.5	18 11 20.85	−21 4 39.0	9.324	0.867	+2.73	+ 9.3

Observed by Feederickson.

(46) *Hestia.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
June 28 13 16 4*	47	25 , 5	+1 13.33	+ 4 2.1	18 45 10.43	−19 2 40.8	9.080	0.867	+2.59	+11.4
28 13 35 52*	48	22 , 5	+1 6.44	− 1 57.1	18 45 9.45	−19 2 41.5	9.205	0.864	+2.59	+11.4
28 13 51 29*	49	23 , 5	+0 24.87	+ 1 53.7	18 45 9.07	−19 2 39.3	9.281	0.860	+2.59	+11.4
July 8 12 36 20*	50	20 , 4	+2 0.80	− 2 31.3	18 35 21.22	−19 13 37.1	9.144	0.866	+2.70	+11.0
8 13 8 2*	51	24 , 5	−1 4.97	− 2 9.4	18 35 20.03	−19 13 40.4	9.306	0.860	+2.70	+11.1

Observed by Frederickson.

(532) *Herculina.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
July 7 12 23 2†	52	20 , 4	−4 54.42	+ 3 22.1	21 36 30.77	−22 51 37.8	n9.430	0.864	+2.15	+18.2
8 13 9 12†	53	15 , 3	−3 44.07	− 0 32.7	21 36 0.89	−23 0 12.3	n9.231	0.880	+2.19	+18.2
15 13 43 39†	54	25 , 5	+2 9.61	− 7 49.2	21 32 1.91	−24 0 7.8	n8.504	0.893	+2.40	+18.7
20 11 52 42†	55	25 , 5	+1 29.03	− 9 3.2	21 28 41.69	−24 42 57.7	n9.339	0.880	+2.52	+18.7
26 12 20 31†	56	30 , 6	−0 53.48	+ 1 10.6	21 24 6.11	−25 35 2.7	n9.011	0.895	+2.64	+18.8
30 13 38 49†	57	20 , 4	−2 27.82	− 6 3.2	21 20 45.46	−26 9 14.3	9.050	0.897	+2.72	+18.8
30 13 58 56†	58	20 , 4	−4 32.09	− 2 2.3	21 20 44.80	−26 9 19.9	9.192	0.893	+2.71	+18.9
Aug. 3 11 30 6†	59	25 , 5	+3 5.92	+ 3 24.2	21 17 24.81	−26 40 57.4	n9.115	0.897	+2.78	+18.2
6 12 2 34†	60	18 , 6	−0 20.25	− 0 45.8	21 14 46.57	−27 4 27.2	n8.428	0.904	+2.84	+18.2
15 12 5 58†	61	30 , 6	−0 45.24	− 7 18.0	21 6 57.04	−28 7 25.8	8.894	0.905	+2.94	+17.5
17 10 42 57†	62	25 , 5	−1 23.52	− 2 8.3	21 5 18.86	−28 19 25.7	n8.944	0.906	+2.95	+17.3
18 10 31 31†	63	25 , 5	+0 47.10	+ 3 51.7	21 4 29.44	−28 25 22.0	n9.012	0.905	+2.96	+17.0
22 9 42 53†	64	25 , 5	−2 3.85	+ 3 53.0	21 1 17.38	−28 47 15.3	n9.227	0.900	+2.97	+16.7
22 10 7 17†	65	25 , 5	−3 36.01	+ 4 56.7	21 1 16.56	−28 47 23.7	n9.061	0.905	+2.97	+16.7
30 9 32 23†	66	28 , 6	−1 47.53	+ 5 31.0	20 55 29.48	−29 23 15.6	n9.044	0.907	+2.98	+15.6
31 9 42 0†	67	20 , 4	−3 42.50	+ 4 3.8	20 54 50.17	−29 26 59.4	n8.895	0.910	+2.97	+15.6
Sept. 6 9 37 40†	68	30 , 6	+2 18.22	+ 0 20.8	20 51 21.21	−29 45 35.4	n8.427	0.912	+2.93	+14.6

Observed by Hammond.

(234) *Barbara.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
July 8 14 11 10*	69	19 , 4	−0 53.23	− 4 40.3	19 1 44.57	− 2 43 15.9	9.416	0.760	+2.51	+12.6
8 14 26 39*	70	20 , 4	−1 4.41	− 7 28.8	19 1 44.21	− 2 43 22.7	9.448	0.760	+2.51	+12.6
8 14 42 28*	71	20 , 4	−1 34.64	− 7 15.8	19 1 43.48	− 2 43 28.5	9.484	0.758	+2.51	+12.6

Observed by Frederickson.

(90) *Antiope.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Sept. 8 11 23 40†	72	25 , 5	+2 33.56	+ 1 46.4	23 35 35.41	− 6 28 3.9	n9.088	0.793	+2.80	+19.4
8 11 41 51†	73	25 , 5	+1 52.62	+ 2 22.8	23 35 34.89	− 6 28 8.3	n8.938	0.794	+2.79	+19.4
8 11 56 39†	74	25 , 5	−1 27.57	+ 2 4.1	23 35 34.42	− 6 28 12.0	n8.758	0.794	+2.78	+19.3
14 11 32 1†	75	25 , 5	+2 45.79	− 0 48.3	23 31 5.88	− 6 56 1.2	n8.703	0.798	+2.84	+19.5
14 11 49 28†	76	25 , 5	−1 29.76	+ 3 57.2	23 31 5.45	− 6 56 4.6	n8.187	0.798	+2.84	+19.5

Observed by Hammond.

(79) *Eurynome.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Sept. 9 12h 49m 53s*	77	23 , 5	−1m 12.06s	+ 5′ 59.6″	0h 21m 1.20s	+ 6° 48′ 47.0″	n8.526	0.667	+2.85s	+16.8″
14 14 3 5*	78	24 , 5	+1 49.14	+ 3 58.1	0 17 46.42	+ 6 17 2.8	9.195	0.678	+2.91	+17.6
14 14 25 17*	79	25 , 5	−2 16.87	− 5 45.3	0 17 45.48	+ 6 16 55.2	9.296	0.680	+2.91	+17.4
14 14 53 12*	80	24 , 5	−0 31.09	+ 2 15.6	0 17 44.80	+ 6 16 18.2	9.393	0.684	+2.91	+17.5
24 11 0 40*	81	20 , 4	+0 11.56	− 1 41.0	0 10 22.29	+ 5 2 18.1	n9.049	0.690	+2.97	+18.7
29 13 1 16*	82	30 , 6	+3 2.84	− 4 47.8	0 6 19.65	+ 4 19 29.8	9.236	0.699	+2.98	+19.1
29 13 40 56*	83	27 , 6	+0 50.84	− 2 50.6	0 6 18.26	+ 4 19 13.8	9.386	0.703	+2.98	+19.1
29 14 4 31*	84	25 , 5	+1 11.91	+ 0 34.5	0 6 17.45	+ 4 19 6.0	9.450	0.705	+2.98	+19.1

Observed by Frederickson.

(356) *Liguria.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Sept. 12 13 55 53†	85	15 , 5	+0 10.82	− 3 44.4	0 12 23.60	+ 1 47 10.9	9.138	0.724	+2.84	+18.0
12 14 5 44†	86	25 , 5	−0 19.35	− 0 44.3	0 12 23.42	+ 1 47 9.5	9.193	0.724	+2.84	+18.0
12 14 21 11†	87	25 , 5	−1 37.98	− 0 0.9	0 12 22.87	+ 1 47 8.5	9.266	0.724	+2.84	+18.0
25 11 42 12†	88	25 , 5	−1 43.20	+ 6 17.1	0 0 19.60	+ 1 26 30.8	n7.213	0.726	+2.94	+19.1
28 11 51 54†	89	25 , 5	+1 26.61	−10 44.8	23 57 23.72	+ 1 20 56.7	8.673	0.727	+2.95	+19.3
28 12 13 0†	90	25 , 5	+2 21.28	+ 0 8.7	23 57 22.76	+ 1 20 55.5	8.949	0.727	+2.94	+19.3

Observed by Frederickson.

(305) *Gordonia.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Sept. 21 12 6 17†	91	25 , 5	−0 19.80	+ 2 7.0	23 46 49.06	+ 2 26 34.3	8.623	0.716	+2.93	+19.3
21 12 22 43†	92	25 , 5	+0 40.95	+ 1 57.8	23 46 48.66	+ 2 26 32.6	8.873	0.716	+2.93	+19.4
21 12 44 45†	93	25 , 5	+2 5.14	− 1 39.9	23 46 47.87	+ 2 26 26.7	9.071	0.717	+2.93	+19.4
26 11 26 25†	94	25 , 5	−1 7.34	+ 2 51.4	23 43 10.14	+ 1 57 4.4	7.949	0.721	+2.98	+19.0
28 10 34 45†	95	25 , 5	+2 24.29	+ 7 55.3	23 41 44.91	+ 1 45 21.8	n8.878	0.723	+2.93	+19.8
28 10 55 35†	96	25 , 5	−2 16.26	+ 3 43.8	23 41 44.28	+ 1 45 18.5	n8.534	0.723	+2.94	+19.7

Observed by Frederickson.

1905 *QU* = (406) [1895 *CB*].

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Sept. 21 8 45 22†	97	25 , 5	−1 58.03	− 7 2.4	22 33 24.99	− 5 21 9.8	n9.315	0.781	+2.82	+20.3
22 8 53 48†	98	20 , 4	+3 45.98	− 1 27.7	22 32 46.74	− 5 23 38.8	n9.262	0.783	+2.81	+20.3
24 10 28 39†	99	25 , 5	+1 31.05	+ 3 17.8	22 31 31.84	− 5 28 30.9	8.314	0.787	+2.80	+20.3
27 10 34 50†	100	25 , 5	+2 10.44	+ 4 21.1	22 29 53.18	− 5 35 1.7	8.777	0.788	+2.78	+20.2
27 10 49 52†	101	25 , 5	−1 31.51	+ 3 21.4	22 29 52.87	− 5 35 3.2	8.953	0.787	+2.79	+20.2
Oct. 3 9 23 53†	102	30 , 6	+3 17.16	− 7 23.2	22 27 15.48	− 5 45 22.7	n8.475	0.789	+2.73	+20.1

Observed by Hammond.

(351) *Yrsa.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Sept. 22 11 54 36†	103	25 , 5	+1 27.98	− 6 58.4	0 28 13.29	−11 54 32.3	n8.757	0.832	+2.81	+18.4
22 12 27 44†	104	23 , 5	−1 25.86	− 6 32.5	0 28 12.12	−11 54 40.9	8.020	0.832	+2.80	+18.3
26 12 22 54†	105	25 , 5	+0 33.07	−10 33.8	0 24 57.35	−12 17 43.5	8.595	0.834	+2.84	+18.3
26 12 56 24†	106	25 , 5	+1 42.10	− 7 4.8	0 24 56.18	−12 17 49.5	9.030	0.832	+2.84	+18.3

Observed by Frederickson.

(82) *Alkmene.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Sept. 30 12 8 54†	107	25 , 5	+1 57.79	+ 9 18.0	1 51 21.54	+11 5 36.0	n9.118	0.616	+3.05	+13.2
30 12 30 15†	108	25 , 5	+0 47.31	− 5 55.6	1 51 21.24	+11 5 37.0	n8.948	0.613	+3.06	+13.1
Oct. 8 10 55 54†	109	25 , 5	−0 40.48	+ 3 52.1	1 45 12.23	+10 38 30.0	n9.297	0.629	+3.15	+14.1
23 9 53 16†	110	20 , 4	−4 20.08	− 0 38.3	1 31 59.78	+ 9 37 31.8	n9.255	0.640	+3.24	+15.3
23 10 27 38†	111	20 , 4	−5 39.17	− 8 43.6	1 31 58.62	+ 9 37 29.5	n9.056	0.634	+3.24	+15.7

Observed by Frederickson.

(169) *Zelia.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Sept. 26 12 55 12*	112	30 , 6	−0 45.10	− 2 46.3	0 51 33.09	+ 8 25 37.3	8.703	0.647	+3.02	+16.7
Oct. 3 13 11 52†	113	25 , 5	+2 5.33	+ 0 32.8	0 44 26.42	+ 8 8 23.6	9.180	0.656	+3.07	+17.6
3 13 33 36†	114	25 , 5	−3 13.94	− 0 52.8	0 44 25.43	+ 8 8 21.4	9.284	0.659	+3.07	+17.3
8 12 9 29†	115	25 , 5	−2 37.47	+ 6 38.5	0 39 23.79	+ 7 54 15.3	8.890	0.655	+3.09	+18.0
8 12 38 30†	116	23 , 5	+1 10.55	− 9 18.8	0 39 22.45	+ 7 54 10.4	9.129	0.657	+3.09	+18.2
12 11 10 28†	117	25 , 5	+1 4.05	+ 1 3.9	0 35 29.76	+ 7 42 29.3	n7.200	0.656	+3.09	+18.4
12 11 27 30†	118	24 , 5	+0 38.36	+ 6 5.5	0 35 28.96	+ 7 42 29.1	8.516	0.656	+3.09	+18.4

Sept. 26, observed by Rice. Other observations by Frederickson.

(211) *Isolda.*

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Oct. 4 10 29 16†	119	25 , 5	+1 46.69	+ 3 4.9	0 26 0.68	+ 9 10 46.0	n9.107	0.641	+3.08	+18.6
4 10 48 37†	120	25 , 5	+1 0.73	+ 0 0.4	0 26 0.07	+ 9 10 42.5	n8.953	0.639	+3.08	+18.6
11 10 31 16†	121	8 , 8	+0 14.10	− 0 44.3	0 20 42.56	+ 8 32 5.1	n8.770	0.646	+3.08	+19.1
11 11 10 50†	122	25 , 5	−1 12.15	− 5 23.9	0 20 41.26	+ 8 31 55.7	8.323	0.646	+3.08	+19.1
11 11 48 12†	123	24 , 5	−1 41.87	− 6 33.5	0 20 40.15	+ 8 31 48.8	8.982	0.648	+3.08	+19.1

Observed by Frederickson.

1905 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				(212) Medea.						
	h m s		m s	′ ″	h m s	° ′ ″			s	″
Oct. 4 11 49 37†	124	30 , 6	−1 18.90	+ 1 39.9	0 20 44.61	+ 8 4 10.7	8.640	0.652	+3.06	+18.7
14 10 3 11†	125	25 , 5	+1 2.47	− 1 25.8	0 13 21.48	+ 7 21 31.2	n8.884	0.662	+3.04	+19.4
14 10 39 35†	126	30 , 6	+2 3.30	− 8 31.5	0 13 20.36	+ 7 21 23.9	n7.537	0.660	+3.04	+19.5
Observed by Frederickson.										
				(224) Oceana.						
Oct. 4 13 54 22†	127	25 , 5	+1 1.06	− 8 46.6	0 35 16.27	+ 6 54 46.2	9.400	0.678	+3.06	+18.1
5 9 56 13†	128	22 , 5	−2 38.29	+ 6 45.6	0 34 30.53	+ 6 51 41.8	n9.296	0.674	+3.06	+18.1
8 9 52 37†	129	30 , 6	+0 35.43	+ 2 28.5	0 31 47.55	+ 6 40 45.4	n9.250	0.674	+3.06	+18.5
13 10 13 47†	130	8 , 8	−0 9.21	− 3 40.9	0 27 23.45	+ 6 22 27.7	n8.958	0.674	+3.06	+18.7
22 12 33 57†	131	25 , 5	+1 49.49	−10 42.9	0 20 12.28	+ 5 51 24.1	9.414	0.690	+3.03	+19.3
22 12 45 16†	132	5 , 1	−0 49.87	− 9 30.4	0 20 11.58	+ 5 51 26.4	9.444	0.691	+3.04	+19.1
Observed by Frederickson.										
				(483) Seppina.						
Oct. 21 10 13 50†	133	25 , 5	+2 1.44	+ 7 40.3	1 38 53.09	− 3 18 29.1	n9.222	0.768	+3.04	+14.8
22 10 41 58†	134	25 , 5	+2 59.83	− 0 38.7	1 38 13.88	− 3 25 50.0	n9.018	0.770	+3.05	+14.9
29 9 39 38†	135	20 , 4	−4 5.98	− 2 32.4	1 33 52.35	− 4 12 24.3	n9.212	0.775	+3.06	+14.1
Nov. 4 10 16 12†	136	25 , 5	−1 35.88	+ 0 10.5	1 30 22.07	− 4 46 47.5	n8.589	0.782	+3.06	+14.0
Observed by Hammond.										
				(257) Silesia.						
Oct. 31 10 18 7†	137	25 , 5	+1 51.12	+ 0 48.0	2 16 58.08	+13 52 43.5	n9.206	0.579	+3.42	+12.7
31 10 38 58†	138	25 , 5	−2 41.04	+ 4 30.2	2 16 57.39	+13 52 41.7	n9.078	0.574	+3.43	+12.4
Nov. 2 10 37 12†	139	25 , 5	+2 35.88	− 1 36.0	2 15 16.50	+13 47 5.5	n9.018	0.574	+3.43	+13.0
2 11 3 19†	137	10 , 10	+0 8.47	− 4 55.2	2 15 15.45	+13 47 0.5	n8.708	0.570	+3.44	+12.9
Oct. 31, observed by Frederickson. Other observations by Hammond.										
				(189) Phthia.						
Nov. 1 10 1 19†	140	25 , 5	+1 45.42	− 8 48.2	2 11 38.74	+11 8 46.6	n9.239	0.620	+3.37	+13.1
2 9 49 12†	141	10 , 10	−0 4.88	+ 6 21.7	2 10 45.71	+11 1 40.3	n9.272	0.623	+3.37	+13.1
11 11 9 31†	142	25 , 5	+1 28.80	− 1 22.7	2 3 8.52	+10 0 18.7	8.766	0.627	+3.36	+13.6
14 8 56 19†	143	25 , 5	+0 33.94	+ 4 59.5	2 0 57.41	+ 9 42 32.0	n9.252	0.639	+3.36	+13.6
14 9 8 56†	144	25 , 5	+0 56.34	+ 2 36.0	2 0 57.01	+ 9 42 29.3	n9.190	0.636	+3.36	+13.6
14 9 25 28†	145	25 , 5	+1 56.14	+ 3 29.8	2 0 56.50	+ 9 42 24.5	n9.089	0.634	+3.36	+13.7
Nov. 1, observed by Frederickson. Other observations by Hammond.										
				(511) Davida.						
‡Nov. 10 10 19 29†	146	25 , 5	+2 17.22	+ 0 43.0	1 42 32.69	−14 17 42.1	n7.978	0.846	+2.97	+11.4
‡ 10 10 41 24†	147	25 , 5	−2 57.59	− 7 18.0	1 42 32.02	−14 17 39.4	8.553	0.846	+2.98	+11.3
10 12 1 48†	148	8 , 8	+0 6.62	+ 4 5.1	1 42 29.75	−14 17 33.2	9.292	0.837	+2.97	+11.3
14 12 32 55*	149	25 , 5	+1 11.40	+ 3 24.8	1 39 59.78	−14 7 2.9	9.452	0.824	+2.96	+10.9
14 12 58 22*	150	25 , 5	−1 53.23	− 8 37.7	1 39 59.00	−14 7 3.3	9.509	0.816	+2.97	+10.8
Observed by Frederickson.										
				(419) Aurelia.						
Nov. 14 10 51 23†	151	25 , 5	+0 36.67	− 1 21.2	3 25 3.14	+18 23 30.8	n9.091	0.495	+3.71	+ 7.1
14 11 5 22†	152	25 , 5	−0 30.72	− 5 13.3	3 25 2.25	+18 23 29.9	n8.976	0.491	+3.72	+ 7.0
29 12 28 35†	153	25 , 5	−0 39.76	+ 2 15.7	3 11 14.55	+17 15 50.3	9.348	0.538	+3.77	+ 8.5
Nov. 14, observed by Hammond. Nov. 29, observed by Frederickson.										
				(344) Desiderata.						
Nov. 21 9 44 41†	154	25 , 5	+2 49.86	+ 0 14.6	4 52 14.83	+32 38 1.9	n9.595	0.319	+4.25	− 1.9
21 10 2 31†	155	25 , 5	−2 36.47	+ 1 16.8	4 52 13.83	+32 38 5.9	n9.560	0.275	+4.24	− 2.5
26 13 6 54†	156	25 , 5	+1 4.56	+ 7 57.5	4 46 16.45	+32 52 4.8	9.002	9.983	+4.35	− 1.2
Observed by Frederickson.										
				(11) Parthenope.						
Nov. 21 13 30 56*	157	20 , 4	−3 58.90	− 3 12.7	3 45 46.14	+12 34 28.4	9.326	0.606	+3.65	+ 5.0
21 14 8 52*	158	25 , 5	+1 28.81	− 2 55.8	3 45 44.65	+12 34 22.8	9.444	0.620	+3.64	+ 5.5
Observed by Frederickson.										
				(16) Psyche.						
Nov. 23 14 59 25*	159	24 , 5	+2 37.92	− 0 23.2	4 23 38.54	+16 37 56.4	n8.473	0.520	+3.77	+ 2.1
Observed by Frederickson.										
				(29) Amphitrite.						
Nov. 23 13 25 12*	160	25 , 5	−2 48.78	− 1 7.6	4 0 35.56	+30 22 24.8	9.328	0.196	+4.23	+ 3.5
23 13 50 57*	161	25 , 5	−3 50.32	− 1 37.1	4 0 31.22	+30 22 22.7	9.425	0.242	+4.23	+ 3.4
Observed by Frederickson.										

1905 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
				(405) *Thia.*						
d h m s			m s	′ ″	h m s	° ′ ″			s	″
Nov. 25 10 16 36†	162	25 , 5	−3 23.91	+ 4 23.4	4 39 46.40	+24 21 13.5	n9.417	0.414	+4.01	− 0.5
26 9 18 39†	163	25 , 5	−0 48.95	+ 2 13.6	4 38 47.34	+24 16 18.5	n9.547	0.478	+4.03	− 0.1
Dec. 4 11 35 45†	164	25 , 5	−4 49.09	− 5 9.8	4 30 19.38	+23 32 24.2	n7.454	0.362	+4.12	+ 0.7
Observed by FREDERICKSON.										
				(322) *Phæo.*						
Nov. 29 10 15 6†	165	25 , 5	−1 11.09	− 3 35.0	4 49 43.82	+23 5 17.6	n9.400	0.437	+4.03	− 1.2
29 10 31 39†	166	25 , 5	+1 49.07	− 4 16.4	4 49 43.24	+23 5 11.5	n9.341	0.422	+4.04	− 0.8
Dec. 4 9 39 16†	167	25 , 5	−0 22.22	+ 4 35.6	4 44 42.73	+22 36 31.1	n9.432	0.458	+4.08	− 0.3
4 10 3 12†	168	25 , 5	−1 1.39	−11 21.5	4 44 41.71	+22 36 25.9	n9.354	0.436	+4.09	− 0.4
4 10 22 28†	169	20 , 4	−3 32.39	− 0 30.2	4 44 40.85	+22 36 21.7	n9.274	0.421	+4.08	− 0.6
Observed by FREDERICKSON.										

* Observed with the 12-inch equatorial. † Observed with the 26-inch equatorial. ‡ W.M.T. may be 1 minute greater.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	12 16 38.99	+ 8 20 49.5	Leipzig II, A.G. 6095	44	18 22 28.63	−20 51 37.8	Cincinnati 1885, 3017
2	12 21 43.37	+ 8 25 11.3	" " " 6120	45	18 8 4.90	−21 5 2.7	American Eph. 1905
3	12 17 40.25	+ 8 34 38.6	" " " 6100	46	18 8 8.18	−21 5 27.8	Frisby's Yarnall 7843
4	12 13 37.24	+ 8 35 33.0	" " " 6074	47	18 43 54.51	−19 6 54.3	Algiers, A.G. Zones
5	12 17 54.67	+ 8 47 53.9	" " " 6104	48	18 41 0.42	−19 0 55.8	" " "
6	12 19 1.02	+ 8 52 15.1	" " " 6108	49	18 44 41.61	−19 4 44.4	" " "
7	12 13 39.08	+ 9 12 47.6	" " " 6075	50	18 33 17.72	−19 11 16.8	" " "
8	12 15 11.17	+ 9 5 46.9	" " " 6087	51	18 36 22.30	−19 11 42.1	" " "
9	12 26 15.62	− 4 31 43.7	Strassburg A.G. Zones	52	21 41 23.04	−22 55 18.1	Cordoba, G.C. 29773
10	12 20 57.03	− 4 20 5.1	" " "	53	21 39 42.77	−22 59 57.8	" " 29745
11	12 18 22.20	− 4 26 48.8	" " "	54	21 29 49.90	−23 52 37.3	" " 29554
12	12 15 24.13	− 3 58 57.8	" " "	55	21 27 10.14	−24 34 13.2	" " 29497
13	12 16 58.39	− 3 51 29.8	" " "	56	21 24 56.95	−25 36 32.1	" " 29469
14	12 7 15.30	− 3 7 1.4	" " "	57	21 23 10.56	−26 3 29.9	" " 29420
15	12 11 24.21	− 2 42 19.2	" " "	58	21 25 14.18	−26 7 36.5	" " 29473
16	12 6 0.77	− 2 36 44.9	" " "	59	21 14 16.11	−26 44 39.8	Paris 30036
17	12 8 25.30	− 2 51 12.0	" " "	60	21 15 3.98	−27 3 59.6	Cordoba, G.C. 29259
18	12 24 52.39	− 2 47 26.3	" " "	61	21 7 39.34	−28 0 25.3	" " 29093
19	12 23 33.54	− 2 14 3.0	Nicolajew, A.G. 3421	62	21 6 39.43	−28 17 34.7	" " 29068
20	12 11 15.42	− 1 30 44.8	" " 3394	63	21 3 39.38	−28 29 30.7	" " 29001
21	12 10 8.28	− 0 47 54.9	" " 3382	64	21 3 18.26	−28 51 25.0	" " 28992
22	12 8 44.86	− 0 42 29.6	" " 3376	65	21 4 49.60	−28 52 37.1	" " 29030
23	14 32 59.16	−14 32 32.8	Washington A.G. Zones	66	20 57 14.03	−29 29 2.2	" " 28831
24	14 28 16.41	−14 35 9.5	" " "	67	20 58 29.70	−29 31 18.8	" " 28863
25	14 29 13.43	−14 23 1.3	" " "	68	20 49 0.06	−29 46 10.8	" " 28668
26	14 29 40.64	−14 23 13.0	" " "	69	19 2 35.29	− 2 38 48.2	Strassburg, A.G. Zones
27	14 34 5.26	−14 15 43.2	" " "	70	19 2 46.11	− 2 36 6.5	" " "
28	14 22 48.51	−13 57 22.9	Camb., U.S., A.G. Zones	71	19 3 15.61	− 2 36 25.3	" " "
29	14 14 5.03	−13 58 11.1	" " "	72	23 32 59.05	− 6 30 9.7	Wien, A.G. 8359
30	14 19 26.29	−13 39 2.7	" " "	73	23 33 39.48	− 6 30 50.5	" " 8362
31	14 38 31.96	− 3 32 36.4	Strassburg, A.G. Zones	74	23 36 59.21	− 6 30 35.4	" " 8374
32	14 39 39.22	− 3 13 51.1	" " "	75	23 28 17.25	− 6 55 32.4	" " 8343
33	14 42 40.94	− 3 34 35.3	" " "	76	23 29 32.85	− 7 0 21.3	" " 8347
34	14 31 58.62	− 3 51 57.6	" " "	77	0 22 40.41	+ 6 42 30.6	Leipzig II, A.G. 134
35	14 25 3.47	− 3 49 25.3	" " "	78	0 15 54.37	+ 6 12 47.1	" " 91
36	14 23 4.60	− 3 57 42.2	" " "	79	0 19 59.14	+ 6 22 23.1	" " 113
37	14 22 6.95	− 3 59 48.3	" " "	80	0 17 10.80	+ 6 14 15.1	" " 97
38	14 21 9.28	− 4 1 33.5	" " "	81	0 10 7.76	+ 5 3 40.4	½ [Albany, A.G. 33 + Leipzig II, A.G. 50.]
39	14 23 19.69	− 4 47 11.9	" " "	82	0 3 13.83	+ 4 23 58.5	Albany, A.G. 8
40	14 21 50.94	− 4 44 44.7	" " "	83	0 5 24.44	+ 4 21 45.3	" " 19
41	18 33 13.43	−21 7 19.8	Algiers, A.G. Zones	84	0 5 2.56	+ 4 18 12.1	" " 17
42	18 34 22.78	−21 3 3.5	" " "	85	0 12 9.94	+ 1 50 37.3	" " 42
43	18 24 10.85	−21 0 49.6	" " "	86	0 12 39.93	+ 1 47 35.8	" " 44

*	α	δ	Authority
	h m s	° ′ ″	
87	0 13 58.01	+ 1 46 51.4	Albany, A.G. 48
88	0 1 59.86	+ 1 19 54.3	½ [Albany, A.G. 3 + Nicolajew, A.G. 1]
89	23 55 54.16	+ 1 31 22.2	½ [Albany, A.G. 8226 + Nicolajew, A.G. 5935.]
90	23 54 58.54	+ 1 20 27.5	½ [Albany, A.G. 8221 + Nicolajew, A.G. 5928]
91	23 47 5.93	+ 2 24 8.0	Albany, A.G. 8179
92	23 46 4.78	+ 2 24 15.4	“ “ 8175
93	23 44 39.80	+ 2 27 47.2	“ “ 8167
94	23 44 14.50	+ 1 53 54.0	Paris 34279
95	23 39 17.69	+ 1 37 6.7	Albany, A.G. 8147
96	23 43 57.60	+ 1 41 15.0	“ “ 8162
97	22 35 20.20	− 5 14 27.7	Warschau 5708
98	22 28 57.95	− 5 22 31.4	“ 5687
99	22 29 57.99	− 5 32 9.0	“ 5692
100	22 27 39.96	− 5 39 43.0	Wien, A.G. 8043
101	22 31 21.59	− 5 38 44.9	“ “ 8062
102	22 23 55.59	− 5 38 19.6	“ “ 8029
103	0 26 42.50	−11 47 52.3	Camb., U.S., A.G. Zones
104	0 29 35.18	−11 48 26.7	“ “ “ “
105	0 24 21.44	−12 7 28.0	“ “ “ “
106	0 23 11.24	−12 11 3.0	“ “ “ “
107	1 49 20.70	+10 56 4.8	Leipzig I, A.G. 569
108	1 50 30.87	+11 11 19.5	“ “ “ 574
109	1 45 49.56	+10 34 23.8	½ [Leipzig I, A.G. 554 + Paris 2298.]
110	1 36 16.62	+ 9 37 54.8	Leipzig II, A.G. 633
111	1 37 34.55	+ 9 45 57.4	“ “ “ 643
112	0 52 15.17	+ 8 28 6.9	“ “ “ 323
113	0 42 18.02	+ 8 7 33.2	“ “ “ 260
114	0 47 36.30	+ 8 8 56.9	“ “ “ 296
115	0 41 58.17	+ 7 47 18.8	“ “ “ 257
116	0 38 8.81	+ 8 3 11.0	“ “ “ 232
117	0 34 22.62	+ 7 41 7.0	“ “ “ 209
118	0 34 47.51	+ 7 36 5.2	“ “ “ 213
119	0 24 10.91	+ 9 7 22.5	“ “ “ 140
120	0 24 56.26	+ 9 10 23.5	“ “ “ 145
121	0 20 25.38	+ 8 32 30.3	“ “ “ 116
122	0 21 50.33	+ 8 37 0.5	“ “ “ 126
123	0 22 18.94	+ 8 38 3.2	“ “ “ 132
124	0 22 0.45	+ 8 2 12.1	“ “ “ 129
125	0 12 15.97	+ 7 22 37.6	“ “ “ 64
126	0 11 14.02	+ 7 29 35.9	“ “ “ 56
127	0 31 12.15	+ 7 3 14.7	“ “ “ 205
128	0 37 5.76	+ 6 44 38.1	“ “ “ 226
129	0 31 9.06	+ 6 37 58.4	Leipzig II, A.G. 185
130	0 27 29.60	+ 6 25 49.9	“ “ “ 158
131	0 18 19.76	+ 6 1 47.7	“ “ “ 103
132	0 20 58.41	+ 6 0 37.7	“ “ “ 122
133	1 36 48.61	− 3 26 24.2	Strassburg, A.G. Zones
134	1 35 11.00	− 3 25 26.2	“ “ “
135	1 37 55.27	− 4 10 6.0	“ “ “
136	1 31 54.89	− 4 47 12.0	“ “ “
137	2 15 3.54	+13 51 42.8	Leipzig I, A.G. 680
138	2 19 35.00	+13 47 59.1	“ “ 698
139	2 12 37.19	+13 48 28.5	“ “ 671
140	2 9 49.95	+11 17 21.7	“ “ 660
141	2 10 47.22	+10 55 5.5	“ “ 668
142	2 1 36.36	+10 1 27.8	Leipzig II, A.G. 797
143	2 0 20.11	+ 9 37 18.9	“ “ 790
144	1 59 57.31	+ 9 39 39.7	Leipzig II, A.G. 789
145	1 58 57.00	+ 9 38 41.0	“ “ 781
146	1 40 12.50	−14 18 36.5	Washington, A.G. Zones
147	1 45 26.63	−14 10 32.7	“ “ “
148	1 42 20.16	−14 21 49.6	“ “ “
149	1 38 45.42	−14 10 38.6	“ “ “
150	1 41 49.26	−13 58 36.4	“ “ “
151	3 24 21.76	+18 24 44.9	Berlin A, A.G. 932
152	3 25 29.25	+18 28 36.2	“ “ 935
153	3 11 50.54	+17 13 26.1	“ “ 877
154	4 49 20.72	+32 37 49.2	Leiden, A.G. 1798
155	4 54 46.06	+32 36 51.6	“ “ 1828
156	4 45 7.54	+32 44 8.5	“ “ 1785
157	3 49 41.39	+12 37 36.1	Leipzig I, A.G. 1137
158	3 44 12.20	+12 37 13.1	“ “ 1112
159	4 20 56.85	+16 38 17.5	Berlin A, A.G. 1176
160	4 3 20.11	+30 23 28.9	½ [Cambridge (Eng.), 2002 + Leiden 1568.]
161	4 4 20.31	+30 23 56.4	½ [Cambridge (Eng.), 2005 + Leiden 1578.]
162	4 43 6.30	+24 16 50.6	Berlin B, A.G. 1530
163	4 39 32.26	+24 14 5.0	“ “ 1512
164	4 35 4.35	+23 37 33.3	“ “ 1488
165	4 50 50.88	+23 8 53.8	“ “ 1577
166	4 47 50.13	+23 9 28.7	“ “ 1549
167	4 45 0.87	+22 31 55.8	“ “ 1536
168	4 45 39.01	+22 47 47.8	“ “ 1538
169	4 48 9.16	+22 36 52.5	“ “ 1552

Nos. 64, 124, 169, 211, 212, 224, 257, 189, 405 and 322 were found photographically. The star places from the Cambridge (U.S.), Algiers, and Strassburg A.G. Zones were furnished through the courtesy of the Directors of the Observatories at those places.

NOTE ON STELLAR PARALLAX,

BY A. HALL.

In the interesting volume recently published by the Yale Observatory, we have determinations of the parallaxes of 163 stars with large proper motions. The parallaxes are remarkably small, and 36 of them are negative. The greatest negative parallax is $-0''.13$, which occurs with three stars. If we consider the negative values as caused by errors of observation, arising from changes in the images of the stars at different seasons of the year, from changes of temperature, &c., we are led to take $\pm 0''.13$ as the limits of such errors. This would leave only 17 parallaxes as real. But the great preponderance of small positive parallaxes, those of 110 stars, gives a probability that some of them are real. The general result is, I think, to cast doubt on all the determinations of parallax hitherto made. The Yale values are found from observations made with the greatest care, and with an instrument well adapted to such work. These results bring back to me an idea that occurred to me when engaged in a few determinations of this kind; which is, that we ought to have a more powerful method of observing. If we could measure from a line in the spectrum of a star we might gain much in accuracy. But whether such an arrangement is within the power of optics I do not know.

1906 *September* 20.

MAXIMA OF LONG-PERIOD VARIABLES.

By IDA WHITESIDE.

The times of maxima of the following twenty stars were determined from the single light-curves deduced from observations made with a four-inch Dolland telescope:

1113. *U Arietis.*

Thirteen observations of this star were made, between November 25, 1905, and March 31, 1906. The curve shows that the star reached a maximum of $7^M.5$ on about the date predicted in Chandler's *Ephemerides of Long-Period Variables* (Jan. 5, 1906). The curve is not very well determined just at the maximum, but shows that it was not far from the time predicted.

5338. *U Bootis.*

The curve for this star is also rather indeterminate, but indicating a maximum of $9^M.7$ on or near May 1, 1906. The series of observations, ten in number, extended from March 28 to June 27, 1906.

5190. *R Camelopardalis.*

Eight observations of this star, made between Jan. 25 and March 28, 1906, all followed maximum. The curve shows, however, that a maximum of about $8^M.2$ occurred before, and probably near Jan. 25, 1906. The time predicted was Feb. 3, 1906.

1623. *T Camelopardalis.*

The maximum of this star was found to be $8^M.2$ on March 5, 1906, quite near the predicted time, March 1, 1906. This was determined from ten well-distributed observations made between Jan. 25 and April 16, 1906.

2735. *U Canis Majoris.*

Only seven observations of this star were made, but they give a fairly well-defined curve. This shows a maximum of $9^M.2$ on March 7, 1906. The observations extended from Feb. 23 to April 13, 1906. The maximum was predicted for April 10, 1906.

2942. *RT Cygni.*

This star reached a maximum of $6^M.5$ on May 27, 1906. The curve was determined by thirteen observations, made between the dates April 28 and Aug. 15, 1906. The predicted time of maximum was June 10, 1906.

7444. *T Delphini.*

Ten observations of this star were made, covering the time from June 27 to Sept. 14, 1906. The curvature of the derived curve is very slight, but shows that a maximum of $9^M.9$ occurred on July 29, 1906. The predicted time was June 6, 1906, but the form of the curve is such that the maximum could not have occurred before the date given, July 29.

5770. *R Herculis.*

From thirteen observations of this star the maximum was found to be $9^M.2$ on July 10, 1906. The star was watched from May 19 to Sept. 6, 1906, and the maximum could not have occurred as early as June 26, the predicted date.

6044. *S Herculis.*

A maximum of this star was reached on May 9, 1906, at which time the star had attained a magnitude of $6^M.8$. The observations, nine in number, extended from April 18 to July 11, 1906. The curve is not very satisfactory, and the maximum may have occurred nearer the time predicted, May 22, 1906.

5950. *W Herculis.*

Eight observations of this star were made, between April 18 and July 11, 1906. They give a well-defined curve, which shows a maximum of $8^M.6$ on May 12, 1906. The date predicted was June 8, 1906.

3170. *S Hydrae.*

A maximum of $7^M.8$ occurred on Feb. 21, 1906. The star was observed ten times, from Jan. 25 to April 18, 1906. The curve is very well defined. The date predicted for the maximum was March 27, 1906.

3493. *R Leonis.*

From ten observations of this star the maximum was found to be $5^M.7$ on April 2, 1906. The observations extended from March 12 to May 18, 1906. The predicted date of maximum was May 3, 1906.

3567. *V Leonis.*

This star had a maximum of $9^M.0$ on Feb. 26, 1906. The twelve observations were made between Jan. 18 and April 18, 1906, and gave a well-determined maximum. The predicted time for the maximum was March 14, 1906.

1222. *R Persei.*

The maximum of this star, $8^M.8$, occurred on Jan. 30, 1906. There were fourteen observations, extending from Nov. 25, 1905 to March 10, 1906. The maximum was predicted for Jan. 25, 1906.

1717. *V Tauri.*

Seven observations of this star determined the maximum as $8^{M}.8$ on March 22, 1906. The predicted time was April 6, 1906. Though there are only a few observations, they extend from Feb. 23 to April 18, 1906, and give a well-determined maximum.

906. *R Trianguli.*

The maximum of this star as deduced from thirteen observations was $6^{M}.0$ on Jan. 26, 1906. The observations covered the time from Dec. 16, 1905, to March 28, 1906. The time predicted for the maximum was Feb. 17, 1906.

3825. *R Ursae Majoris.*

From eighteen observations the maximum of this star was determined as $7^{M}.7$ on March 4, 1906. The observations extended from Jan. 18 to May 29, 1906. The observed maximum was fifteen days ahead of that predicted (March 19, 1906).

4557. *S Ursae Majoris.*

Fourteen observations of this star were made, between Feb. 23 and June 27, 1906. These give a maximum of $8^{M}.0$ on April 11, 1906. The form of the curve is a little doubtful, and the predicted date, April 3, 1906, is very possibly correct.

4511. *T Ursae Majoris.*

A maximum of this star occurred on or very near the date predicted for it, July 17, 1906. The star then had a magnitude of $8^{M}.5$. Eight observations were made, extending from May 29 to Aug. 16, 1906. The star was but very little fainter at the time of the last observation than at maximum, and may possibly have become brighter still, later on, but it was impossible to observe it further.

4521. *R Virginis.*

Seven observations of this star, made between May 22 and July 26, 1906, show a well-determined maximum of $6^{M}.8$ on June 29. The time predicted for the maximum was July 14, 1906, but the curve fell very rapidly after the maximum, being $0^{M}.6$ fainter on the predicted than on the observed maximum.

South Cambridge, N.Y., 1906 October 10.

OBSERVATIONS OF COMET *e* 1906 (KOPFF),

MADE AT THE GOODSELL OBSERVATORY WITH THE 16-INCH REFRACTOR.

BY H. C. WILSON.

[Communicated by WM. W. PAYNE, Director.]

1906 Northfield M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Sept. 6 9 10 59	1	7 , 6	−0 10.51	−5 13.6	22 38 17.28	+9 23 39.0	n9.403	0.718	+2.61	+17.7
7 8 46 41	2	12 , 6	+0 41.31	−4 4.0	22 37 36.10	+9 17 56.1	n9.449	0.724	+2.61	+17.8
24 10 2 45	3	9 , 4	+1 14.77	+4 22.6	22 28 22.63	+7 23 13.5	n8.391	0.722	+2.56	+19.6
26 9 25 6	4	11 , 6	−0 20.68	+0 41.6	. . . .	. . . .	n8.892	0.725	+2.55	+19.7

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	22 38 25.18	+9 28 34.9	Mun.$_1$31470. Mun.$_2$12650	3	22 27 5.30	+7 18 31.3	Göttingen 6230
2	22 36 52.18	+9 21 42.3	DM. +9.5058. Micrometer comparison with No. 1.	4	22 28 2.4	+7 8 –	DM. +6°5035

The observations Sept. 6 and 26, were made with difficulty in bright moonlight. Those on Sept. 7 and 24 were made under more favorable circumstances, and are fairly accurate.

CONTENTS.

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DEVELOPMENT OF THE TWO PRINCIPAL NON-SECULAR TERMS IN THE RADIUS-VECTOR OF A PLANET WHICH ARE INDEPENDENT OF THE MEAN LONGITUDE OF THE DISTURBING PLANET.

By SIMON NEWCOMB.

1. In determining the indirect action of a planet on the moon there are some terms in the longitude and radius-vector of the earth which may be required to a degree of precision even higher than that required in tabulating the motion of the earth itself. This is especially the case with the constant of mean distance of the earth from the sun, to the inverse cube of which the disturbing action of the sun on the moon is proportional. In a less degree the remark may be true of the annual term, which gives rise to terms in the longitude of the moon independent of the purely lunar arguments. It therefore seems desirable not to depend solely upon the numerical values of the terms in question which have been derived in the construction of solar and other tables, but to develop the required quantities in an algebraic form. This I have done for the constant term of $\log r$, and for the coefficients depending on the mean anomaly. The terms depending on higher multiples of the mean anomaly than the first are deemed to be entirely insensible for any practical astronomical purpose. It is also the case that the perihelion and eccentricity of the earth's orbit are so adjusted as to represent all the terms in the longitude which depend upon the mean anomaly simply. The problem before us is, therefore, that of deriving the terms in the radius-vector after the adjustment in question has been made.

The developments have been carried to terms of the second order in the eccentricities and mutual inclination, this being deemed amply sufficient for the purpose. To reduce to a minimum the complexity and possible confusion arising from the double classification of the two planets as disturbed and undisturbed — outer and inner — the formulas will first be developed on the hypothesis that the outer planet is the disturbing one. Its elements are therefore represented by accents. The transformation when the disturbing planet is an inner one is in the present problem very simple.

To insure the correctness of the developments a complete duplicate computation of most of the formulas has been made by my assistant, Miss ISABEL MARTIN.

2. *Notation.* In the main the usual astronomical notation is adopted, and so far as this is the case no definitions are necessary. But there are certain modifications which I have used in nearly all my work on the subject, and which I have found so conducive to simplicity and to accuracy of thought that I cannot but feel they should be more generally adopted. They are largely based on the general principle that, so far as practicable, algebraic symbols should represent pure numbers, independent of the special concrete units that may be adopted. This advantage is especially marked in the case of the mean distance, a linear quantity of which I use the logarithm, based on an arbitrary unit never necessary to define, instead of the line itself. I shall represent the partial derivative as to $\log a$ by the symbol D, so that we shall have symbolically,

$$D = a\frac{\partial}{\partial a} = aD_a$$

I also use the CAUCHY notation D_x for a derivative as to x. To simplify still farther the processes of development and integration, the differential coefficients as to the time are taken as to nt, the mean longitude or mean anomaly of the disturbed planet. The symbol α is used for the ratio of the mean distances, taken less than unity. I also put

Δ, the linear distances of the planets.

$$R = \frac{1}{\Delta}$$

A_i, coefficients determined by the development

$$\frac{a'}{\Delta_0} = a'R_0 = \tfrac{1}{2}\sum_{-\infty}^{+\infty} A_i \cos i\,(l'-l)$$

when e and e' vanish.

μ, the sum of the masses of the sun and the disturbed planet.

$$M' = \frac{m'}{\mu}\frac{a}{a'} = \frac{m'}{\mu}\alpha$$

$$\rho = \log r$$

As R is above defined it is of dimensions -1 as to length. The actual numerical developments are therefore those of $a'R$, in order that they may be purely numerical.

π, the longitude of the perihelion, is used in the development as if counted from the common node, but, in the present problem, the point from which it is counted is indifferent; I therefore put, for brevity,

$$H = \pi - \pi'$$

3. The development of R which I have used is that found in Astronomical Papers of the *American Ephemeris*, Vol. V, Pt. I. The notation there used is partly taken from Vol. III of the same series, where the signification of the coefficients A is fully given. Selecting only the special terms necessary to our purpose, we find them to be

$$(1)\quad \left\{\begin{aligned}
a'R = {} & \tfrac{1}{2}\left(P_0^0 + (e^2+e'^2)\,P_0^2\right. \dots\dots\dots\dots & (i = 0)\\
& + (P_1^1 + e^2 P_1^3 + e'^2 P_{1,0}^{1,2})\,e\cos g \dots\dots & (i = 0)\\
& + P_2^2\,e^2\cos 2g \dots\dots\dots\dots\dots & (i = 0)\\
& + P_{2,-1}^{2,1}\;e^2 e'\cos(\pi'-\pi+g) \dots\dots & (i = +1)\\
& + (P_{0,1}^{0,1} + e^2 P_{0,1}^{2,1})\,e'\cos(\pi-\pi'+g) \dots & (i = -1)\\
& + P_{1,1}^{1,1}\,ee'\cos(\pi-\pi'+2g) \dots\dots\dots & (i = -1)\\
& + P_{-1,1}^{1,1}\;ee'\cos(\pi-\pi') \dots\dots\dots\dots & (i = -1)
\end{aligned}\right.$$

The symbolic values of the coefficients P are:—

$$(2)\quad \left\{\begin{aligned}
P_0^0 &= A_0\\
P_0^2 &= \tfrac{1}{4}(D+D^2)\,A_0\\
P_1^1 &= -\tfrac{1}{2}\,D\,A_0\\
P_2^2 &= +\tfrac{1}{8}(-3D+D^2)\,A_0\\
P_1^3 &= \tfrac{1}{16}(3D+D^2-D^3)\,A_0\\
P_{1,0}^{1,2} &= -\tfrac{1}{8}(D^2+D^3)\,A_0\\
P_{0,1}^{0,1} &= +\tfrac{1}{2}(-1+D)\,A_1\\
P_{0,1}^{2,1} &= \tfrac{1}{8}(4-5D+0D^2+D^3)\,A_1\\
P_{1,1}^{1,1} &= \tfrac{1}{4}(-2+3D-D^2)\,A_1\\
P_{-1,1}^{1,1} &= +\tfrac{1}{4}(2-D-D^2)\,A_1\\
P_{2,-1}^{2,1} &= \tfrac{1}{16}(1-2D+0D^2+D^3)\,A_1
\end{aligned}\right.$$

There is also a term of the second order in the inclination depending on twice the argument of the latitude; but this term is omitted because it will be multiplied by the eccentricities in the actual development, and will therefore be of an order higher than the second.

It is also to be remarked that the symbols A_0 and A_1 differ from the usual ones in that they include the terms depending on the mutual inclination of the orbits. Their special values are found in the work already referred to.

4. Taking the orbit of the planet as the plane of reference the equations to be integrated may be written in the form

$$\left.\begin{aligned}
D_t a &= 2\frac{m'}{\mu}\,a\,\frac{\partial R}{\partial l}\\
D_t l_0 &= -2\frac{m'}{\mu}\,aDR + \frac{m'}{2\mu}\,ea\,\frac{\partial R}{\partial e}\\
D_t e &= -\frac{m'}{\mu}\,\frac{\cos\varphi}{e}\,a\,\frac{\partial R}{\partial \pi} - \frac{m'}{2\mu}\,ea\,\frac{\partial R}{\partial g}\\
eD_t\pi &= \frac{m'}{\mu}\cos\varphi\;a\,\frac{\partial R}{\partial e}
\end{aligned}\right\}\quad (3)$$

Taking these expressions as the base of our work, the formation of the partial derivatives of R as to the elements and the integration are processes so simple that no details need be presented, unless such as may enable the reader to detect any error or imperfection in the work. The results of the two processes can be most succinctly mastered by noting that by the integration, the inequalities of each element (the secular variation of e and π being omitted as not entering into our problem), come out in the general form

$$\left.\begin{aligned}
\delta\log a &= \delta_0 a + a_1\cos g + a_2\cos 2g + \dots\\
\delta e &= \delta_0 e + e_1\cos g + e_2\cos 2g + e_1'\cos(H+g)\\
&\quad + e_2'\cos(H+2g) + e_1''\cos(-H+g)\\
\delta\pi &= \delta_0\pi + \pi_1\sin g + \pi_2\sin 2g + \pi_1'\sin(H+g)\\
&\quad + \pi_2'\sin(H+2g) + \pi_1''\sin(-H+g)\\
\delta l &= \delta_0 l + l_1\sin g + l_2\sin 2g + l_1'\sin(H+g)\\
&\quad + l_2''\sin(H+2g)
\end{aligned}\right\}\quad (4)$$

δ_0 meaning the arbitrary constant of integration, and e_1, π_1, etc., being coefficients to be determined.

The constant $\delta_0 a$ is determined by the condition that the actual mean motion as derived from observation shall be related to the undisturbed a by the condition

$$a^3 n^2 = \mu$$

The constants of e, π and l are to be determined by the condition that the undisturbed values of these elements shall represent the terms of the actual longitude of the planet which depend on g simply.

The actual mean longitude of the planet being

$$l_0 + \int n\,dt$$

the actual mean motion is

$$D_t l_0 + n$$

For the constant term of $D_t l_0$ we find, from the given equations,

$$\begin{aligned}
\delta D_t l_0 = {} & -Mn\left\{DP_0^0 + (e^2+e'^2)DP_0^2 + 2ee'\cos H\,DP_{-1,1}^{1,1}\right\} \quad (5)\\
& + Mn\left\{\frac{e^2}{2}P_0^2 + \frac{ee'}{2}\cos H P_{-1,1}^{1,1}\right\}
\end{aligned}$$

The arbitrary constant $\delta_0 a$ is now determined by the condition $\tfrac{3}{2}n\delta_0 a - \delta D_t l_0 = 0$

giving

(6) $\delta_0 a = -\frac{2}{3} M \{ DP_0^0 + (e^2+e'^2)\, DP_0^2 + 2ee' \cos H\, DP^{1,1}_{-1,1} \} + \frac{1}{3} M \{ e^2 P_0^2 + ee' \cos H\, P^{1,1}_{-1,1} \}$

The process of differentiating the seven terms of R already given, substituting the partial derivatives in (3), and integrating, need not be set forth in detail. Carrying it through, putting $1-\frac{1}{2}e^2$ for $\cos\varphi$, and dropping all terms above the second order, we find the following values of the coefficients which enter the forms (4), the factor M, common to all the coefficients, being omitted.

$$
\begin{aligned}
a_1 &= 2eP_1^1 \\
a_1' &= 2e' P^{0,1}_{0,1} \\
e_1 &= P_1^1 + e^2(P_1^3 - P_1^1) + e'^2 P^{1,2}_{1,0} \\
e_2 &= eP_2^2 \\
e_1' &= -\frac{ee'}{2} P^{0,1}_{0,1} \\
e_2' &= \tfrac{1}{2} e' P^{1,1}_{1,1} \\
e_1'' &= 2ee' P^{2,1}_{2,-1} \\
e\pi_1 &= P_1^1 + e^2(3P_1^3 - \tfrac{1}{2}P_1^1) + e'^2 P^{1,2}_{1,0} \\
&= e_1 + 2e^2 P_1^3 + \tfrac{1}{2} e^2 P_1^1 \\
e\pi_2 &= eP_2^2 = e_2 \\
e\pi_1' &= 2ee' P^{2,1}_{0,1} \\
e\pi_2' &= \tfrac{1}{2} e' P^{1,1}_{1,1} = e_2' \\
e\pi_1'' &= 2ee' P^{2,1}_{2,-1} = e_1'' \\
l_1 &= -e(\tfrac{3}{2} P_1^1 + 2DP_1^1) \\
l_2 &= -e^2(P_2^2 + DP_2^2) \\
l_1' &= -e'(3P^{0,1}_{0,1} + 2DP^{0,1}_{0,1}) \\
l_2' &= -ee'(\tfrac{5}{4} P^{1,1}_{1,1} + DP^{1,1}_{1,1})
\end{aligned}
$$

For convenience I also express $e\delta g = e\delta l - e\delta\pi$ in the same form as the elements, putting

$$
e\delta g = g_1 \sin g + g_2 \sin 2g + g_1' \sin(H+g) + g_2' \sin(H+2g) + g_1'' \sin(-H+g)
$$

where

$$g_1 = el_1 - e\pi_1, \text{ etc.}$$

5. We next express $\delta\rho$ in terms of δ of elements,

$$
(7)\quad \left\{
\begin{aligned}
\delta\rho = \delta a + \tfrac{1}{2} e\delta e &- (1-\tfrac{9}{8}e^2)\cos g\,\delta e \\
&- \tfrac{3}{2} e \cos 2g\,\delta e \\
&+ (1-\tfrac{3}{8}e^2)\sin g\, e\delta g \\
&+ \tfrac{3}{2} e \sin 2g\, e\delta g
\end{aligned}
\right.
$$

and substitute the symbolic expressions for δa, etc., leading to

$$
(8)\quad \left\{
\begin{aligned}
\delta\rho = \delta_0 a &+ \tfrac{1}{2} e\delta_0 e + \tfrac{1}{2}(g_1 - e_1) + \tfrac{9}{4} e(g_2 - e_2) - \frac{e^2}{16}(3g_1 - 9e_1) \\
&+ \{\tfrac{3}{4} e(g_2' - e_2') + \tfrac{1}{2}(g_1' + g_1'' - e_1'')\} \cos H \\
&+ \{a_1 - \delta_0 e - \tfrac{1}{4} ee_1 + \tfrac{3}{4} eg_1 + \tfrac{1}{2}(g_2 - e_2)\} \cos g \\
&+ \{a_1' + \tfrac{1}{2}(g_2' - e_2')\} \cos(H+g) + e\delta_0 g \sin g
\end{aligned}
\right.
$$

In order to determine the arbitrary constants of e and π, it is necessary to express δr in the same way as $\delta\rho$. Corresponding to (7) we have

$$
\left.
\begin{aligned}
\delta r = \delta l &+ 2\sin g\,\delta e + \tfrac{5}{4}\sin 2g\, e\delta e \\
&+ 2\cos g\, e\delta g + \tfrac{5}{4}\cos 2g\, e^2\delta g
\end{aligned}
\right\} \quad (9)
$$

Substituting the symbolic values of δl, etc., and retaining only constants and terms to the second order in g we find

$$
\left.
\begin{aligned}
\delta r = &\{l_1 + 2\delta_0 e + g_2 - e_2 + \tfrac{3}{4} e(e_1 - g_1)\} \sin g \\
&+ 2e(\delta_0 l - \delta_0\pi)\cos g \\
&+ (l_1' + g_2' - e_2')\sin(H+g) \\
&+ \{g_1' - e_1' - g_1'' + e_1'' + \tfrac{3}{4} e(g_2' - e_2')\} \sin H + \delta_0 l
\end{aligned}
\right\} \quad (10)
$$

Noting the relations between g_1, e_1, etc., we find the coefficient of $\sin g$ in (10) to be

$$2\delta_0 e + l_1 - 2e_2 + \tfrac{3}{4} ee_1 + (l_1' + g_2' - e_2')\cos H$$

which, equated to 0, gives $\delta_0 e$.

The constant term of δr, equated to 0, gives for $\delta_0 l$ a quantity of the second order in e and e'. This being again multiplied by e in $\delta\rho$, is dropped.

We find the coefficients of $\cos g$ in δr to be

$$-2e\delta_0\pi + (l_1' + 2g_2')\sin H$$

which, equated to 0, gives $\delta_0\pi$.

The value of $\delta_0 a$ is given in (6).

6. The values of the arbitrary constants and of the other terms of $\delta\rho$ are now to be expressed in terms of $P^{n,n'}_{j,j'}$ through the values of a_1, e_1, etc., already given. Then, the P's are to be replaced by the A's through (2).

Using the form

$$\delta\rho = \delta_0\rho + \delta_1\rho \cos H + \rho_c \cos g + \rho_s \sin g$$

or, restoring the factor $\frac{m'}{\mu} a$

$$\delta\rho = \frac{m'}{\mu} a \{\rho_0 + \rho_1 \cos H + (\rho_{0,c} + \rho_{1,c}\cos H)\cos g + \rho_{1,s} \sin H \sin g\}$$

we find

$$
\left.
\begin{aligned}
\rho_0 &= -\tfrac{1}{2} DA_0 + \frac{e^2}{48}(3D - 8D^2 - 2D^3)A_0 - \frac{e_2'}{24}(D^2 + D^3)A_0 \\
\rho_1 &= \frac{ee'}{24}(-7D + 5D^2 + 2D^3)A_1 \\
\rho_{0,c} &= \frac{e}{8}(3D + 2D^2)A_0 \;;\; \rho_{1,s} = \frac{e'}{4}(1 - D^2)A_1 \\
\rho_{1,c} &= \frac{e'}{4}(-1 + D^2)A_1 = -\rho_{1,s}
\end{aligned}
\right\} \quad (11)
$$

This is the solution for the action of an outer on an inner planet.

7. *Action of an Inner on an Outer Planet.* This action is derivable from the preceding by an interchange of the accented with the unaccented elements and operations. Δ being symmetrical, the required interchange is effected by writing, in the fundamental equations

$$\frac{a}{\Delta} \quad \text{for} \quad \frac{a'}{\Delta}$$

$$D' = \frac{\partial}{\partial \log a'} = -(1+D) \text{ for } D$$

Now, in the value of R as actually developed and just used, let

$$a'R = P\text{e} \cos N$$

be any one term of R. The corresponding term in the action of an inner on an outer planet is

$$aR = P'\text{e}' \cos N'$$

the accent indicating an interchange of accented and unaccented elements. Expressing $\delta\rho'$ in the form

$$\delta\rho' = m\{\rho_0' + \rho_1' \cos H' + (\rho'_{0,c} + \rho'_{1,c} \cos H') \cos g' + \rho'_{1,s} \sin H' \sin g'\}$$

where $\qquad H' = \pi' - \pi = -H$

we find

$$\rho_0' = \tfrac{1}{8}(1+D)A_0 - \frac{e'^2}{48}(9+13D+2D^2-2D^3)A_0$$

$$(12) \qquad\qquad + \frac{e^2}{24}(D+2D^2+D^3)A_0$$

$$\rho_1' = \frac{ee'}{24}(10+11D-D^2-2D^3)A_1$$

$$\rho'_{0,c} = \frac{e'}{8}(-1+D+2D^2)A_0 \quad ; \quad \rho'_{1,c} = -\frac{e}{4}(2D+D^2)A_0$$

$$\rho'_{1,s} = \frac{e}{4}(2D+D^2) = -\rho'_{1,c}$$

These last results have been reached by an independent development, in order that the known relation between the two actions may serve as a verification of the accuracy of each. The transformation of the expressions (11) into (12), or *vice versa*, by substituting $-(1+D)$ for D is very simple.

8. If it be desired to use the more familiar coefficients $b_n^{(i)}$ instead of A_0 and A_1, and the derivatives as to a or α instead of those as to $\log a$, it may be done by the substitution

$$A_0 = b_1^{(0)} - \sigma^2 \alpha\, b_3^{(1)}$$
$$A_1 = b_1^{(1)} - \tfrac{1}{2}\sigma^2 \alpha\,(b_3^{(0)} + b_3^{(2)})$$

where $\sigma = \sin \frac{1}{2}$ mutual inclination.

To introduce the derivatives as to a or α instead of the logarithmic derivatives, we have the forms

$$D = \alpha D_\alpha$$
$$D^2 = \alpha D_\alpha + \alpha^2 D_\alpha^{\,2}$$
$$D^3 = \alpha D_\alpha + 3\alpha^2 D_\alpha^{\,2} + \alpha^3 D_\alpha^{\,3}$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

OBSERVATIONS OF MINOR PLANETS AND COMETS,

MADE WITH THE 26-INCH EQUATORIAL AT U.S. NAVAL OBSERVATORY,

By J. C. HAMMOND and M. FREDERICKSON.

[Communicated by Rear-Admiral Asa Walker, U.S.N., Superintendent.]

1906 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
				(95) *Arethusa.*						
Jan. 5 10ʰ 8ᵐ 43ˢ	1	25 , 5	+0ᵐ 44.86ˢ	+ 3′ 36.6″	6ʰ 27ᵐ 19.38ˢ	+11° 40′ 3.4″	n9.200	0.611	+0.74ˢ	− 8.9″*
5 10 19 26	2	24 , 5	+0 9.64	− 4 20.0	6 27 18.99	+11 40 3.6	n9.139	0.608	+0.74	− 8.9*
5 10 26 54	3	25 , 5	−0 11.59	+ 1 17.7	6 27 18.64	+11 40 3.2	n9.090	0.607	+0.74	− 8.9*
6 10 59 14	4	25 , 5	−1 32.48	+ 2 36.2	6 26 25.12	+11 38 2.4	n8.687	0.603	+0.75	− 9.0*
				(347) *Pariana.*						
Jan. 9 10 22 53	5	25 , 5	−1 35.36	+ 6 55.9	6 27 44.82	+28 37 45.8	n9.051	0.209	+0.83	− 8.0†
18 10 6 29	6	25 , 5	+1 48.76	− 4 58.6	6 18 39.31	+29 23 11.1	n8.696	0.159	+0.86	− 7.0†
18 11 1 19	7	25 , 5	−2 38.68	− 0 42.9	6 18 37.25	+29 23 23.0	8.880	0.165	+0.86	− 7.1†
20 14 22 24	8	23 , 5	+1 29.30	− 2 8.6	6 16 42.11	+29 32 46.7	9.663	0.499	+0.86	− 6.9†
28 9 15 3	9	20 , 5	+0 30.20	− 5 29.8	6 10 49.93	+30 2 25.1	n8.777	0.130	+0.82	− 6.1†
28 10 7 24	10	8 , 8	−0 4.86	+ 0 31.6	6 10 48.74	+30 2 32.5	8.784	0.130	+0.83	− 6.1†
				(68) *Leto.*						
Jan. 10 9 58 14	11	25 , 5	+2 29.32	+ 0 30.5	8 32 44.85	+30 20 55.2	n9.601	0.392	+0.68	− 9.9*
10 10 12 17	12	25 , 5	+0 18.58	+ 0 15.2	8 32 44.45	+30 20 57.1	n9.576	0.364	+0.67	−10.0*
				(17) *Thetis.*						
Jan. 28 13 11 3	13	25 , 5	−2 36.58	+ 2 11.7	8 50 57.00	+18 22 32.7	9.022	0.493	+0.88	− 9.4†
29 10 31 7	14	25 , 5	+1 26.99	− 7 28.2	8 50 1.09	+18 27 50.0	n9.329	0.514	+0.90	− 9.4†
29 11 1 15	15	25 , 5	+2 16.58	− 4 33.9	8 50 2.97	+18 27 56.5	n9.186	0.499	+0.90	− 9.4†
Feb. 1 12 51 19	16	25 , 5	−1 25.03	− 6 18.1	8 11 0.01	+19 4 2.7	9.133	0.484	+0.95	− 9.5†
5 11 24 14	17	25 , 5	+2 55.01	+ 0 6.5	8 43 4.43	+19 9 30.1	n8.548	0.470	+0.95	− 9.5†
15 9 59 50	18	25 , 5	+0 1.88	+ 3 0.6	8 33 15.21	+20 3 18.0	n9.048	0.459	+0.97	− 9.1†
15 10 12 5	19	25 , 5	−0 31.99	− 3 7.8	8 33 44.63	+20 3 19.6	n8.936	0.455	+0.97	− 9.1†

1906 Wash'n M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
				(184) *Dejopeja.*						
	h m s		m s	′ ″	h m s	° ′ ″			s	″
Jan. 30 13 34 11	20	25 , 5	+0 31.29	+ 7 34.4[4]	7 55 30.22	+21 43 39.4[4]	9.442	0.478	+0.92	− 9.0†
31 12 31 29	21	25 , 5	−1 47.47	− 5 57.7	7 54 43.41	+21 45 35.9	9.220	0.434	+0.93	− 9.0†
Feb. 2 11 59 40	22	25 , 5	+2 18.92	+ 2 58.0	7 53 9.16	+21 49 7.3	9.079	0.420	+0.93	− 9.0†
2 12 24 16	23	25 , 5	+1 22.68	− 8 24.0	7 53 8.31	+21 49 8.9	9.232	0.434	+0.93	− 9.0†
2 12 49 59	24	25 , 5	+3 31.07	− 5 45.4	7 53 7.52	+21 49 9.4	9.345	0.452	+0.93	− 9.0†
				(543) *Charlotte* [1904 *OT*].						
Feb. 13 9 33 24	25	25 , 5	−1 10.71	− 3 29.8	8 31 35.79	+15 24 14.4	*n*9.237	0.557	+0.97	− 9.6†
13 9 48 14	26	25 , 5	−1 6.21	+ 2 50.6	8 31 35.26	+15 24 13.2	*n*9.157	0.552	+0.97	− 9.6†
22 11 42 49	27	25 , 5	+3 31.51	− 1 55.1	8 25 8.65	+15 31 17.4	9.242	0.555	+0.93	− 9.7†
22 12 3 2	28	25 , 5	−2 20.20	− 4 9.3	8 25 7.82	+15 31 18.1	9.328	0.563	+0.94	− 9.6†
22 12 33 24	29	25 , 5	−1 51.38	− 7 29.9	8 25 7.07	+15 31 18.9	9.427	0.578	+0.94	− 9.6†
				(364) *Isara.*						
Feb. 13 12 25 29	30	20 , 4	−1 14.40	− 2 58.2	9 32 8.06	+21 31 28.8	8.756	0.418	+0.99	− 9.2†
17 10 44 28	31	25 , 5	+3 3.52	− 0 27.4	9 28 5.53	+21 59 2.6	*n*9.067	0.416	+1.01	− 9.1†
19 10 43 46	32	25 , 5	+1 1.51	− 1 27.6	9 26 6.34	+22 11 54.5	*n*8.990	0.407	+1.02	− 9.1†
19 11 0 55	33	24 , 5	−0 48.80	− 4 8.2	9 26 5.55	+22 11 59.6	*n*8.786	0.402	+1.02	− 9.1†
28 12 53 20	34	25 , 5	−2 12.72	− 2 54.6	9 18 4.33	+22 59 11.9	9.420	0.446	+1.01	− 8.3†
28 13 12 42	35	25 , 5	−2 1.22	− 0 47.5	9 18 3.69	+22 59 15.7	9.473	0.465	+1.01	− 8.3†
				(118) *Peitho.*						
Feb. 15 12 43 19	36	25 , 5	−2 56.38	+ 3 32.6	9 41 4.29	+28 34 36.5	8.992	0.206	+1.01	− 8.9†
27 12 8 16	37	20 , 4	−0 25.14	+ 7 10.7	9 29 17.15	+28 52 19.4	9.180	0.217	+1.04	− 7.6†
27 12 34 54	38	25 , 5	+1 26.99	+ 5 20.2	9 29 16.30	+28 52 19.3	9.319	0.250	+1.04	− 7.6†
27 12 57 37	39	25 , 5	−1 47.72	+ 6 17.0	9 29 15.38	+28 52 17.0	9.406	0.285	+1.04	− 7.6†
				(335) *Roberta.*						
Feb. 15 14 12 19	40	25 , 5	+1 30.97	− 0 16.0	8 9 32.08	+17 40 43.6	9.600	0.610	+0.93	− 9.4†
17 8 53 46	41	25 , 5	−2 7.34	− 3 11.4	8 8 10.78	+17 48 15.4	*n*9.245	0.516	+0.93	− 9.4†
19 8 45 8	42	25 , 5	−2 5.42	− 1 1.7	8 6 43.89	+17 56 24.4	*n*9.242	0.514	+0.91	− 9.4†
19 9 5 25	43	25 , 5	−1 56.27	− 4 3.3	8 6 43.37	+17 56 28.3	*n*9.128	0.506	+0.92	− 9.3†
19 9 23 37	44	20 , 4	−0 7.53	+ 0 45.9	8 6 42.63	+17 56 32.1	*n*8.987	0.500	+0.91	− 9.4†
				(178) *Belisana.*						
Feb. 16 8 53 30	45	25 , 5	−0 27.09	+ 1 26.1	10 38 58.18	+11 51 0.8	*n*9.608	0.670	+0.96	− 8.8†
25 10 44 42	46	25 , 5	+0 30.91	+ 4 53.7	10 30 25.82	+12 40 39.7	*n*9.232	0.598	+1.05	− 9.1†
25 11 8 36	47	25 , 5	+1 0.35	+ 0 56.9	10 30 24.97	+12 40 44.9	*n*9.093	0.592	+1.05	− 9.1†
				(550) [1904 *PL*].						
Feb. 16 10 52 46	48	20 , 4	+0 16.79	− 4 7.6	9 42 58.93	+ 0 36 36.8	*n*9.109	0.735	+1.07	− 9.6†
16 11 11 30	49	25 , 5	−2 26.97	+ 4 18.7	9 42 58.26	+ 0 36 41.4	*n*8.964	0.734	+1.07	− 9.6†
23 11 53 23	50	25 , 5	+0 40.90	+ 6 19.6	9 36 37.54	+ 1 2 16.6	8.767	0.730	+1.09	−10.4†
Mar. 5 9 47 57	51	23 , 5	+2 37.50	+ 3 29.9	9 28 22.94	+ 1 43 29.4	*n*8.984	0.724	+1.05	−11.2†
5 10 12 6	52	25 , 5	−2 44.81	+ 2 28.4	9 28 22.11	+ 1 43 32.8	*n*8.687	0.723	+1.06	−11.1†
				(359) *Georgia.*						
Feb. 19 13 57 32	53	25 , 5	−1 24.35	− 5 54.1	9 10 30.56	+23 56 47.9	9.517	0.466	+1.01	− 8.8†
23 13 0 19	54	25 , 5	+0 36.59	+ 1 35.4	9 7 3.74	+24 2 23.8	9.418	0.422	+1.01	− 8.4†
				(514) [1903 *MB*].						
Mar. 4 11 49 2	55	25 , 5	−0 16.93	+ 3 23.2	10 22 12.01	+ 4 34 39.2	8.476	0.693	+1.11	− 9.7†
4 12 0 36	56	5 , 1	+0 54.02	+10 12.4	10 22 11.67	+ 4 34 40.9	8.725	0.693	+1.11	− 9.7†
5 11 40 10	57	23 , 5	−1 16.92	− 8 25.0	10 21 26.74	+ 4 38 56.5	8.353	0.692	+1.11	− 9.7†
21 10 42 52	58	25 , 5	+2 18.64	+ 0 39.3	10 10 46.28	+ 5 44 53.3	8.733	0.680	+1.04	−10.1†
21 10 59 56	59	25 , 5	−1 38.80	+ 1 34.9	10 10 46.10	+ 5 44 56.4	8.945	0.681	+1.05	−10.0†
21 11 23 19	60	25 , 5	−1 50.15	− 6 9.4	10 10 45.41	+ 5 45 1.1	9.126	0.682	+1.05	−10.0†
				(333) *Badenia.*						
Mar. 22 13 1 22	61	25 , 5	+0 15.08	+ 1 40.2	10 40 15.13	+ 9 28 16.0	9.426	0.653	+1.09	− 9.0†

1906 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.
(332) *Siri.*									
h m s			m s	′ ″	h m s	° ′ ″			s ″
Apr. 1 13 32 8	62	25 , 5	+0 43.61	− 5 53.1	12 17 13.05	+ 0 10 8.6	9.338	0.739	+1.27 − 7.7†
1 13 57 37	63	25 , 5	−0 49.55	− 5 55.9	12 17 12.08	+ 0 10 13.1	9.417	0.739	+1.27 − 7.7†
3 10 31 54	64	25 , 4	+0 12.98	− 5 10.3	12 15 40.96	+ 0 18 20.5	n9.056	0.737	+1.27 − 7.8†
3 10 50 50	65	25 , 5	+1 37.60	− 6 11.5	12 15 40.35	+ 0 18 24.7	n8.884	0.737	+1.27 − 7.8†
3 11 6 45	66	25 , 5	−1 10.65	− 3 13.2	12 15 39.81	+ 0 18 29.6	n8.654	0.737	+1.27 − 7.7†
(484) *Pittsburghia.*									
Apr. 2 10 43 45	67	25 , 5	+0 38.56	− 4 21.1	12 16 34.59	+15 35 31.4	n9.016	0.544	+1.20 − 6.6*
2 11 5 42	68	25 , 5	+3 27.59	+ 3 14.2	12 16 33.97	+15 35 37.4	n8.770	0.540	+1.20 − 6.6*
(86) *Semele.*									
Apr. 3 13 43 5	69	24 , 5	+1 32.68	+ 1 34.0	11 49 30.92	+ 8 12 12.5	9.473	0.671	+1.19 − 7.7†
3 14 7 5	70	20 , 4	+1 54.42	+ 7 57.3	11 49 30.41	+ 8 12 13.1	9.522	0.678	+1.19 − 7.7†
(324) *Bamberga.*									
Apr. 6 11 10 39	71	25 , 5	−1 7.20	− 4 38.0	12 46 20.76	−17 46 0.4	n8.894	0.863	+1.50 − 7.1*
6 11 24 17	72	5 , 1	−3 29.82	+ 4 46.2	12 46 20.39	−17 45 57.9	n8.698	0.864	+1.51 − 7.0*
10 10 23 51	73	25 , 5	−0 15.65	+ 4 26.8	12 42 53.29	−17 30 10.0	n9.129	0.859	+1.52 − 7.6*
13 10 3 44	74	25 , 5	+0 30.89	− 1 32.5	12 40 19.14	−17 17 26.5	n9.164	0.857	+1.51 − 8.1*
15 11 25 32	75	25 , 5	+3 28.41	+ 1 51.0	12 38 35.02	−17 8 21.6	8.633	0.861	+1.51 − 8.6*
(278) *Paulina.*									
Apr. 10 11 41 2	76	25 , 5	−1 52.05	+ 5 2.2	13 42 36.07	− 1 22 45.7	n8.974	0.753	+1.35 − 5.2†
16 13 25 50	77	25 , 5	+1 53.95	− 2 7.8	13 37 10.35	− 1 10 33.7	9.227	0.750	+1.39 − 5.4†
16 13 45 52	78	25 , 5	−2 2.11	− 6 27.6	13 37 9.42	− 1 10 33.1	9.312	0.750	+1.39 − 5.4†
(407) *Arachne.*									
Apr. 13 10 51 25	79	25 , 5	−2 19.82	+ 3 28.1	12 15 11.94	−14 5 9.0	7.441	0.845	+1.43 − 9.0*
15 10 18 48	80	25 , 5	+0 27.31	− 0 24.8	12 14 13.35	−13 53 40.6	n8.653	0.844	+1.42 − 9.4*
15 10 34 42	81	25 , 5	+2 25.58	+ 4 7.3	12 14 13.00	−13 53 38.0	n8.090	0.844	+1.41 − 9.5*
18 11 43 27	82	25 , 5	+1 17.24	− 2 38.7	12 11 58.45	−13 35 46.1	9.190	0.837	+1.39 − 9.7*
(65) *Cybele.*									
May 21 8 41 52	83	25 , 5	−0 8.60	− 3 33.2	14 47 8.60	−11 8 48.5	n9.399	0.814	+1.74 − 2.6*
21 8 53 32	84	25 , 5	−1 57.67	+ 8 34.2	14 47 8.32	−11 8 46.2	n9.362	0.816	+1.75 − 2.5*
25 10 19 13	85	25 , 5	+2 33.01	+ 0 31.7	14 44 35.37	−10 58 2.6	n8.460	0.826	+1.74 − 2.7†
(554) *Peraga.*									
Apr. 23 12 51 49	86	25 , 5	−1 17.59	+ 2 15.4	16 5 29.13	−24 49 10.9	n9.150	0.889	+1.55 + 2.7†
23 13 8 18	87	20 , 4	+0 5.67	+ 3 8.2	16 5 28.48	−24 49 11.4	n9.052	0.892	+1.56 + 2.6†
27 12 13 23	88	25 , 5	−2 32.85	+ 0 47.0	16 2 28.10	−24 42 51.1	n9.100	0.890	+1.64 + 2.4†
May 4 13 35 36	89	25 , 5	+1 3.56	+ 4 0.8	15 56 21.57	−24 27 33.3	8.789	0.893	+1.78 + 1.3†
4 13 51 5	90	25 , 5	−1 56.81	+ 0 26.4	15 56 20.89	−24 27 32.2	8.978	0.892	+1.78 + 1.4†
10 12 7 30	91	25 , 5	+2 17.57	+ 4 24.9	15 50 36.21	−24 10 46.0	n8.826	0.892	+1.88 + 0.6†
20 12 10 2	92	25 , 5	−2 36.17	− 2 45.6	15 40 18.06	−23 35 23.9	8.670	0.890	+1.98 − 0.3†
22 12 15 30	93	25 , 5	+3 28.13	+ 3 16.9	15 38 13.45	−23 27 30.9	8.903	0.889	+2.00 − 0.8†
23 12 53 7	94	25 , 5	+2 58.52	+ 1 41.8	15 37 9.92	−23 23 21.7	9.230	0.881	+2.01 − 1.0†
23 13 22 36	95	25 , 5	+3 4.18	+ 6 24.7	15 37 8.71	−23 23 15.6	9.359	0.873	+2.01 − 0.9†
(443) *Photographica.*									
May 21 9 37 17	96	25 , 5	+1 10.76	− 3 18.3	14 31 12.12	− 8 27 40.1	n9.068	0.807	+1.67 − 3.0*
21 9 49 19	97	25 , 5	+0 50.39	+ 2 24.7	14 31 11.78	− 8 27 39.2	n8.970	0.808	+1.67 − 3.0*
25 11 8 17	98	25 , 5	−2 31.77	− 3 13.7	14 28 17.86	− 8 13 8.2	9.011	0.806	+1.68 − 2.7†
25 11 28 43	99	25 , 5	+2 19.96	+ 0 18.6	14 28 18.31[1]	− 8 13 3.7	9.153	0.804	+1.66 − 3.0†
(92) *Undine.*									
May 22 10 25 11	100	25 , 5	+2 0.64	− 6 27.9	15 53 39.39	− 9 53 46.7	n9.243	0.813	+1.80 + 0.4*
June 2 11 10 14	101	25 , 5	−2 1.92	− 0 2.3	15 41 55.51	− 9 52 49.4	8.219	0.819	+1.88 + 0.5†
2 11 27 31	102	25 , 5	−0 15.06	− 3 47.2	15 41 54.90	− 9 52 48.9	8.712	0.819	+1.88 + 0.5†
(190) *Ismene.*									
May 22 11 6 19	103	25 , 5	+1 29.79	+ 2 17.9	15 56 45.56	−13 9 6.6	n9.015	0.837	+1.85 + 0.4*
25 12 51 38	104	25 , 5	+1 34.88	+ 8 4.5	15 54 49.13	−13 2 12.8	9.140	0.835	+1.87 + 0.4†
(106) *Dione.*									
May 23 10 41 17	105	25 , 5	+0 31.86	+ 2 52.2	16 22 13.18	−21 51 30.6	n9.306	0.871	+1.97 + 1.8*

1906	Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(289) *Nenetta*.						
	h m s			m s	′ ″	h m s	° ′ ″			s	″
May 24	10 15 6	106	30 , 6	+1 57.97	− 1 31.6	15 24 23.00	−10 12 33.6	n8.811	0.824	+1.80	− 0.8†
29	12 9 50	107	24 , 5	+0 59.08	+ 6 15.8	15 20 26.06	−10 25 18.8	9.183	0.818	+1.82	− 0.9†
					(374) *Burgundia*.						
June 29	12 1 57	108	25 , 5	−0 28.75	− 3 30.2	17 57 5.97	−11 11 59.3	8.879	0.830	+2.20	+ 7.2†
					Comet 1905 *c* (Giacobini).						
1905 Dec. 7	17 20 12	109	20 , 4	−0 56.39	− 7 41.1	14 27 34.03	+20 26 23.3	n9.627	0.598	+0.71	− 9.3[2]
11	17 27 51	110	25 , 4	−1 36.12	− 7 8.5	14 47 24.14	+18 30 12.5	n9.618	0.614	+0.77	− 8.1[2]
1906 Jan. 6	17 55 1	111	25 , 5	+0 55.57	+ 5 26.0	17 36 7.08	− 3 23 34.9	n9.632	0.751	−1.83	+ 6.4*
9	18 40 47	112	20 , 1	+2 47.84	− 7 0.8	18 0 40.67	− 7 1 33.4	n9.605	0.768	−1.84	+ 6.0†
Feb. 15	7 11 27[3]	113	25 , 5	+3 21.99	− 7 56.8	0 29 3.39	−15 46 44.0	9.636	0.782	−1.33	−12.3†
16	6 56 51	114	25 , 5	+1 0.18	−11 4.7	0 36 5.20	−15 0 11.3	9.618	0.790	−1.29	−12.5†
17	6 56 57	115	25 , 5	+0 58.09	+ 3 11.0	0 42 56.37	−14 13 32.1	9.613	0.789	−1.26	−12.6†
24	7 12 51	116	25 , 5	+2 42.63	+ 0 44.5	1 24 16.81	− 9 4 1.7	9.607	0.775	−1.10	−12.8†
26	7 11 58	117	20 , 1	+0 33.33	+ 2 20.3	1 35 0.77	− 7 43 11.7	9.602	0.771	−1.06	−12.9†
Mar. 22	7 56 16	118	25 , 5	−2 15.22	+ 1 2.1	3 2 27.07	+ 4 2 21.5	9.642	0.728	−0.90	−11.8†
					Comet 1905 IV (1906 *b*).						
Mar. 5	12 29 6	119	25 , 5	−2 14.88	− 3 27.3	11 35 1.88	+ 1 42 36.3	n8.438	0.724	+1.12	− 7.9†
6	11 42 38	119	10 , 2	−2 39.70	− 2 14.5[5]	11 34 37.06	+ 1 43 49.0[5]	n9.044	0.724	+1.12	− 8.0†
17	14 37 24	120	30 , 6	−0 8.70	− 0 6.5	11 29 48.38	+ 2 0 42.6	9.485	0.725	+1.19	− 8.7†
20	12 27 26	120	25 , 5	−1 21.72	+ 4 12.8	11 28 35.37	+ 2 5 1.8	8.997	0.720	+1.20	− 8.8†
21	12 31 1	120	25 , 5	−1 46.46	+ 5 41.8	11 28 10.63	+ 2 6 30.8	9.060	0.720	+1.20	− 8.8†
22	13 45 14	120	25 , 5	−2 11.97	+ 7 12.2	11 27 45.12	+ 2 8 1.2	9.410	0.723	+1.20	− 8.8†
31	13 44 41	121	25 , 5	+0 31.26	− 3 41.9	11 24 26.11	+ 2 19 28.7	9.500	0.724	+1.18	− 8.9†
Apr. 1	11 42 17	121	25 , 5	+0 16.40	− 2 43.1	11 24 8.25	+ 2 20 27.5	9.049	0.718	+1.18	− 8.9†
3	12 17 27	121	25 , 5	−0 21.08	− 0 41.1	11 23 30.76	+ 2 22 29.5	9.288	0.719	+1.17	− 8.9†
12	10 8 29	122	25 , 5	+1 41.72	+ 2 33.9	11 21 18.99	+ 2 28 30.1	8.244	0.716	+1.12	− 8.8†
13	12 4 0	123	25 , 5	+1 10.46	− 7 45.4	11 21 7.03	+ 2 28 51.9	9.387	0.720	+1.12	− 8.8†
16	12 25 16	122	25 , 5	+1 1.23	+ 3 34.1	11 20 38.48	+ 2 29 30.3	9.474	0.722	+1.10	− 8.8†
17	10 31 17	124	25 , 5	−0 29.15	+ 6 18.8	11 20 31.37	+ 2 29 33.8	9.038	0.716	+1.09	− 8.7†
27	11 3 1	124	25 , 5	−1 0.54	+ 2 47.1	11 19 59.90	+ 2 26 2.6	9.372	0.720	+1.01	− 8.2†
May 19	10 58 19	125	25 , 5	+1 9.50	+ 1 8.0	11 23 54.73	+ 1 50 8.6	9.548	0.729	+0.81	− 7.1†
					Comet 1906 *c* (Ross).						
Mar. 20	7 17 39	126	15 , 3	−1 5.35	+ 1 21.5	2 12 36.00	− 4 44 58.5	9.644	0.751	−1.10	−12.6*
21	7 21 42	127	20 , 4	−3 19.92	− 7 51.4	2 15 47.24	− 3 39 38.9	9.646	0.748	−1.08	−12.5*
22	7 22 16	128	20 , 4	−1 13.49	− 3 37.5	2 18 52.53	− 2 36 3.5	9.646	0.746	−1.08	−12.3*
23	7 26 27	129	25 , 5	−0 40.63	− 6 50.6	2 21 53.36	− 1 33 50.1	9.648	0.743	−1.08	−12.1*
Apr. 2	7 44 33	130	9 , 2	−1 0.01	+14 17.2	2 48 19.65	+ 7 31 57.3	9.662	0.730	−1.04	−10.9*
3	7 51 55	131	8 , 2	+1 2.27	− 3 29.2	2 50 41.52	+ 8 19 48.2	9.664	0.731	−1.04	−10.7*

* Observed by Hammond. † Observed by Frederickson. [2] Observed by W. W. Dinwiddie. [3] Observed on 12-inch equatorial.

[4] Δδ and App. δ probably in error by 10″, the value of a revolution of the micrometer. Corrected values : Δδ +7′ 44″.4 : App. δ +21° 43′ 49″.4.

[5] Δδ and App. δ probably in error by 10″, the value of a revolution of the micrometer. " " −2′ 4″.6 : + 1° 43′ 58″.9.

Mean Places of Comparison-Stars for the beginning of the year.

The star places from the Cambridge (U.S.) Zones were furnished through the courtesy of the Director of the Observatory at that place.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	6 26 33.78	+11 36 35.7	Leipzig I, A.G. 2315	9	6 10 18.91	+30 8 1.0	½ (Cambridge, Eng., 3078 + ½ Leiden, 2534)
2	6 27 8.61	+11 44 32.5	" " 2322	10	6 10 52.77	+30 2 7.0	½ (Cambridge, Eng., 3089 + ½ Leiden, 2541)
3	6 27 29.49	+11 38 51.1	" " 2328	11	8 30 14.85	+30 20 34.6	Leiden, A.G. 3580
4	6 27 56.85	+11 35 35.2	" " 2333	12	8 32 25.20	+30 20 51.9	" " 3592
5	6 29 19.35	+28 30 57.9	Camb. (Eng.) A.G. 3341	13	8 53 32.70	+18 20 30.4	Berlin (A), A.G. 3617
6	6 16 49.69	+29 28 16.7	" " 3175	14	8 48 36.20	+18 35 27.6	" " 3572
7	6 21 15.07	+29 24 13.0	" " 3231	15	8 47 15.49	+18 32 39.8	" " 3561
8	6 15 11.95	+29 35 2.2	" " 3153	16	8 45 24.09	+19 11 0.3	" " 3545

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
17	8 40 8.44	+19 9 33.4	Berlin (A), A.G. 3510	75	12 35 5.07	−17 10 4.0	Washington A.G. Zones
18	8 33 42.36	+20 0 26.5	Berlin (B), " 3469	76	13 11 26.77	− 1 27 12.7	Nicolajew, A.G. 3643
19	8 34 18.65	+20 6 36.5	" " 3475	77	13 35 15.01	− 1 8 20.5	" " 3623
20	7 54 58.01	+21 36 14.0	" " 3211	78	13 39 10.14	− 1 4 0.1	" " 3628
21	7 56 29.95	+21 51 42.6	" " 3221	79	12 18 3.33	−14 8 28.1	Washington, A.G. Zones
22	7 50 49.31	+21 46 18.3	" " 3184	80	12 13 44.62	−13 53 6.4	Rad. 1890, 3194
23	7 51 44.70	+21 57 41.9	" " 3192	81	12 11 16.01	−13 57 35.8	Camb. (U.S.), A.G. Zones
24	7 49 35.52	+21 55 3.8	" " 3172	82	12 10 39.82	−13 32 57.7	Rad. 1890, 3177
25	8 32 45.56	+15 27 53.8	Berlin (A), A.G. 3436	83	14 47 15.46	−11 5 12.7	Camb. (U.S.), A.G. Zones
26	8 32 40.50	+15 21 32.2	" " 3435	84	14 49 4.24	−11 17 17.9	" " " "
27	8 21 36.18	+15 33 22.2	" " 3335	85	14 42 0.59	−10 58 31.6	Paris 18203
28	8 27 27.08	+15 35 37.0	" " 3390	86	16 6 45.17	−24 51 29.0	Cordoba Zones XVI^h 364
29	8 27 0.51	+15 38 58.4	" " 3385	87	16 5 21.25	−24 52 22.2	" " " 260
30	9 33 21.47	+21 34 36.2	Berlin (B), A.G. 3808	88	16 4 59.31	−24 43 40.5	" " " 236
31	9 25 1.00	+21 59 39.1	" " 3771	89	15 55 16.23	−24 31 35.4	Cordoba Zones XV^h 3745
32	9 25 3.81	+22 13 31.2	" " 3772	90	15 58 15.92	−24 28 0.0	" " " 3963
33	9 26 53.33	+22 16 16.9	" " 3787	91	15 48 16.76	−24 15 11.5	Cape 1890, 1885
34	9 20 16.04	+23 2 14.8	" " 3752	92	15 42 52.25	−23 32 38.0	Cordoba Gen. Cat. 21408
35	9 20 3.90	+23 0 11.5	" " 3751	93	15 34 43.32	−23 30 47.0	" " " 21234
36	9 43 59.66	+28 31 12.8	Camb., Eng., A.G. 5099	94	15 34 9.39	−23 25 2.5	" " " 21222
37	9 29 41.25	+28 45 16.3	" " 5017	95	15 34 2.52	−23 29 39.4	" " " 21216
38	9 27 48.27	+28 47 6.7	" " 5005	96	14 29 59.69	− 8 24 18.8	Vienna, A.G. 5126
39	9 31 2.06	+28 45 37.6	" " 5024	97	14 30 19.72	− 8 30 0.9	" " 5130
40	8 8 0.18	+17 41 9.0	Berlin (A), A.G. 3241	98	14 30 47.95	− 8 9 51.8	" " 5134
41	8 10 17.19	+17 51 36.2	" " 3255	99[1]	14 25 56.69	− 8 13 19.3	" " 5114
42	8 8 18.40	+17 57 35.5	" " 3248	100	15 51 36.95	− 9 47 19.2	" " 5545
43	8 8 38.72	+18 0 40.9	" " 3246	101	15 46 55.55	− 9 52 47.6	" " 5526
44	8 6 49.25	+17 55 55.6	" " 3232	102	15 45 8.08	− 9 49 2.2	" " 5518
45	10 39 24.31	+11 19 43.5	Leipzig I, A.G. 4099	103	15 55 13.92	−13 11 24.9	Camb. (U.S.), A.G. Zones
46	10 29 53.83	+12 35 55.1	" " 4057	104	15 53 12.38	−13 10 17.7	Rad. 1890, 4127
47	10 29 23.57	+12 39 57.1	" " 4056	105	16 21 39.35	−21 54 24.6	" " 4264
48	9 42 41.07	+ 0 10 51.0	Nicolajew, A.G. 2937	106	15 22 23.23	−10 38 1.2	Paris 19152
49	9 45 24.16	+ 0 32 32.3	" " 2948	107	15 19 25.16	−10 31 33.7	Camb. (U.S.), A.G. Zones
50	9 35 55.55	+ 0 55 37.4	½ (Nic. 2914 + Alb. 3845)	108	17 57 32.52	−11 38 36.3	" " " "
51	9 25 44.39	+ 1 40 10.7	Albany, A.G. 3797	109	14 28 29.71	+20 31 16.7	Berlin (B), A.G. 5096
52	9 31 5.86	+ 1 41 15.5	" " 3823	110	14 48 59.46	+18 37 29.1	" (A), " 5382
53	9 11 53.90	+24 2 50.8	Berlin (B), A.G. 3710	111	17 35 13.34	− 3 29 7.3	Warsaw 4167
54	9 6 26.11	+24 0 56.8	" " 3678	112	17 57 54.67	− 6 54 38.6	Vienna, A.G. 6072
55	10 22 27.83	+ 4 31 25.7	Albany, A.G. 4042	113	0 25 42.73	−15 38 34.9	Washington, A.G. Zones
56	10 21 16.54	+ 4 24 38.2	" " 4036	114	0 35 6.31	−14 48 54.1	" " "
57	10 22 42.55	+ 4 47 31.2	½ (Albany 4046 + Leipzig II, 5485)	115	0 41 59.54	−14 16 30.5	" " "
58	10 8 26.60	+ 5 44 24.1	Leipzig II, A.G. 5398	116	1 22 5.28	− 9 4 33.4	Vienna, A.G. 294
59	10 12 23.85	+ 5 43 31.5	" " 5417	117	1 34 28.50	− 7 45 19.1	" " 338
60	10 12 34.51	+ 5 51 20.5	" " 5418	118	3 4 13.19	+ 4 1 31.2	Albany, A.G. 908
61	10 39 28.96	+ 9 23 44.8	" " 5571	119	11 37 15.64	+ 1 46 11.5	" " 4335
62	12 16 28.14	+ 0 16 9.4	Nicolajew, A.G. 3399	120	11 29 55.89	+ 2 0 57.8	" " 4308
63	12 18 0.36	+ 0 16 16.7	" " 3403	121	11 23 50.67	+ 2 23 19.5	" " 4288
64	12 15 26.71	+ 0 22 38.6	" " 3398	122	11 19 36.15	+ 2 26 5.0	" " 4264
65	12 14 1.48	+ 0 24 44.0	" " 3393	123	11 19 55.15	+ 2 36 46.1	" " 4265
66	12 16 49.19	+ 0 21 50.5	" " 3400	124	11 20 59.43	+ 2 23 23.7	" " 4271
67	12 15 54.83	+15 39 59.1	Berlin (A) A.G. 4648	125	11 22 41.42	+ 1 49 7.7	" " 4281
68	12 13 5.18	+15 32 29.8	" " " 4635	126	2 13 42.45	− 4 46 7.4	Warsaw 378
69	11 17 57.05	+ 8 7 46.2	Leipzig II, A.G. 5946	127	2 19 38.24	− 3 31 35.0	" 389
70	11 47 34.80	+ 8 1 23.5	" " " 5945	128	2 20 7.10	− 2 32 13.7	½ (Pulk. 1875, 584+585)
71	12 17 26.46	−17 41 15.3	Washington, A.G. Zones	129	2 22 35.07	− 1 26 47.4	Nicolajew, A.G. 491
72	12 19 48.70	−17 50 37.1	" " "	130	2 49 20.70	+ 7 17 51.0	Leipzig II, A.G. 1072
73	12 13 7.42	−17 34 29.2	" " "	131	2 49 40.29	+ 8 23 28.1	" " " 1076
74	12 39 46.71	−17 15 45.9	" " "				

[1] Comparison-star (99) has a decided proper motion in α.

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A DETERMINATION OF THE SUN'S MOTION RELATIVE TO THE FAINTER STARS,

BY GEORGE C. COMSTOCK.

In No. 558 of the *Astronomical Journal*, I have published provisional results of an examination of the proper motions of certain faint stars, ninth to twelfth magnitude, including among these results a determination of the elements of the sun's motion. That determination was undertaken primarily as a test of the observed proper motions themselves, to ascertain whether these quantities represented real motions of the stars or were only the residual effects of errors in the data employed, it being commonly supposed that the average proper motion of such faint stars would prove insensible. Although the number of proper motions at that time available, only sixty-eight stars, was extremely limited, they furnished elements of the sun's motion that, as respects both direction and velocity, were in satisfactory accord with the best determinations from other data, and showed also that the resulting mean distance of the stars was in excellent agreement with an extrapolation of KAPTEYN's empirical formula (*Pub. Groningen*, No. 8, p. 24), which represents stellar parallax as a function of magnitude and proper motion.

Encouraged by these indications, I have continued my determinations of the proper motions of faint stars by the methods of the paper cited, and whenever necessary, have myself determined the proper motions of the bright companion stars, using for this purpose all the material accessible to me, including a determination of the places of these stars made with the meridian circle of the Washburn Observatory by Mr. A. S. FLINT, about the epoch 1904.0. In every case, the adopted positions and motions of all the comparison-stars are either taken directly from NEWCOMB's *Fundamental Catalogue*, or are determined with reference to the system represented by that catalogue for the equinox 1850.0. The proper motions of the faint stars are therefore referred to the same system, and a discussion of probable errors shows that for all stellar magnitudes here included, the adopted proper motions are approximately of the same order of precision as the proper motions of the fifth and sixth magnitude stars of the NEWCOMB catalogue.

Nominally there are available at the present time proper motions of 216 stars, included with very few exceptions, between the eighth and twelfth or thirteenth magnitudes. But certain of these stars are clearly binaries in which the observed motion is orbital in character, while in other cases it is impossible to determine with certainty the true character of the motion. After deducting all such cases there remain 149 stars whose observed proper motions are supposed to arise from undisturbed translation through space, and upon these it is now proposed to base a determination of the sun's motion.

While the amount of data must still be designated as small, it is unusually homogeneous and precise in character, and the individual stars have not been chosen through any selective criteria that I am able to recognize as impairing their right to stand as fair samples of the stars included between the eighth and twelfth magnitudes.

It seems to be frequently assumed that the solar motion is a determinate vector quantity whose elements may be ascertained from any group of suitably chosen material, *e.g.*, the particular set of proper motions here under consideration. As an immediate consequence of this assumption, differing results for the elements of the solar motion, as obtained by different investigators, are construed as indicating either inadequate data or erroneous methods of treating the data. Neither the assumption nor its consequences commend themselves to the present writer, who, holding to the concept that all motions with which we are concerned are relative, a change of the position of one thing with respect to another, finds no *à priori* ground for holding that the motion of the sun derived from one group of stars should agree either in direction or amount with the motion referred to a different set of bodies. If *à posteriori*, such an agreement shall be found to exist, it furnishes

evidence of the relative immobility of the two sets of stars to which the solar motion has been referred, and constitutes a substantial addition to our knowledge of the stellar system. On the other hand, discordant results derived from diverse sets of stars similarly treated, tend to show systematic motion of one group relative to the other.

With reference to the purely relative character of the stellar motions it seems not only needless, but distinctly disadvantageous, to introduce explicitly the concept of a moving sun whose velocity enters into and is a part of each observed proper motion. Such a solar motion must be referred to some assumed origin of coordinates, and no such attainable origin can be more determinate or more fixed than the sun itself. I therefore adopt the sun as origin, and proceed to inquire the motion of the stars relative to it. I shall designate the mean motion of any group of stars relative to the sun as its flux, and treat the determination of this flux as the substance of the problem in hand. It is apparent that when this quantity is determined, the solar motion relative to the group of stars employed will be immediately given by changing the sign of the flux. It appears to be impossible to derive elements of the flux or of the solar motion without recourse to hypothesis, and of the several hypotheses hitherto employed, that one seems to be held in most favor which affirms that the stellar motions are promiscuously and indiscriminately directed toward all points of the celestial compass. More rigorously stated: If through the sun there be drawn lines severally parallel to the direction of the supposed absolute motion of every star in the heavens, the points of intersection of these lines with a sphere of unit-radius will be scattered, with approximate uniformity, over the surface of the sphere. The adoption of this hypothesis eliminates certain embarrassing terms, and renders practicable a solution of the problem. The assumption, however, has been vigorously attacked, and as its necessary truth is by no means obvious, I have abstained from employing it, and tentatively, I substitute in its place a limited form of the following proposition:

Hypothesis I. Any widely extended group of stars having the sun near its center has no systematic flux relative to any other similar group.

The particular form in which the hypothesis is applied will be best appreciated from the accompanying figure which represents a projection of the heavens upon the plane of the equator. The radial lines are hour circles, making angles of 30° one with another, and it is assumed that the mean of the stars observed in the region 1, 1, is at rest relative to the mean of the stars in the region *A*, *A*, etc. The extent to which this hypothesis is confirmed or invalidated by the data here treated will be subsequently examined.

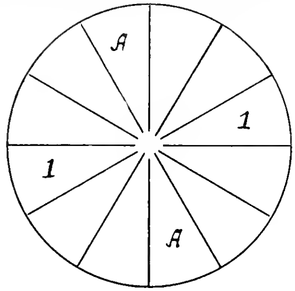

FIG. 1.

Hypothesis II As a second hypothesis, also subject to verification *à posteriori*, I assume that in the mean the parallaxes of stars of known magnitude and proper motion may be represented with a sufficient approximation to accuracy by an expression of the form given by Kapteyn, and cited above.

These two hypotheses lead to the following very simple analysis of the problem:

Let there be given the observed proper motions of a group of n stars, and let the positions of these stars be referred to a system of rectangular coordinates whose origin is at the sun, and whose axes are parallel to those employed for the measurement of right-ascensions, α, and declinations, δ. The coordinates of each star are given by the equations:

$$x = r\cos\delta\cos\alpha$$
$$y = r\cos\delta\sin\alpha$$
$$z = r\sin\delta$$

The velocities of the stars in directions parallel to the coordinate axes are obviously,

$$\begin{aligned} x' &= -r\cos\delta\sin\alpha\,.\,\frac{d\alpha}{dt} - r\sin\delta\cos\alpha\,.\,\frac{d\delta}{dt} + \cos\delta\cos\alpha\,.\,\frac{dr}{dt} \\ y' &= +r\cos\delta\cos\alpha\,.\,\frac{d\alpha}{dt} - r\sin\delta\sin\alpha\,.\,\frac{d\delta}{dt} + \cos\delta\sin\alpha\,.\,\frac{dr}{dt} \\ z' &= \qquad\qquad\qquad\qquad + r\cos\delta\,.\,\frac{d\delta}{dt} + \sin\delta\,.\,\frac{dr}{dt} \end{aligned} \tag{1}$$

The equations of the component velocities of the center of gravity of the stars referred to the sun are

$$X' = \frac{\Sigma m\,x'}{\Sigma m} \qquad Y' = \frac{\Sigma m\,y'}{\Sigma m} \qquad Z' = \frac{\Sigma m\,z'}{\Sigma m}$$

but in consequence of our entire ignorance of the masses, m, these equations cannot be employed, and I substitute for them the simpler expressions which result when we assume the masses to be all equal:

$$(2) \qquad X' = \frac{1}{n}\Sigma x' \qquad Y' = \frac{1}{n}\Sigma y' \qquad Z' = \frac{1}{n}\Sigma z'$$

I call the motion thus represented, the flux of the system of stars relative to the sun.

Introducing the abbreviations

$$r\cos\delta\frac{d\alpha}{dt} = A' \qquad r\frac{d\delta}{dt} = D' \qquad \frac{dr}{dt} = \rho$$

and changing the signs of the components of the flux, we obtain the following equations for the sun's motion:

$$(3)$$
$$\begin{aligned} nX'' &= \Sigma\{\ \ \sin\alpha . A'\sin 1'' + \sin\delta\cos\alpha . D'\sin 1'' - \cos\delta\cos\alpha . \rho\} \\ nY'' &= \Sigma\{-\cos\alpha . A'\sin 1'' + \sin\delta\sin\alpha . D'\sin 1'' - \cos\delta\sin\alpha . \rho\} \\ nZ'' &= \Sigma\{\qquad\qquad\qquad -\cos\delta . D'\sin 1'' - \sin\delta . \rho\} \end{aligned}$$

These equations constitute a solution of the problem, but the method in which they are to be applied calls for some discussion. The α and δ are known quantities, as are also the proper motions $\frac{d\alpha}{dt}$ and $\frac{d\delta}{dt}$. The radius-vector may be assumed known from the KAPTEYN formula above cited: the considerable accidental errors that may inhere in individual values of r thus derived being largely compensated in the mean, and systematic deviation of the formula from the truth, affecting only the scale upon which the resulting X'', Y'', Z'' are expressed. The quantities thus enumerated completely determine A' and D', which therefore become known quantities whose values are to be computed for each star employed. The radial velocities, ρ, are, however, quite unknown, and must therefore be eliminated. The form of the equations shows that this elimination may be secured through the hypothesis of an indiscriminate distribution of velocities, above considered, and also indicates that other hypotheses may equally lead to such elimination, *e.g.*, if the velocities were all equal, whether directed toward or from the sun, they would be eliminated from the first two equations, in a summation extending throughout the twenty-four hours of right-ascension, through the effect of the factors $\cos\alpha$, $\sin\alpha$. Similarly, the radial velocities will be eliminated from the third equation through the factor $\sin\delta$, if the summation be made throughout a region bisected by the equator. We have here a special case of a more general proposition, viz.: The radial velocities of any group of stars symmetrically situated with respect to a plane passing through the sun have a vanishing effect upon that component of the sun's motion that is normal to the plane in question. If, for example, we determine the coordinate Y'', from stars lying near the hour circle 0^h, 12^h, every radial velocity in the group will be multiplied by the small factor $\sin\alpha$, the products $\cos\delta \sin\alpha . \rho$ will tend to be as often positive as negative, and the mean of the small terms thus produced will tend toward zero as a limit.

If we agree to determine the velocity parallel to each coordinate axis solely from stars lying near the great circle normal to that axis, we may omit the terms in ρ without sensible error and substitute in place of the complete forms given above the following equivalents for them:

$$(4) \qquad \begin{aligned} -nY'' &= \Sigma\{\cos(\alpha-\alpha') A' - \sin\delta\sin(\alpha-\alpha') . D'\}\sin 1'' \\ -nZ'' &= \Sigma\ \cos\delta\ .\ D'\sin 1'' \end{aligned}$$

Concerning these equations we note that:

(*a*) The $\alpha - \alpha'$ here substituted in place of α denotes a horizontal coordinate no longer measured from the vernal equinox, but from one-half of the great circle, that half in right-ascension α', near which the stars are assumed to lie.

(*b*) The same reasoning that shows the elimination of the ρ terms from the equation in Y'' equally indicates that the D' terms will be there eliminated, and they might have been dropped from Equations (4). They are, however, retained in order that their actual effect upon the summation may show how nearly in fact the elimination is effected, and thus given some clue to the probable residual effect of the ρ terms.

(*c*). The equation in Z'' is in terms limited to stars near the equator, since here only is there assurance that the ρ terms will be eliminated. It may, however, be applied to stars of higher declination to determine whether there is for them such an elimination of the ρ terms, through the supposed chance distribution of velocities, as to give results in agreement with the equatorial zone.

(*d*) If the heavens be supposed divided into any even number of lunes by means of hour circles symmetrically distributed throughout the twenty-four hours of right-ascension, and if α' denote the right-ascension of the median line of any one of these lunes, then will the equation in Y'', if applied to the lunes α' and $\alpha' + 12^h$, determine the projection of the sun's velocity upon that radius of the equator, whose right-ascension is $\alpha' + 6^h$. By dividing the sky into a great number of lunes we may determine the projection of the sun's velocity upon as many such radii as may be desired, and every pair of radii at right angles to each other will furnish a value of the projection of the sun's velocity upon the plane of the equator. The agreement, one with another, of the several values thus determined will tend to confirm the fundamental hypothesis of the relative immobility of the several groups of stars employed. (*Hypothesis I*), while a considerable divergence among these values, if not attributable to inadequate data, will tend to discredit that hypothesis.

(*e*) If S represent the projection of the sun's velocity

upon the plane of the equator, and A be the corresponding right-ascension, we shall have

$$X = S \cos A \quad , \quad Y = S \sin A$$

Let Y_1'' and Y_2'' denote values of Y'' corresponding to the lunes in right-ascension α' and $\alpha' + 6^h$, then we may obtain by transformation of coordinates,

$$(5) \qquad \begin{aligned} X &= -Y_1'' \sin \alpha' - Y_2'' \cos \alpha' \\ Y &= +Y_1'' \cos \alpha' - Y_2'' \sin \alpha' \end{aligned}$$

These values of X and Y, together with the value of Z directly given by Equations (4), completely determine the elements of the sun's motion, subject to the limitations imposed by the fundamental hypotheses. A method of control upon *Hypothesis I* has already been indicated, and the validity of *Hypothesis II*, with respect to the mean distances of the stars, may be examined as follows: The linear velocity of the sun, V, and the coordinates of the apex of the solar motion A, D, are obviously given by the equations,

$$(6) \qquad \begin{aligned} X &= V \cos D \cos A \\ Y &= V \cos D \sin A \\ Z &= V \sin D \end{aligned}$$

If the scale of distances is properly represented by KAPTEYN's formula (extrapolated) the value of V resulting from these equations should agree with the spectroscopic determinations of that velocity, provided the bright stars from which the latter determination has been made do not themselves possess a considerable systematic motion with respect to the faint ones here considered. This control upon the mean validity of *Hypothesis II* is applied in a subsequent part of the paper.

TABLE I. — MEAN STELLAR DISTANCES.

μ	H	μ	H
0″	$+\infty$	10″	9.29
1	0.00	11	9.26
2	9.79	12	9.24
3	9.67	13	9.21
4	9.58	14	9.19
5	9.51	15	9.17
6	9.45	16	9.15
7	9.40	17	9.13
8	9.36	18	9.11
9	9.33	19	9.10
10	9.29	20	9.08

m	K	K'
7	0.49	0.49
8	0.53	0.55
9	0.57	0.61
10	0.62	0.68
11	0.66	0.74
12	0.71	0.81
13	0.75	0.87

In applying the method above outlined to the proper motions here to be discussed, I have derived the assumed distances of the stars, r, from the above table in which H and K are auxiliary quantities depending respectively, upon μ, the total proper motion of the star in question, expressed in seconds of arc per century, and m, the star's photometric magnitude. If the distance r is expressed in units equal to 20,626,481 times the semi-axis major of the earth's orbit (the distance corresponding to a parallax of 0″.01), we shall find the logarithm of the distance given by the sum of the quantities H and K. The parallax, π, follows immediately from the value of r.

$$\log r = H + K \quad (\text{KAPTEYN})$$
$$\log \pi = 8.00 - \log r$$

For illustration, I apply the table to the second star of my list, a faint companion of *α Andromedae*, and representing the parallax of the star by π find as follows:

$$\mu = 1''.9 \qquad H = 9.81 \qquad \log r = 0.47$$
$$m = 10.9 \qquad K = 0.66 \qquad \pi = 0''.003$$

The application of the table is facilitated by use of the relation: Any ten-fold increase of μ produces a diminution of 0.71 in the value of H. Thus the value of H for $\mu = 50''$ is $9.51 - 0.71 = 8.80$. By means of values of $\log r$ furnished by the above table, each proper motion in right-ascension and declination has been converted into the corresponding A' or D', and from these there have been formed the several quantities entering into the second members of Equations (4).

The sky being divided into sections by symmetrically placed hour circles, as shown in Fig. 1, the terms arising from stars falling in each of these sections were separately summed, and the resulting sums are shown in the following Table II, where α' now denotes the right-ascension of the median line of each lune, n is the number of stars observed in that lune, and the remaining quantities are sufficiently indicated by the headings of the table. Each value of Y'', Z'', shown in the table is the simple mean of the values furnished by the two opposing lunes, the results from which are presented on the corresponding line. Although there is some disparity in the numbers of stars observed in opposite lunes, it appears best to combine them in this manner in order to eliminate the effect of a possible error in the adopted precession constant. A reference to Equation (4) will show that if the signs of the numbers in the second and third columns of Table II be changed before the means are formed, the resulting means will represent component velocities of the sun parallel to the plane of the equator, and in directions whose right-ascensions are severally 90° greater than the values of α' presented in the first column. The printed values of Y'' have been thus derived. The ( ) about the Z'' indicate that the values here given are subject to revision, since stars at all distances from the equator have entered into these values, and the radial velocities, ρ, are not here eliminated.

TABLE II.

α'	$\cos(\alpha-\alpha')\,A'$	$-\sin\delta\,\sin(\alpha-\alpha')\,D'$	$\cos\delta\,.\,D'$	n	α'	$\cos(\alpha-\alpha')\,A'$	$-\sin\delta\,\sin(\alpha-\alpha')\,D'$	$\cos\delta\,.\,D'$	n	Y''	(Z'')	n
0^h	+0.71	−0.06	−1.78	16	12^h	−1.09	−0.05	−1.99	14	−0.90	+1.88	30
2	+0.17	+0.05	−1.38	15	14	−2.13	−0.07	−2.37	7	−1.21	+1.87	22
4	+1.25	+0.01	−3.00	10	16	−4.24	+0.06	−2.61	11	−2.72	+2.86	21
6	+0.77	−0.08	−0.97	12	18	−2.05	−0.06	−2.76	11	−1.40	+1.86	23
8	−2.28	+0.04	−3.82	17	20	−0.81	−0.05	−2.83	15	+0.69	+3.32	32
10	−1.68	−0.22	−2.54	9	22	+1.53	−0.11	−1.77	12	+1.66	+2.16	21

We may note the following features presented by the table:

(*a*) The terms $\sin\delta\sin(\alpha-\alpha')\,D'$ are uniformly small, indicating a nearly complete automatic elimination of the declination terms from the value of Y'', and thereby confirming the *à priori* reason for supposing a similar elimination of the radial velocities. Every value of the term $\cos\delta\,.\,D'$ indicates a motion of the sun toward the north, and the range among the twelve results is so small that the numbers must be regarded as indices of a real phenomenon. The proper motions upon which they are based must equally be regarded as real quantities, the supposition that they may be only residual errors of a fortuitous character being put out of court by the unanimity of their testimony. Also, the sequence in the values of Y'' is obviously of a systematic character, and if we derive from these numbers through Equation (5) the velocities of the sun parallel to the x and y axes originally introduced, we shall find the following values, to which are added those of Z, obtained from the discussion that follows:

TABLE III.

Region	α'	n	X	Y	Z
1	0^h, 6^h	53	+1.40	−0.90	+2.62
2	2, 8	54	+0.01	−1.38	+2.78
3	4, 10	42	+1.50	−2.78	+2.75
Mean		149	+0.97	−1.69	+2.72

While the several results here shown agree one with another in respect of sign, the values of X and Y are more divergent than could be desired if *Hypothesis I* is to be justified by them. On the other hand they are not so divergent as to disprove the hypothesis, and I am constrained, therefore, to leave in abeyance the question of its complete validity until a larger body of data shall be available for the purpose. I adopt the simple mean of the several values as the definitive result for the X and Y velocities of the solar motion, and proceed to derive the Z velocity from a grouping of the proper motions in declination different from that exhibited in Table III.

Let the sky be divided by parallels of declination into the zones indicated in the second column of Table IV, and let values of Z be determined from summation within each zone. These values, together with the number of stars upon which they depend, n, will be found in Table IV under the rubric Z_1. If, instead of forming the simple means of the quantities furnished by the several zones, we subdivide each zone into lunes, as above, and give equal weight to the result from each lune, we find the quantities shown under the rubric Z_2.

TABLE IV.

Zone	Limits of Declination		Z_1	Z_2	n
I	−20° to	+20°	+2.83	+2.68	54
II	+20	+45	+2.71	+2.68	69
III	+45	+90	+0.7	+0.7	26

Concerning both Z_1 and Z_2, it is to be noticed that the effect of radial velocities is eliminated in Zone I, but is present in increasing measure in Zones II and III, except in so far as it may be eliminated by the chance distribution of velocities sometimes postulated. The close agreement between the results of Zones I and II affords some support to the hypothesis in question, and the discordant result shown under III does not militate severely against it, since the weight of the value here obtained is extremely small. I adopt as a definitive result $Z = +2.70$.

For the determination of the apex of the solar motion I have applied Equations (6), not only to the mean values of X, Y, Z above given, but also to the results furnished by the individual groups of stars designated 1, 2, 3, in Table III. The resulting coordinates and the linear velocity V, are as follows:

TABLE V.

Region	A	D	V	n
1	333°	+59°	3.06	53
2	270	+63	3.11	54
3	298	+41	4.19	42
Mean	300	+54	3.38	149

The factor $\sin 1''$ which appears in Equations (4), having been omitted in the numerical work whose results are

above shown, it must be here introduced, and with respect to it and to the fact that V is a centennial motion, we find that the values of V as given, represent the annual motion of the sun in radii of the earth's orbit. We have to compare with these values KAPTEYN's interpretation of CAMPBELL's spectroscopic result for the velocity of the solar motion, 18.45 *km/sec*, which is equivalent to $V = 3.90$ in the units employed above. It appears from the disparity between this number and the mean value of V above given, 3.38, that the assumed distances of the stars need to be increased about 15 per cent. upon the average, and as a matter of formal adjustment this may be done by adding 0.06 to the numbers in the column K of Table I. This, however, assumes the relative flux of the bright and faint stars to be insignificant, a hypothesis that stands in much need of confirmation. The numbers K' in the last column of this table have been derived by applying to K a graduated correction whose mean value is $+0.06$, but which vanishes for stars of the seventh magnitude, and brighter, in order that KAPTEYN's direct determination of the mean parallax of these stars may not be disturbed by results from the fainter stars here discussed. If K' be substituted for K in the distance formula,

$$\log r = H + K'$$

the resulting values will correspond to the mean distance of the stars as furnished by the present investigation. In order to derive such mean values of the distances I have grouped the total proper motions, μ, with respect to stellar magnitude, m, and the resulting mean values with the corresponding mean distance and mean parallax of the stars as furnished by the corrected Table I, are shown in Table VI.

TABLE VI.

m	μ	n	$\log r$	π
	$''$			$''$
8.3	3.45	35	0.20	0.0063
9.5	3.15	43	0.30	.0050
10.5	2.99	45	0.38	.0042
11.5	2.63	20	0.48	0.0033

While the numbers in this table can make no pretence of being definitive values of the mean parallax corresponding to stars of the magnitude, m, shown in the first column, I think that there can be little doubt that they represent fairly well the order of magnitude in question, and indicate that the stars of the ninth, tenth and eleventh magnitudes are much nearer to the earth than has been commonly assumed. I regard the result as a substantial justification of *Hypothesis II*.

It appears from PICKERING's investigations that the fainter stars of the galaxy present spectra that are predominantly of the first type, and as stars of this type are supposed to be at a greater distance than corresponding stars of the second type, there arises some presumption that the fainter galactic stars will be as a class more distant than similar stars outside the galaxy. To determine whether this is in fact the case for stars as bright as those here considered, I have grouped the proper motions, μ, with respect to galactic latitude, β, and find the results shown in Table VII.

TABLE VII.

β	n	μ	μ'	m'	μ''
°		$''$	$''$		$''$
+78	10	4.6	. .	(9.2)	(4.3)
63	10	5.0	4.63	9.6	4.53
45	12	4.3	4.77	9.6	4.67
35	15	5.0	3.90	9.7	3.82
25	12	2.4	3.20	9.9	3.17
15	9	2.2	2.20	10.0	2.20
+ 5	13	2.0	2.17	9.8	2.12
− 5	14	2.3	2.47	9.5	2.34
15	12	3.1	2.70	9.5	2.57
25	16	2.7	2.77	9.9	2.74
37	11	2.5	3.03	10.0	3.03
−52	9	3.9	. .	(9.7)	(3.8)

The zone in galactic latitude 63° contains one star of abnormally rapid motion, 49″ per century, and if this be included, the mean result for the zone is $\mu = 7''.1$; if it be rejected, $\mu = 3''.0$. I have adopted for the zone the mean of these two results. The column under the rubric μ' contains adjusted values of μ obtained by taking the mean of three contiguous values and placing it opposite the middle one of the three. The column m' contains the mean magnitudes of the stars similarly adjusted, and as there is some diversity in these numbers I adopt from Table VI the mean rate of diminution of μ, considered as a function of m, $0''.26$ per magnitude, and with this value reduce the several values of μ' to what they should be if m' were in each case 10.0. These reduced values are shown in the column μ''.

The sequence of the numbers in each μ column is so pronounced that I am constrained to regard it as real, and as indicating that the galaxy is in fact a region of minimum proper motions and parallaxes for stars of the tenth magnitude. The numbers of the table are well represented by the supposition that the proper motions in galactic latitude 60° are about twice as great as those in the galaxy, but the data are inadequate to determine the exact law of increase, whether it be symmetrical with respect to the galaxy or more pronounced upon the north side of that plane, as is suggested by the values of μ''.

The chief results obtained in the present paper may be summarized as follows:

I. At least as far down the photometric scale as the

twelfth magnitude the stars present measurable proper motions, *e.g.*, two or three seconds per century.

2. The mean distance, and therefore the mean luminosity of these stars is considerably less than has been commonly assumed.

3. The proper motions, and presumably the parallaxes, of stars fainter than the ninth magnitude show a progressive increase with increasing distance from the galaxy.

4. We may provisionally employ the hypothesis that groups of stars widely distributed through space, but having a common center, possess no considerable flux relative one to the other.

5. Kapteyn's empirical formula representing stellar parallax as a function of proper motion and magnitude may be extended to stars of the eleventh magnitude, and probably to fainter ones, without serious error in the mean result.

6. The motion of the sun relative to the more distant stars (10^m and 11^m) is not widely different from its motion relative to the brighter lucid stars and the flux of the nearer parts of the stellar system with respect to those more remote is therefore small.

The interpretation to be placed upon the discordance between the position of the stellar apex here obtained and those otherwise derived, is reserved for another paper.

Washburn Observatory, 1906 *December* 11.

MEASURES OF DOUBLE STARS,

By JOHN A. MILLER and W. E. HOWARD.

The stars in the list that follows are selected from those that have been noted as double by the Cambridge (England) observers while making the observations for the *Katalog der Astronomischen Gesellschaft*, Zone +25° to +30°. With few exceptions, no star is included measures of which have been recently published.

The measures were made with the twelve-inch refractor of the Kirkwood Observatory, Indiana University. The position-angle given for any one night is the mean of four or more settings, and the distance is the mean of six or more settings for double distance.

The magnitudes given are the mean of estimates made by the observer at the time the measures were made. The letter H. means that the measures were made by Mr. Howard, and the letter M. that they were made by myself.

A few of the stars noted as double by the Cambridge observers seemed to us single. We have examined each of those stars on at least three nights, before deciding they were single; these are noted in the body of the list that follows.

The positions given are for 1875.0.

BD. 24°29. A.G. 124. $9^M.0$; $9^M.3$.
$\alpha = 0^h\ 12^m\ 13^s.10$; $\delta = +25°\ 3'\ 20''.1$.

t	θ °	ρ ″	
1903.739	325.5	6.43	
.772	324.7	6.91	
1903.755	325.1	6.67	H.

BD. 29°78. A.G. 309. $9^M.0$; $9^M.7$.
$\alpha = 0^h\ 26^m\ 19^s.32$; $\delta = +29°\ 25'\ 40''.4$.

1903.799	36.2	4.89	
.818	34.0	4.47	
1903.808	35.1	4.68	M.

BD. 25.78. A.G. 329.
$\alpha = 0^h\ 28^m\ 18^s.99$; $\delta = +25°\ 45'\ 18''.9$.

1903.655	Duplicity uncertain	
.733		
1903.694		M.

BD. 27°609. A.G. 1934.
$\alpha = 3^h\ 50^m\ 36^s.27$; $\delta = +27°\ 35'\ 4''.0$.

1903.799	15.1	6.16	
.821	12.3	6.34	
1903.810	13.7	6.25	H.

B.D. 29°1037. A.G. 2781. $7^M.3$; $9^M.5$.
$\alpha = 5^h\ 47^m\ 22^s.08$; $\delta = +29°\ 56'\ 16''.9$.

t	θ °	ρ ″	
1904.128	231.2	4.93	
.151	231.1	4.37	
1904.139	231.1	4.65	M.

BD. 26°1167 A.G. 3103. $9^M.0$; $9^M.5$.
$\alpha = 6^h\ 10^m\ 1^s.81$; $\delta = +26°\ 14'\ 28''.4$.

1904.128	72.9	21.78	
.186	73.4	21.94	
.252	73.0	21.47	
1904.189	73.1	21.73	H.

BD. ———. A.G. 3655. $9^M.0$; $9^M.5$.
$\alpha = 6^h\ 52^m\ 22^s.75$; $\delta = +27°\ 56'\ 22''.9$.

1904.128	191.7	10.00	
.151	189.2	10.17	
.258	193.6	10.23	
1904.179	191.5	10.13	M.

BD. 26°1497. A.G. 3843. $8^M.5$; $9^M.0$.
$\alpha = 7^h\ 7^m\ 21^s.61$; $\delta = +26°\ 13'\ 4''.6$.

1904.359	350.6	26.06	
.389	350.8	25.97	
.406	350.8	27.37	
1904.385	350.7	26.80	H.

BD. 26°1573. A.G. 3996. $8^M.6$; $8^M.7$.
$\alpha = 7^h\ 22^m\ 50^s.94$. $\delta = +26°\ 2'\ 2''.3$.

t	θ °	ρ ″	
1904.351	. . .	1.67	
1905.164	163.7	1.29	
.173	168.5	1.44	
1904.876	166.1	1.37	M.

B.D. 25°1691. A.G. 4005. $9^M.0$; $10^M.0$.
$\alpha = 7^h\ 23^m\ 21^s.72$; $\delta = +25°\ 25'\ 28''.8$.

1905.091	108.4	3.27	
.162	107.6	3.31	
1905.126	108.0	3.29	M.

BD. 27°1403. A.G. 4009.
$\alpha = 7^h\ 23^m\ 39^s.71$; $\delta = +27°\ 52'\ 54''.9$.
This is double, but too close to measure.

BD. 28°1543. A.G. 4335. $9^M.0$; $9^M.5$.
$\alpha = 7^h\ 57^m\ 23^s.02$; $\delta = +28°\ 48'\ 29''.5$.

1905.162	109.9	2.71	
.170	103.9	2.60	
1905.166	106.9	2.65	M.

BD. 26°1759. A.G. 4450. $9^M.0$; $9^M.1$.

$\alpha = 8^h\,9^m\,44^s.49$; $\delta = +26°\,45'\,5''.3$.

t	θ (°)	ρ (″)	
1904.334	324.2	5.82	
.389	322.0	6.47	
1905.091	322.2	6.02	
1904.605	322.8	6.13	M.

BD. 26°2015. A.G. 5111. $9^M.0$; $9^M.5$.

$\alpha = 9^h\,43^m\,19^s.59$; $\delta = +26°\,37'\,23''.3$.

1904.225	313.9	5.25	
.334	311.0	. .	
.389	309.6	4.76	
1904.316	311.5	5.00	H.

BD. 26°2161. A.G. 5579. $8^M.5$; $8^M.9$.

$\alpha = 10^h\,52^m\,53^s.71$; $\delta = +26°\,6'\,30''.7$.

1904.225	75.8	5.51	
.351	75.2	5.00	
.408	75.1	5.00	
1904.328	75.4	5.18	M.

BD. 27°2044. A.G. 5830. $8^M.5$; $9^M.2$.

$\alpha = 11^h\,33^m\,4^s.89$; $\delta = +27°\,38'\,56''.0$.

1904.351	87.4	4.80	
.225	88.0	5.60	
.408	86.8	5.22	
1904.328	87.4	5.21	H.

BD. 29°2368. A.G. 6354. $8^M.8$; $9^M.2$.

$\alpha = 13^h\,0^m\,52^s.40$; $\delta = +29°\,15'\,53''.7$.

1904.225	336.0	18.85	
.406	337.1	17.89	
.408	334.8	18.16	
1904.346	335.9	18.30	M.

BD. 27°2423. A.G. 6982. $9^M.5$; $10^M.0$.

$\alpha = 14^h\,43^m\,36^s.52$; $\delta = +26°\,58'\,48''.1$.

1904.458	333.0	9.10	
.460	336.5	9.71	
1904.459	334.7	9.40	H.

BD. 28°2506. A.G. 7446. $9^M.0$; $10^M.0$.

$\alpha = 15^h\,55^m\,58^s.08$; $\delta = +28°\,27'\,49''.8$.

1904.458	229.8	5.85	
.460	228.9	5.74	
1904.459	229.3	5.79	M.

BD. 27°2666. A.G. 7754.

$\alpha = 16^h\,34^m\,34^s.01$; $\delta = +27°\,16'\,10''.2$.

Observed on two nights ; could not see double.

BD. 28°2607. A.G. 7798. 8^M ; 9^M.

$\alpha = 16^h\,40^m\,6^s.67$; $\delta = +28°\,35'\,15''.0$.

1903.804	160.6	5.27	
.807	160.7	5.05	
.810	160.7	5.12	
1903.807	160.7	5.15	H.

BD. 29°3160. A.G. 8583. 9^M ; $9^M.5$.

$\alpha = 17^h\,54^m\,34^s.87$; $\delta = +29°\,30'\,4''.7$.

t	θ (°)	ρ (″)	
1903.804	190.5	10.99	
.807	190.2	11.14	
.810	190.0	10.93	
1903.807	190.2	11.02	M.

BD. 29°3235. A.G. 8817. $8^M.6$; $9^M.0$.

$\alpha = 18^h\,14^m\,38^s.0$; $\delta = +29°\,52'\,1''.0$.

1903.804	173.9	18.31	
.807	173.7	18.25	
.810	174.8	18.01	
1903.807	174.1	18.19	H.

BD. 26°3245. A.G. 8882. $9^M.1$; $9^M.5$.

$\alpha = 18^h\,20^m\,8^s.12$; $\delta = +26°\,49'\,59''.7$.

1903.804	14.2	3.74	
.807	13.8	3.59	
.810	16.2	3.62	
1903.807	14.7	3.65	M.

BD. 28°3018. A.G. 9016.

$\alpha = 18^h\,27^m\,39^s.38$; $\delta = +28°\,18'\,5''.5$.

Single ; seeing good ; sky black.

BD. 28°3036. A.G. 9085. $9^M.2$; 12^M.

$\alpha = 18^h\,32^m\,56^s.16$; $\delta = +28°\,16'\,15''.4$.

1903.884	172.6	28.51	
.895	177.6	29.71	
1903.889	175.1	29.07	H.

BD. 26°3316. A.G. 9096. $9^M.6$; $9^M.7$.

$\alpha = 18^h\,33^m\,36^s.85$; $\delta = +26°\,59'\,45''.2$.

1903.804	227.8	2.86	
.807	227.2	2.56	
.884	227.5	2.91	
1903.832	227.5	2.78	H.

BD. 29°3419. A.G. 9424. $9^M.1$; $10^M.1$.

$\alpha = 18^h\,53^m\,19^s.10$; $\delta = +29°\,14'\,25''.3$.

1903.807	302.4	5.43	
.895	302.4	5.62	
.931	304.1	5.42	
1903.878	302.9	5.49	M.

BD. 28°3180. A.G. 9562. $9^M.1$; $10^M.4$.

$\alpha = 19^h\,0^m\,5^s.79$; $\delta = +28°\,53'\,3''.2$.

1903.804	184.4	6.07	
.807	184.0	5.65	
.881	184.1	5.79	
1903.832	184.2	5.83	M.

BD. 26°3471. A.G. 9673. $9^M.4$; $10^M.1$.

$\alpha = 19^h\,6^m\,17^s.38$; $\delta = +26°\,53'\,5''.9$.

1903.884	143.9	3.38	
.895	144.7	3.01	
.903	146.9	3.01	
1903.894	144.8	3.13	H.

BD. 26°3491. A.G. 9732. $9^M.1$; $10^M.0$.

$\alpha = 19^h\,9^m\,7^s.99$; $\delta = +26°\,13'\,35''.6$.

t	θ (°)	ρ (″)	
1903.884	295.2	17.96	
.903	296.4	17.84	
.914	294.7	18.14	
1903.900	295.4	17.98	M.

BD. 28°3253. A.G. 9743. $9^M.1$; $9^M.8$.

$\alpha = 19^h\,9^m\,40^s.14$; $\delta = +28°\,40'\,53''.1$.

1903.903	259.7	4.57	
.914	258.9	4.66	
.922	257.7	4.51	
1903.913	258.8	4.58	H.

θ is 180° too large.

BD. 29°3534. A.G. 9762. 9^M ; 10^M.

$\alpha = 19^h\,10^m\,41^s.94$; $\delta = +29°\,11'\,45''.1$.

1903.903	127.4	9.01	
.914	127.0	9.10	
.922	126.0	9.39	
1903.913	126.8	9.16	M.

BD. 27°3367. A.G. 9907. $9^M.2$; $9^M.7$.

$\alpha = 19^h\,18^m\,39^s.24$; $\delta = +27°\,31'\,16''.0$.

1903.903	227.1	8.64	
.914	226.8	8.72	
.922	225.6	8.81	
1903.913	226.5	8.72	H.

BD. 29°3608. A.G. 10012. $8^M.7$; $10^M.3$.

$\alpha = 19^h\,23^m\,14^s.92$; $\delta = +29°\,26'\,14''.2$.

1903.933	226.2	2.25	
.955	228.2	2.38	
.964	229.3	2.34	
1903.951	227.9	2.13	M.

BD. 29°3623. A.G. 10057. $8^M.5$; $9^M.6$.

$\alpha = 19^h\,26^m\,8^s.44$; $\delta = +29°\,26'\,47''.2$.

1903.922	36.2	29.72	
.931	37.0	. .	
.933	37.0	29.24	
1903.929	36.7	29.32	M.

BD. 29°3656. A.G. 10149. $9^M.5$; $10^M.0$.

$\alpha = 19^h\,30^m\,36^s.35$; $\delta = +29°\,28'\,5''.5$.

1903.879	287.4	2.83	
.928	290.4	3.17	
.920	290.3	3.07	
1903.909	289.4	3.35	H.

BD. 25°3914. A.G. 10251. $9^M.2$; $9^M.5$.

$\alpha = 19^h\,34^m\,59^s.77$; $\delta = +25°\,54'\,7''.7$.

1903.928	310.7	15.23	
.942	307.7	16.04	
.953	306.7	14.23	
1903.941	308.3	15.16	H.

BD. 24°3865. A.G. 10368. $9^{M}.2$; 10^{M}.

$\alpha = 19^h\ 40^m\ 19^s.30$; $\delta = +24°\ 54'\ 20''.1$.

t	θ	ρ	
	°	″	
1903.879	96.2	5.45	
.953	98.1	5.12	
.961	94.1	5.53	
1903.931	96.1	5.37	M.

B.D. 24°3911. A.G. 10496. $9^{M}.1$; $9^{M}.7$.

$\alpha = 19^h\ 46^m\ 16^s.77$; $\delta = +25°\ 2'\ 32''.8$.

1903.953	195.3	18.66	
.980	195.4	18.86	
.983	194.3	18.56	
1903.972	195.0	18.69	H.

B.D. 29°3795. A.G. 10577. $8^{M}.8$; $10^{M}.5$.

$\alpha = 19^h\ 49^m\ 23^s.70$; $\delta = +29°\ 41'\ 55''.3$.

1903.950	206.3	2.11	
.953	201.9	2.40	
.955	201.1	2.36	
1903.953	203.3	2.29	M.

BD. 26°3768. A.G. 10770. $9^{M}.2$; $12^{M}.2$.

$\alpha = 19^h\ 57^m\ 51^s.28$; $\delta = +26°\ 15'\ 24''.8$.

1903.953	92.5	15.61	
.980	93.1	15.20	
.983	93.8	16.40	
1903.972	93.1	15.73	H.

BD. 25°4130. A.G. 10938. $9^{M}.5$; $10^{M}.5$.

$\alpha = 20^h\ 5^m\ 10^s.44$; $\delta = +25°\ 9'\ 9''.7$.

1903.882	192.9	5.29	
.953	190.1	5.04	
.955	190.4	5.05	
1903.930	191.1	5.13	H.

BD. 28°3664. A.G. 10992. $8^{M}.8$; $9^{M}.3$.

$\alpha = 20^h\ 7^m\ 14^s.79$; $\delta = +29°\ 0'\ 20''.4$.

1903.953	306.1	4.02	
.955	303.0	3.97	
.971	305.4	4.05	
.983	304.0	3.77	
1903.915	304.6	3.95	M.

BD. 28°3668. A.G. 11006. $9^{M}.3$; $9^{M}.7$.

$\alpha = 20^h\ 7^m\ 49^s.16$; $\delta = +28°\ 50'\ 0''.7$.

1903.971	235.2	18.36	
.983	235.0	18.47	
1904.008	233.4	18.45	
1903.987	234.5	18.43	H.

BD. 27°3716. A.G. 11248. $9^{M}.4$; 10^{M}.

$\alpha = 20^h\ 18^m\ 56^s.65$; $\delta = +27°\ 58'\ 31''.9$.

t	θ	ρ	
	°	″	
1903.971	248.9	30.94	
.983	250.2	30.50	
1904.011	251.1	30.61	
1903.988	250.1	30.68	M.

BD. 28°3738. A.G. 11253. $9^{M}.2$; $9^{M}.4$.

$\alpha = 20^h\ 19^m\ 13^s.09$; $\delta = +28°\ 58'\ 41''.3$.

1903.971	244.5	2.11	
.983	241.2	2.58	
1904.011	242.2	2.61	
1903.988	242.6	2.43	H.

BD. 27°3739. A.G. 11332. $8^{M}.7$; $9^{M}.7$.

$\alpha = 20^h\ 22^m\ 20^s.12$; $\delta = +27°\ 38'\ 45''.5$.

1903.942	69.2	24.29	
.983	69.4	24.66	
1904.011	69.9	. .	
1903.978	69.5	24.47	M.

BD. 29°4053. A.G. 11375. $9^{M}.3$; $9^{M}.8$.

$\alpha = 20^h\ 23^m\ 57^s.21$; $\delta = +19°\ 31'\ 28''.3$.

1903.971	331.8	8.02	
1904.008	332.2	8.15	
.011	332.4	8.11	
1903.997	332.1	8.09	H.

BD. 27°3850. A.G. 11693. $9^{M}.0$; $10^{M}.5$.

$\alpha = 20^h\ 39^m\ 33^s.23$; $\delta = +27°\ 58'\ 0''.4$.

1903.971	23.4	2.49	
1904.011	18.5	2.98	
1903.991	20.9	2.73	M.

BD. 29°4160. A.G. 11694. $9^{M}.2$; 10^{M}.

$\alpha = 20^h\ 39^m\ 40^s.33$; $\delta = +29°\ 54'\ 26''.7$.

1903.928	179.5	20.12	
.942	179.6	19.94	
.971	179.1	20.39	
1903.947	179.6	20.16	H.

BD. ———. A.G. 12127. $9^{M}.3$; $9^{M}.7$.

$\alpha = 21^h\ 1^m\ 6^s.53$; $\delta = +29°\ 2'\ 58''.8$.

1903.928	259.1	5.94	
.942	258.6	5.07	
.971	260.3	5.24	
1903.947	259.3	5.42	M.

BD. 28°4192. A.G. 12895. $9^{M}.5$; 10^{M}.

$\alpha = 21^h\ 41^m\ 51^s.17$; $\delta = +28°\ 29'\ 12''.4$.

1903.928	157.3	9.48	
.942	157.3	9.74	
.961	158.3	9.04	
1903.944	157.6	9.42	H.

BD. 24°4498. A.G. 12990. $8^{M}.9$; $9^{M}.2$.

$\alpha = 21^h\ 49^m\ 22^s.63$; $\delta = +24°\ 47'\ 16''.0$.

t	θ	ρ	
	°	″	
1903.928	197.4	5.77	
.942	198.6	6.03	
.971	199.7	6.57	
1903.947	198.6	6.13	M.

BD. 29°4582. A.G. 13178. $7^{M}.8$; $8^{M}.5$.

$\alpha = 22^h\ 0^m\ 7^s.06$; $\delta = +29°\ 17'\ 59''.0$.

1903.791	95.3	9.05	
.966	95.9	8.71	
.971	95.8	8.88	
1903.909	95.6	8.84	H.

BD. 27°4305. A.G. 13402. $9^{M}.2$; $9^{M}.7$.

$\alpha = 22^h\ 17^m\ 52^s.79$; $\delta = +27°\ 23'\ 24''.6$.

1903.791	259.1	10.29	
.942	259.8	11.20	
.953	259.3	11.21	
1903.895	259.3	10.90	H.

BD. 28°4411. A.G. 13570. $9^{M}.1$; $10^{M}.5$.

$\alpha = 22^h\ 30^m\ 26^s.40$; $\delta = +29°\ 5'\ 56''.3$.

1903.928	155.0	23.05	
.942	155.5	23.29	
.961	154.9	23.90	
1903.944	155.4	23.40	H.

BD. 25°4920. A.G. 14033.

$\alpha = 23^h\ 13^m\ 50^s.80$; $\delta = +25°\ 30'\ 53''.1$.

Observed on two nights. No companion visible nearer than 46″.12. Seeing very good. M.

BD. 28°4656. A.G. 14329. $8^{M}.8$; $8^{M}.9$.

$\alpha = 23^h\ 46^m\ 22^s.78$; $\delta = +28°\ 37'\ 57''.2$.

1903.928	268.0	6.27	
.942	269.9	6.59	
.969	269.1	6.09	
1903.946	269.0	6.32	M.

BD. 26°4727. A.G. 14394.

$\alpha = 23^h\ 54^m\ 0^s.36$; $\delta = +26°\ 13'\ 25''.5$.

Single.

BD. 27°467. A.G. 14424. $8^{M}.8$; $9^{M}.6$.

$\alpha = 23^h\ 57^m\ 28^s.95$; $\delta = +27°\ 17'\ 2''.9$.

1903.928	289.1	5.26	
.942	288.0	5.48	
.969	289.0	5.03	
1903.946	288.7	5.25	H.

Kirkwood Observatory, Bloomington, Ind., 1906 *July* 31.

ON THE PERIOD OF THE VARIABLE STAR 120.1906.

By NAOZO ICHINOHE.

This star (*B.D.* +47°692) is also one of CERASKI's variables, and was announced in *A.N.* 4126 as an *Algol*-type star, with a range from $8^M.3$ to $9^M.5$ or 10^M. I began to observe this star on Nov. 8, and on that very night observed it to have the magnitude of $9^M.4$; but I could not follow the star long, as it clouded over two and a half hours after my first observation, when I had obtained only three estimates.

I could observe this star on Nov. 9, 10, 13, 15, 18, 22, 23 and 27, but the star always showed its normal brightness; when I pointed the telescope to this star on the 28th of November, I noticed the brightness to be a little fainter than usual, and on further observation I saw that it was still decreasing. Its magnitude was estimated eight times on this night; the first estimate was made at $11^h\ 27^m$ G.M.T., and the last at $23^h\ 13^m$, so that the observations cover about half a day, but I was not able to see the increase of its light. Hence I could not determine the epoch of minimum exactly; still I believe that it was pretty near to its minimum, because the last estimation was about $9^M.5$.

Prof. CERASKI noted two determinations of minimum in *A.N.* These two and my two observations show that the period must be a multiple or sub-multiple of $6^d.85$. But taking all my observations into consideration, I think that it must be $6^d.85$ itself. Then, roughly speaking, its formula for the minimum will be:

$$1906 \text{ Nov. } 29.0 + 6^d.85\ E.$$

The duration of the eclipse is about one whole day.

Yerkes Observatory, 1906 December 10.

COMET *g* 1906 (*THIELE, Nov. 10*).*

Elements and ephemeris by STRÖMGREN (*Supplement to A.N. 4134*), computed from observations on Nov. 10, 12 and 14.

ELEMENTS.

T = 1906 Nov. 22.3858 Berlin M.T.

$$\left.\begin{aligned}\omega &= 9^\circ\ 47'.54\\ \Omega &= 85\ \ \ 4.36\\ i &= 57\ \ \ 9.57\end{aligned}\right\}1906.0$$

$\log q = 0.08450$

EPHEMERIS: Berlin midnight.

1906	α	δ	log Δ	Br.
Dec. 1	10^h 14^m 58^s	+40° 18.4		
2	22 30	41 34.3	9.8036	1.3
3	30 12	42 47.9		
4	38 4	43 59.2		
5	46 4	45 7.9		
6	10 54 13	+46 13.9	9.8141	1.2

Observation by MORGAN: 1906 Nov. 12.8624 Gr. M.T. $\alpha = 9^h\ 25^m\ 36^s.1$. $\delta = +15^\circ\ 4'\ 50''$.

* *From Supplement to No. 590.*

COMET *h* 1906 (*METCALF, Nov. 16*).*

OBSERVATIONS.

Greenwich M.T.	α	δ	Observer
1906 November 16.5917	4^h 4^m 16^s	−2° 44′ 48″	Metcalf
16.6988	4 11.4	−2 46 55	Hammond
19.5965	4 3 30.7	−3 25 1	Wilson

* *From Supplement to No. 590.*

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ON THE ACTION OF THE PLANETS ON THE MOON.

By SIMON NEWCOMB.

I have recently brought to substantial completion, and have in press, a determination of the action of the planets on the moon, which may be regarded as a continuation and completion of my work on that subject published in 1894.* It has been carried through under the auspices of the Carnegie Institution, and with the aid of Dr. FRANK E. ROSS.

I propose in advance of the publication to present a brief resume of the methods and results, designed not only to make them more easily known than can be done in the pages of an extended memoir, but to facilitate the work of the reader of the memoir who desires to make a study of its contents, and who will find a previous orientation on the subject useful.

1. *Fundamental Methods and Arrangement.* The investigation of 1894 was based on the idea of expressing the moon's coordinates in terms of the osculating elements, both of the earth's orbit around the sun, and of the moon's around the earth, strictly developed from the Lagrangian equations for the variation of the elements. This method unaltered I found to be impracticable of application from the complexity of the developments required, a subject which is fully discussed in the memoir itself. The difficulties were avoided in the present work by regarding the elements of the earth's motion around the sun as given quantities, to be introduced from the beginning in the differential equations. Thus the six elements of the moon's orbit around the earth are alone taken as the independent variable elements. These elements are not however elliptic ones, but the final constants, as constructed by DELAUNAY in his theory.

To a large extent the work may therefore be regarded as a continuation and practical completion of DELAUNAY's theory. It differs in principle from the method adopted by HILL in taking DELAUNAY's complete results as the basis, instead of adding the necessary DELAUNAY operations to the elliptic formulae, as HILL had done.* It may be remarked that the fundamental numbers given by HILL are those which formed the basis of RADAU's work.

When the work was almost complete in a rough form, the completion of BROWN's work on the inequalities due to the action of the sun appeared in print.† A little examination made it evident that in most points this work afforded a much better basis than that of DELAUNAY. The result was successive reconstructions of my own work, by substituting BROWN's results for those of DELAUNAY, in deriving most of the fundamental quantities. I desire to acknowledge the very courteous way in which Professor BROWN assented to this use of his work before the publication was complete, and supplied me with hints in utilizing it.

In one fundamental point, the introduction of the motion of the ecliptic, the idea of the former work was retained. The differential equations of the lunar coordinates were transformed, so that they were referred to the moving plane of the ecliptic as the fundamental plane *ab initio*, instead of the fixed plane of the ecliptic. It was found that a very simple addition to the perturbative function would enable this to be done, and the complexities inherent in the subject to be entirely done away with. It may be stated that the fundamental equations proved to be substantially the same as those derived by HILL.‡ The adopted method of integration was, however, from the nature of the case, entirely different from that of HILL, being at once assimilated with the adopted method.

HILL|| has shown how the perturbative function arising from the action of the planets may each be reduced to a sum of six products of two factors each, one of the factors being a function of the coordinates of the planet or sun.

* *Astronomical Papers of the American Ephemeris*, Vol. V, Part V.

* *Ibid*, Vol. III, Parts II and IV.

† *Memoirs, R.A.S.*, 1899–1905.

‡ "Lunar inequalities due to the motion of the ecliptic," *Annals of Mathematics*, Vol. I.

|| *American Journal of Mathematics*, Vol. VI.

and the other a function of the moon's coordinates only. This was the method adopted by the writer as far back as 1872–74, when his earlier work was really executed, but it remained unpublished by him until 1894.

One advantage of this method is that the terms of the perturbative function for the planetary action can be at once combined with those due to the action of the sun alone, thus simplifying the passage from the action of the sun to that of a planet. It has also the advantage that the direct and indirect actions are treated on a uniform plan, and can be combined at as early a stage in the process as one desires.

The final form to which the differential equations for the variations of the six elements of the moon's orbit were reduced is the following:

For the elements α, $(= \log . a)$, e and γ,

$$\frac{d\alpha}{ndt} = M\left(\alpha_1 \frac{\partial H}{\partial l} + \alpha_2 \frac{\partial H}{\partial \pi} + \alpha_3 \frac{\partial H}{\partial \theta}\right) \tag{1}$$

For the elements l_0, π_0 and θ_0 (mean longitude, longitudes of perigee and of node at the initial epoch):

$$\frac{dl_0}{ndt} = -M\left(\alpha_1 \frac{\partial H}{\partial \alpha} + e_1 \frac{\partial H}{\partial e} + \gamma_1 \frac{\partial H}{\partial \gamma}\right) \tag{2}$$

Regarding these equations as expressing the direct action, M is the mass of the planet, and H the perturbative function, each multiplied by certain constant factors, while α_i, e_i and γ_i $(i = 1, 2, 3)$ are constant numerical factors, derived from the DELAUNAY theory. H is of the form

$$Ax^2 + By^2 + Cz^2 + 2\,Dxy + 2\,Exz + 2\,Fyz \tag{3}$$

x, y and z being the moon's geocentric coordinates. The last two terms of H may be dropped, except in some special cases.

The indirect action is expressed in the same form, substituting m^2 for M, and certain linear functions of the perturbations of the earth's longitude, latitude and radius-vector for A, B, C, etc.

It will be seen from the preceding expression that two distinct developments are required; one of the planetary factors, A, B, etc.; the other of the lunar factors, x^2, y^2, etc., and of their derivatives as to the lunar elements.

2. *Choice of Coordinate Axes.* The most important simplification which I have made in the work arises through referring the coordinates of all three bodies — moon, sun and planet — to an X-axis passing through the mean sun. The lunar coordinates first reached by BROWN's theory are practically rectangular ones, referred to an X-axis passing through the projection of the mean moon on the ecliptic. The reduction of DELAUNAY's longitude to either of these systems is a comparatively simple matter. The passage from BROWN's coordinates to those of the adopted system is merely a transformation of the X-axis through the angle D, the moon's mean elongation from the sun.

3. *Development of Planetary Factors.* The development of the planetary A, B, etc., has formed numerically the heaviest portion of the work. The method of development was suggested by the special nature of the main point in view; the certain determination of any possible inequality of long period hitherto overlooked. The older observations of the moon seem to make it almost certain that there must be at least one inequality, and perhaps several, with a period of two or three centuries, and a coefficient of 5″ or more. It seemed possible that the terms of the second order due to the mutual perturbations of *Venus* and the *Earth* might give rise to such a term. In fact, taking the perturbations individually, it was found that the result might possibly be of this order of magnitude. An apparent objection to the possibility of this conclusion is that the slight effect thus produced can be only a minute fraction of the total action of *Venus*. This is true, but the same thing is also true of all the terms of very long period, notably the Hansenian term, which arises from a very small fraction of the total action of *Venus*, and is sensible only because multiplied by a numerical coefficient of several millions.

The development of A, B, etc., for *Venus* and *Mars* is extremely complex even when the perturbations of the planet are neglected. Much more so must it be if account is taken of these perturbations. Moreover, the development as executed by DELAUNAY and RADAU, which proceeds according to the powers of the eccentricity and mutual inclination, apart from its complexity, can scarcely fail to leave doubt as to the inclusion of all sensible terms. The apparent alternative is the one which I adopted in my first investigation; that of developing the coefficients by the CAUCHY-HANSEN method, in which the eccentric anomaly is the independent variable. But a difficulty is here met with which is surmountable only by computing a very complex set of terms with a high degree of numerical precision, because the required coefficients come out as minute differences between large quantities.

These considerations led me to adopt for *Venus* and *Mars* a development by purely mechanical quadratures. This method permitted the perturbations to be taken account of without a great addition to the labor. An additional advantage was that almost the entire numerical work was one of purely mechanical computation.

This method was not necessary in the action of *Jupiter*; but it was nevertheless employed on account of its theoretical simplicity.

4. *Lunar Factors.* The developments of the lunar factors x^2, y^2, etc., and of their derivatives as to α, e, and y, have proved the most troublesome part of the work. The plan

of this part will be best seen by a consideration of the form of H shown in (3). The planetary factors, A, B, etc. in that expression are formed of an infinite series of periodic terms, of which the constituents of the arguments are the mean longitudes of the sun and of the planet. The constituents of the arguments for the lunar factors, x^2, etc., are l, π and θ of the moon, and the sun's mean anomaly, g'. The latter is, therefore, common to the two sets of arguments, planetary and lunar, a fact which however does not complicate the combination. When the expression for H is formed, the combined arguments will be the sums and differences of these separate lunar and planetary arguments. The method employed by Radau was to take these final arguments one by one, and select the combination of lunar and planetary arguments which would form them, thus developing each term of H separately.

Owing to the vast number of terms which might become sensible, and to the advisability of proceeding on a uniform plan, the system adopted in the present work was to effect what was supposed to be a complete development of the planetary factors, and another one of the lunar factors, before proceeding to the multiplications required to form the final terms of H and its derivatives. This process makes it much easier for a student of my work to verify and make use of it. But, as the work progressed, the system as I applied it was found subject to a drawback arising from the exceptional character of many terms whose products would give rise to appreciable inequalities. As a general rule the numerical values of the lunar factors are not required beyond the fifth decimal, and only in exceptional cases beyond the third or fourth. The fifth decimal was therefore taken as the limit of precision aimed at. But, after all the combinations were made, it was found that there were many cases in which this degree of precision did not suffice; and it was necessary to repeat several of the developments *ab initio* to the sixth or even the seventh places of decimals. It was also found that in some special cases the lunar arguments which had been dropped out as unimportant, owing to the minuteness of the terms depending on them, led to terms worthy of computation.

It may also be remarked that, at the outset, it was supposed that the analytic development in *Action* would suffice for the work. This expectation proving ill-founded, the developments of the moon's coordinates given by Delaunay — then those by Hansen — and finally those by Brown were used in the case of those terms in which greater precision was needed.

The want of homogeneity thus arising in the tables could be cured by a fresh development from the fundamental data of Brown and Delaunay; but I do not think any important change would thus result in the expressions for the inequalities of the moon's elements in Part IV. The general outcome of the successive emendations to which this part of the work had to be subjected is that it bears a fragmentary aspect, and that it does not pretend to completeness as to the minute terms of long-period. Although I have endeavored to include in the work every term that can be of practical importance in the lunar theory, I shall feel more confidence in the success of this endeavor when some one else shall have tried independently.

The more important of the results will now be briefly set forth.

5. *The Jovian Evection.* The existence of this term was brought out empirically by the writer in 1875. The coefficient derived from observation was 1″.5. Its origin was discovered by Nieson (now Nevill), who showed it to be due to the action of *Jupiter*, and computed the coefficient 1″.16. It is a result of an inequality of the eccentricity and perigee of the moon having a period of about seventeen years, and produced by *Jupiter* in the same way that the sun produces the evection; hence the term *Jovian Evection*. Nevill computed a term in the mean longitude having the same period, and found a coefficient of 2″.

The next theoretical computation was by Hill, who reduced the coefficient to 0″.90 for the evection and 0″.209 for the mean longitude. Later Radau, in his independent work, reproduces Hill's coefficients. This independent confirmation, apart from the fullness with which Hill's work was given, inspired full confidence in the result.

Such being the case, I was much surprised when Dr. Ross, who made a preliminary computation of the principal terms, using the methods of Hill and Radau, reproduced Nevill's coefficient, 1″.16. This result I withheld from publication until it should be confirmed by my own work. This has been done, my definitive result being 1″.15. The difference from Hill's number arises from the fact that Hill, and doubtless Radau, included in the indirect action only the principal perturbations of the *Earth* produced by *Jupiter*. The yet larger coefficient, 1″.5, which I found from the observations, is accounted for by the correction derived by Hill to Hansen's term in the mean longitude depending on the moon's node. The period of the node and of the Jovian evection differ so little that the two may be combined during a considerable time, and it happened that the combined effect reached its maximum during the period of observations which I discussed.

6. *The Hansenian Inequality of Long Period due to the Action of Venus.* The possibility that this inequality might be modified when the mutual perturbations of *Venus* and the *Earth* were taken into account was a strong incentive toward taking up the work. But this was not found to be the case. The result is:

$$\delta l = 14''.90 \sin(18V - 16g' - g + 228°33')$$

where V is the mean longitude of *Venus* reckoned from the earth's perihelion, and g' and g are the mean anomalies of the sun and moon. Radau's coefficient, when reduced

to the adopted mass of *Venus*, 1 ÷ 408000, is 14″.14. The deviation of the present from previous values is much too small to account for the enigmatical inequality in the moon's mean longitude. The value which I now find differs but little from my value of 1895. In both computations the most important innovation made was in taking account of the solar perturbations of the moon in the development of the lunar factors x^2, etc.

Besides the complete computation of A, B, etc., for this term, in which perturbations were included, I computed the fundamental numbers for a separate determination of that part of the inequality arising from the periodic perturbations of *Venus* and the *Earth*. But as the result showed that this part of the term did not amount to more than a fraction of a second, I have made no attempt to complete the computation. The effect, whatever it may be, is included in my final result as above quoted.

7. *The Secular Acceleration.* The theoretical value of this element to which I am led is markedly smaller than any hitherto derived, being 5″.80. It rests on the value $-8''.595$ of the secular variation of the earth's eccentricity for 1850. It cannot be readily compared with preceding values for which the adopted value of this number is not precisely known. DELAUNAY uses as a datum

$$pne'\Delta e' = -635''\ T^2$$

from which he derived the result 6″.11 in 1859. My numbers give 6″.05 for this integral. Reducing this result of DELAUNAY in proportion to this factor would give 5″.82, in fair agreement with my result. But, in his last paper on the subject, DELAUNAY adds terms depending on the ninth and tenth power of m, which he found to change in sign from the preceding terms of the series, and to increase the result by 0″.066 after the first.

The latest work on this subject with which I am acquainted, is the Inaugural Dissertation of Dr. ALBERT VON BRUNN.* The final result of this very elaborate investigation is 5″.9768. But I cannot find that the adopted secular motion of e' for 1850 used in the work is explicitly stated, and my efforts to infer it from the author's statements, either deductively or inductively, have failed.

It should be said that the parallactic terms have been entirely omitted in my work, and I do not suppose they would be sensible in the present case.

As the practical value of the acceleration to be used includes the effect of the earth's tidal retardation, the amount of which must be determined empirically, it follows that the theoretical value of the acceleration is of less immediate practical interest than it otherwise might be.

8. *Motions of the Perigee and Node.* The parts of these motions due to the action of the planets, as well as of the sun, have been computed by Professor BROWN. In consequence of the unavoidable uncertainty of the earth's ellipticity, the motions to be used in future lunar tables must be derived from observation. The theoretical values are, however, of interest, especially in the case of the perigee, the motion of which would be materially changed if the earth's gravitation were not exactly as the inverse square. The hypothesis as to the sun's gravitation which I have shown would best represent the motion of the perihelion of *Mercury*, when applied to the case of the earth and moon, would result in an increase of the centennial motion of the perigee by about 150″. BROWN's results showed that there could be no such increase, and my own more than confirm this conclusion, as my theoretical result is greater than his. It may be remarked in this connection that the deviation of the sun's gravitation from the inverse square would also result in a further acceleration of the lunar perigee; but the amount of this has not been computed.

My computation of the direct action of the planets from *Venus* to *Saturn* compare with BROWN's as follows:

Action of	Perigee N	Perigee B	Perigee $N-B$	Node N	Node B	Node $N-B$
	″	″	″	″	″	″
Venus	200.31	196.6	+3.7	−81.96	81.9	−0.1
Mars	3.64	3.4	+0.2	− 2.51	2.5	0.0
Jupiter	61.06	58.2	+2.9	−53.89	53.6	−0.3
Saturn	2.91	2.6	+0.3	− 2.63	2.5	−0.1

As the details of BROWN's computations are unpublished I cannot explain the difference. In computing the indirect action, BROWN has used the constants of the earth's log r given in my tables of the sun. I have deemed it desirable in the present problem to use a more rigorous development. This is found in the present volume of the *A. J.*, No. 590. Moreover, I have computed only the combined indirect action of all the planets, conceiving that little importance would attach to the individual values. The modification of BROWN's results thus reached proves to be comparatively small. Of interest is an ulterior correction to the centennial motions, amounting to $-2''.64$ for π, and $-0''.18$ for θ, which arises through the adjustment of the constants of integration, especially of the mean distance or mean motion. The motion of the perigee and node due to the action of the sun alone is based upon the expression of the moon's radius-vector in terms of a constant n, representing the observed mean motion of the moon in longitude. When the action of a planet is taken account of in the usual way, the relation between n and r is taken to remain unchanged. But, in fact, when the arbitrary constants introduced in integrating the planetary action are adjusted, a correction comes in which we may apply indifferently either to n or to r. In which way soever we make this change, it will modify the action of the sun on the perigee and node. We may therefore regard the correction as due either to planetary or solar action, the two being combined in producing it.

* *Die Säkularbeschleunigung des Mondes*, Göttingen, 1905.

OBSERVATIONS OF COMETS,

MADE WITH THE 16-INCH EQUATORIAL OF THE CINCINNATI OBSERVATORY,

By J. G. PORTER, Director.

1906 Cincinnati M.T.	✱	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Comet 1905 VI (1906 a).										
Feb. 1 16h 41m 36s	1	8 , 8	+0m 30.84s	− 3′ 27.1″	16h 12m 52.75s	+57° 41′ 14.0″	n9.749	n0.144	−1.44s	−10.7″
3 12 59 32	2	8 , 8	−0 15.74	− 5 0.2	16 9 4.30	+61 13 18.6	n9.978	0.486	−1.46	−11.4
3 13 33 13	2	8 , 8	−0 19.09	− 2 10.3	16 9 0.95	+61 16 8.5	n9.977	0.336	−1.46	−11.4
8 13 44 34	5	8 , 8	+3 6.76	− 2 21.7	15 47 59.77	+71 25 0.2	n0.135	n9.306	−1.42	−13.4
15 10 13 1	6	4 , 4	−8 51.21	− 4 28.0	12 55 45.18	+84 3 30.4	n0.630	n0.056	+3.47	−12.4
27 10 24 13	7	6 , 6	+3 3.22	− 1 30.8	5 58 48.84	+69 27 24.6	9.951	n0.432	+1.04	+ 7.2
Mar. 17 8 27 30	8	6 , 6	+0 56.93	+ 5 47.3	5 40 31.58	+48 43 42.8	9.614	n9.683	−0.07	+ 2.2
21 7 44 3	9	9 , 6	+1 14.68	− 0 40.8	5 41 19.52	+45 42 35.2	9.505	n9.606	−0.15	+ 1.3
22 8 27 55	10	9 , 6	−1 34.29	− 4 34.2	5 41 37.31	+44 59 9.3	9.628	9.443	−0.17	+ 1.1
Comet 1905 IV (1906 b).										
Mar. 17 10 55 20	11	6 , 8	−0 5.22	− 0 22.1	11 29 51.87	+ 2 0 27.0	n9.037	0.723	+1.19	− 8.6
20 10 48 22	12	6 , 6	+0 12.64	+ 3 55.7	11 28 36.60	+ 2 4 29.5	n8.986	0.722	+1.21	− 8.8
21 9 33 52	12	6 , 6	−0 10.76	+ 5 15.7	11 28 13.20	+ 2 5 49.5	n9.358	0.725	+1.21	− 8.8
22 10 33 0	12	9 , 6	−0 35.87	+ 6 45.3	11 27 48.08	+ 2 7 19.1	n9.043	0.722	+1.20	− 8.8
31 9 56 46	13	7 , 6	+0 37.24	− 3 50.0	11 24 29.11	+ 2 19 20.4	n9.022	0.720	+1.18	− 8.9
Apr. 2 10 2 11	13	10 , 6	−0 1.37	− 1 44.7	11 23 50.50	+ 2 21 25.7	n8.893	0.719	+1.18	− 8.9
3 9 48 47	13	10 , 6	−0 19.51	− 0 47.1	11 23 32.36	+ 2 22 23.3	n8.982	0.719	+1.18	− 8.9
10 7 56 34	14	6 , 6	+0 44.05	+ 4 17.5	11 21 44.61	+ 2 27 32.3	n9.395	0.722	+1.12	− 8.7
11 8 51 23	14	6 , 6	+0 30.92	+ 4 46.8	11 21 31.48	+ 2 28 1.6	n9.154	0.719	+1.12	− 8.7
12 9 52 36	14	6 , 6	+0 18.63	+ 5 15.8	11 21 19.19	+ 2 28 30.6	n8.156	0.718	+1.12	− 8.7
16 9 34 2	14	6 , 6	−0 20.92	+ 6 15.9	11 20 39.64	+ 2 29 30.7	n8.267	0.718	+1.11	− 8.7
19 9 50 59	15	4 , 6	+0 11.03	+ 3 33.2	11 20 18.27	+ 2 29 29.5	8.597	0.718	+1.08	− 8.6
19 9 50 59	14	4 , 6	−0 42.25	+ 6 12.8	11 20 18.28	+ 2 29 27.7	8.597	0.718	+1.09	− 8.6
23 10 24 54	15	6 , 6	+0 24.83	+ 2 28.3	11 20 2.02	+ 2 28 24.9	9.137	0.719	+1.03	− 8.3
23 10 24 54	16	6 , 6	+0 5.67	− 8 14.6	11 20 2.16	+ 2 28 23.0	9.137	0.719	+1.03	− 8.3
26 10 12 24	15	6 , 6	+0 22.13	+ 0 49.6	11 19 59.30	+ 2 26 46.3	9.133	0.719	+1.01	− 8.2
May 13 8 33 56	17	6 , 6	−0 55.80	+ 4 30.0	11 22 9.57	+ 2 3 49.8	8.845	0.722	+0.88	− 7.5
14 8 46 42	17	6 , 6	−0 40.30	+ 2 28.5	11 22 25.06	+ 2 1 48.4	9.011	0.723	+0.87	− 7.4
16 8 50 42	17	6 , 8	−0 7.12	− 1 54.0	11 22 58.22	+ 1 57 26.0	9.095	0.724	+0.85	− 7.3
23 8 59 8	18	6 , 6	+1 19.95	− 3 24.5	11 25 18.72	+ 1 39 46.1	9.273	0.727	+0.76	− 6.8
24 9 11 16	18	6 , 6	+1 43.02	− 6 15.7	11 25 41.78	+ 1 36 54.9	9.333	0.728	+0.75	− 6.8
June 9 9 33 28	19	6 , 4	+0 33.17	+ 1 30.8	11 33 28.34	+ 0 42 5.1	9.527	0.737	+0.65	− 5.9
14 9 11 29	21	6 , 4	+0 13.57	− 7 42.2	11 36 26.71	+ 0 21 37.2	9.518	0.740	+0.61	− 5.5
21 9 5 6	23	6 , 6	+0 46.60	− 9 43.8	11 40 59.92	− 0 9 21.9	9.546	0.743	+0.56	− 5.0
Comet 1906 g.										
Nov. 22 12 42 38	24	8 , 8	+0 39.81	+ 6 21.3	10 16 26.71	+28 18 19.1	n9.710	0.645	+1.77	−19.0
24 14 44 36	25	8 , 6	+1 7.39	− 0 23.5	10 29 11.17	+31 13 48.6	n9.628	0.411	+1.73	−20.3
Dec. 11 13 23 57	26	6 , 4	−0 19.35	+10 26.5	12 36 10.19	+51 5 54.0	n9.861	0.516	+0.61	−25.8
Comet 1906 h.										
Nov. 22 11 41 30	27	6 , 6	−2 26.86	− 9 45.6	4 2 43.58	− 3 58 54.4	n8.506	0.777	+3.10	+ 4.8
24 14 7 51	28	8 , 6	−1 9.41	+ 6 38.2	4 2 10.93	− 4 19 21.4	9.416	0.758	+3.10	+ 4.7
Dec. 12 11 26 25	29	8 , 6	+1 7.12	− 6 25.7	3 59 14.51	− 5 32 42.4	9.006	0.788	+3.17	+ 2.3
18 9 53 16	29	8 , 6	+1 23.47	+ 4 50.9	3 59 30.90	− 5 21 26.5	n8.579	0.787	+3.21	+ 1.6

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	16 12 23.35	+57 44 51.8	Hels.&Gotha, A.G. 8712	16	11 19 55.46	+ 2 36 45.9	Albany, A.G. 4265
2	16 9 21.50	+61 18 30.2	Comp. with 3 and 4	17	11 23 4.49	+ 1 59 27.3	Albany, A.G. 4283
3	16 10 10.82	+61 26 22.8	Hels.&Gotha, A.G. 8698	18	11 23 58.01	+ 1 43 17.4	Albany, A.G. 4290
4	16 14 46.72	+61 7 13.3	Hels.&Gotha, A.G. 8725	19	11 32 54.52	+ 0 40 40.2	Comp. with 20
5	15 44 54.43	+71 27 35.3	O. Arg. 15690	20	11 30 58.40	+ 0 43 47.2	Nic., A.G. 3273
6	13 4 32.92	+84 8 10.8	Carrington 1956	21	11 36 12.53	+ 0 29 24.9	Comp. with 22
7	5 55 44.58	+69 28 48.2	Hels.&Gotha, A.G. 968	22	11 38 31.11	+ 0 42 26.4	Nic., A.G. 3298
8	5 39 34.72	+48 37 53.3	Bonn, A.G. 4708	23	11 40 12.76	+ 0 0 26.9	Nic., A G. 3304
9	5 40 4.99	+45 43 14.7	Bonn, A.G. 4721	24	10 15 45.13	+28 12 16.8	Camb., Eng., A.G. 5311
10	5 43 11.80	+45 3 42.4	Bonn, A.G. 4751	25	10 28 2.05	+31 14 32.4	Leiden, A.G. 4217
11	11 29 55.90	+ 2 0 57.7	Albany, A.G. 4308	26	10 36 58.90	+50 55 53.3	Camb., Mass., A.G. 4121
12	11 28 22.75	+ 2 0 42.6	Comp. with 11	27	4 5 7.34	− 3 49 13.6	Heid. 412, Rad_3 981
13	11 23 50.69	+ 2 23 19.3	Albany, A.G. 4288	28	4 3 17.24	− 4 26 4.3	Schjellerup 1302
14	11 20 59.44	+ 2 23 23.5	Albany, A.G. 4271	29	3 58 4.22	− 5 26 19.0	W.B. 3^h, 1081, War 674
15	11 19 36.16	+ 2 26 4.9	Albany, A.G. 4264				

OBSERVATIONS OF THE SATELLITES OF *SATURN* IN 1906,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By J. C. HAMMOND.

[Communicated by Rear-Admiral ASA WALKER, U.S.N., Superintendent.]

In the following observations, the position-angles were taken about the inner satellite of each pair. Usually eight settings in position-angle and eight measurements of distance were made. A magnifying power of 400 diameters was used, unless otherwise stated. When the distance of *Iapetus* from *Titan* exceeded 450″, the differences of right-ascension and declination of this pair were observed by the method of transits. The printed differences in α and δ in these cases are in the sense *Iapetus* minus *Titan*. They have been corrected for the motion of *Saturn* during the interval between the transits of the satellites. In all cases, corrections have been applied for differential refraction and for the constants of the instrument.

The micrometer was not disturbed during the series, and the parallel remained constant. The adopted value of the correction for parallel (+0°.427) is the mean of eleven determinations. Bright field illumination was used throughout, except in the case of *Hyperion*. The value of one revolution of the micrometer screw is the same as that hitherto used:

$$1 \text{ rev.} = 9''.9316 - 0''.00005\,(t - 50° \text{ Fah.})$$

In the last column is noted the condition of the seeing, the following abbreviations being used: p = poor, f = fair, g = good, and e = excellent seeing.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
				Tethys–Rhea.									
	1906	h m s	°	h m s	″			1906	h m s	°	h m s	″	
1	July 30	13 17 18	94.45	13 17 11	121.29	e	9	Sept. 18	11 16 52	96.42	11 16 57	37.12	g
2	Aug. 15	12 40 40	274.48	12 41 0	128.98	e	10	19	9 19 35	100.50	9 19 25	70.87	g
3	18	11 50 11	98.76	11 50 14	97.38	g	11	21	10 12 25	334.15	10 12 32	5.96	g
4	22	12 15 58	99.14	12 15 50	97.57	f	12	23	10 22 44	99.75	10 22 58	103.41	p
5	Sept. 1	11 27 1	263.01	11 27 9	38.64	f	13	Oct. 7	9 45 25	250.98	9 45 15	18.55	f
6	5	11 52 49	101.07	11 52 59	23.51	f	14	12	9 27 36	184.06	9 27 45	13.16	e
7	6	11 43 10	243.59	11 43 10	18.60	g	15	13	8 47 16	278.59	8 47 7	102.29	g[1]
8	12	11 21 37	277.16	11 21 43	56.37	e	16	14	8 21 51	63.44	8 21 59	6.77	e[2]

(1) Comparisons 4, 4. (2) Comparisons 6, 6.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
				Tethys–Dione.									
1	July 30	13 38 20	99.60	13 38 28	55.39	e[1]	8	Sept. 19	9 54 57	90.13	9 55 14	67.62	g
2	Aug. 15	12 29 19	107.19	12 29 28	10.21	e	9	21	10 2 23	276.70	10 2 38	13.08	g
3	18	11 38 38	98.51	11 38 36	82.17	g	10	Oct. 7	9 26 5	272.83	9 26 13	88.53	f
4	22	12 5 19	273.35	12 5 30	31.50	f	11	12	9 2 26	108.66	9 2 37	45.65	e
5	Sept. 5	11 30 15	283.70	11 30 7	52.01	f	12	13	8 41 44	281.08	8 41 41	75.88	g[1]
6	6	11 56 6	99.78	11 56 10	81.58	g	13	14	8 10 42	97.19	8 10 45	67.49	e[2]
7	12	11 53 27	257.02	11 53 24	31.75	e							

(1) Comparisons 4, 4. (2) Comparisons 6, 6.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
						Dione–Rhea.							
	1906	h m s	°	h m s	″			1906	h m s	°	h m s	″	
1	July 30	13 30 44	90.04	13 30 42	65.68	e^1	8	Sept. 19	10 6 10	189.38	10 10 36	13.08	g^3
2	Aug. 15	12 51 1	275.52	12 51 8	138.33	e	9	19	10 15 21	196.53	. . .	. .	g^4
3	18	12 6 15	98.97	12 6 17	16.58	g	10	Oct. 7	9 35 22	98.72	9 35 32	72.85	fg
4	22	12 39 18	97.12	12 39 22	127.78	f	11	12	9 11 13	271.98	9 11 21	40.25	e
5	Sept. 5	11 40 2	103.18	11 39 59	73.79	f	12	13	8 58 8	271.28	8 58 20	27.32	g
6	6	12 9 22	273.42	12 9 13	99.16	g^2	13	14	8 32 50	281.34	8 32 56	61.16	e^5
7	12	12 2 21	300.46	12 2 20	28.50	e							

(1) Comparisons 8, 7. (2) Comparisons 8, 9. (3) Comparisons 4, 8. (4) Comparisons 4, –. (5) Comparisons 6, 6.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
						Enceladus–Tethys.							
1	July 24	14 46 30	147.43	14 50 39	3.50	p^1	8	Sept. 4	11 6 47	277.31	11 6 51	79.12	p
2	24	14 54 26	158.59	. . .	. .	p^2	9	6	11 30 22	68.88	11 29 49	5.86	g
3	30	13 59 50	274.24	14 0 2	67.67	e	10	12	11 13 10	286.06	11 13 16	15.18	e
4	Aug 15	12 17 46	94.72	12 18 3	83.70	e	11	19	9 36 33	274.01	9 36 49	82.81	g
5	18	11 25 7	272.31	11 28 58	22.05	g^3	12	21	9 51 14	287.33	9 51 13	12.02	g
6	22	11 55 44	86.61	11 55 32	15.13	f	13	Oct. 13	8 35 40	99.44	8 35 51	56.35	g^3
7	Sept. 1	11 47 0	98.70	11 47 10	18.39	f	14	15	7 31 20	245.19	7 31 25	15.73	f^4

(1) Comparisons 4, 8. (2) Comparisons 4, –. (3) Comparisons 4, 4. (4) Comparisons 6, 6; satellites very faint.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
						Mimas–Enceladus.							
1	Aug. 15	12 2 2	264.89	12 2 15	9.83	e^1	4	Sept. 19	8 55 5	95.81	8 55 16	62.33	g^1
2	22	12 28 7	277.84	12 26 32	59.02	f^1	5	Oct. 14	7 48 2	255.97	7 47 47	5.23	e^3
3	Sept. 1	11 16 24	94.83	11 16 23	53.31	f^2							

(1) Comparisons 4, 4. (2) Comparisons 4, 5. (3) Comparisons 4, 5; bright wires.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
						Mimas–Tethys.							
1	Aug. 15	12 8 59	95.75	12 9 8	73.59	e^1	4	Oct. 12	9 19 21	281.02	9 19 16	53.96	e^2
2	Sept. 1	11 7 11	95.78	11 7 17	73.65	f^1	5	14	7 58 22	284.42	7 58 14	44.95	e^3
3	19	9 5 7	265.07	9 5 24	19.02	g^1							

(1) Comparisons 4, 4. (2) Comparisons 5, 4. (3) Comparisons 4, 4; bright wires.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
						Rhea–Titan.							
1	July 19	14 23 57	275.28	14 24 17	96.39	f	8	Sept. 7	10 48 27	352.39	10 48 26	12.82	p
2	30	14 24 30	257.95	14 24 24	49.21	e	9	9	10 53 46	326.17	10 54 12	21.22	f
3	Aug. 15	13 3 47	102.18	13 3 20	112.09	e	10	12	11 41 24	96.73	11 41 28	237.28	e
4	18	12 19 20	274.33	12 16 47	215.63	g	11	19	10 25 46	274.88	10 25 49	186.97	g
5	Sept. 1	11 36 14	266.06	11 36 25	57.87	f	12	Oct. 12	9 37 21	87.56	9 37 18	166.41	e
6	5	12 3 53	277.64	12 4 9	238.35	f	13	13	9 6 54	93.87	9 7 10	261.06	g
7	6	12 22 46	285.97	12 23 2	91.78	g	14	15	7 46 27	100.98	7 46 36	104.89	f^1

(1) Comparisons 6, 6.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
						Titan–Hyperion.							
1	July 21	13 53 34	93.37	13 53 35	176.45	g	4	Sept. 19	10 45 5	39.11	10 45 16	28.66	g^2
2	24	15 8 2	97.69	15 8 11	106.24	p	5	Oct. 8	9 51 34	263.62	9 51 35	77.32	f^3
3	Sept. 14	11 21 24	273.46	11 22 49	287.17	p^1	6	15	8 17 25	346.12	8 16 56	27.51	f^4

(1) Comparisons 4, 4; *Hyperion* very faint. (2) *Hyperion* very faint. (3) Comparisons 6, 6. (4) Comparisons 5, 4.

No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.	No.	Date	Wash. M.T.	p	Wash. M.T.	s	Rem.
						Titan–Iapetus.							
1	July 24	15 22 21	33.36	15 22 26	79.68	p	9	Sept. 9	10 37 54	113.18	10 37 11	84.97	p^2
2	30	11 11 35	296.52	14 11 33	124.81	e	10	11	10 59 4	93.73	10 58 59	55.89	p^2
3	Aug. 18	12 39 2	261.22	12 39 17	404.44	g^1	11	12	11 30 55	82.83	11 31 2	80.90	e
4	Sept. 1	11 58 54	251.43	11 58 40	167.06	f	12	Oct. 12	9 47 41	320.62	9 47 50	52.54	e
5	3	11 39 21	143.80	11 39 38	70.23	p	13	13	9 18 9	292.76	9 18 6	123.25	g
6	4	11 18 11	119.59	11 18 12	123.96	p	14	14	8 44 32	287.49	8 44 38	176.62	e^3
7	5	12 19 45	113.45	12 19 47	159.01	f	15	15	7 59 59	285.78	8 0 14	204.90	f^3
8	7	10 36 22	112.02	10 36 30	148.26	p							

(1) Power 175. (2) Bright wires. (3) Comparisons 6, 6.

No.	Date	Wash. M.T.	Δα	Wash. M.T.	Δδ	Rem.	No.	Date	Wash. M.T.	Δα	Wash. M.T.	Δδ	Rem.
						Titan–Iapetus [rectangular].							
			s		″					s		″	
1	July 19	15 4 14	+35.60	15 4 14	+56.5	f^1	4	Sept. 23	10 41 14	+42.12	10 41 14	+20.2	p^1
2	Aug. 15	13 21 56	−39.12	13 21 56	−27.3	e^1	5	Oct. 7	10 1 1	+32.45	10 1 1	+31.8	f^1
3	Sept. 19	11 9 55	+11.92	11 9 55	+21.3	g^1							

(1) Bright wires; power 175; comparisons 25:5.

THE SUSPECTED VARIABLE STAR B.D. +68°200.

R.A. $2^h 43^m 2^s.9$: Decl. +68° 28′ 28″ (1900).

By J. A. PARKHURST.

In the notes to the last section of the Potsdam Photometric *Durchmusterung*,* Müller and Kempf state that this star varies with a range not exceeding half a magnitude, and a period of about two days. Photographs taken here appear to confirm the variation, and as the star is liable to be used as a comparison star for the new *Algol*-type variable *RZ Cassiopeiae*, a settlement of the question would seem important. The table gives the results from the best plates taken with the two-foot reflecting telescope.

* *Potsdam Publications*, XVI, 254.

Plate No.	Date G.M.T.	Mag.	No. expos. on plate
547	1906 Aug. 14.80	6.36	11
568	Aug. 31.65	5.98	16
605	Sept. 25.68	6.58	15

The comparison stars used, with the adopted magnitudes, are: B.D. +68°184, 9.49, and +69°171, 7.80. The spectrum of the latter is a typical Type I (Draper Catalogue A), while that of the suspected variable may be stated as a solar type with the hydrogen lines unusually strong.

(Star near edge of plate, magnitudes therefore uncertain by 0.1 or 0.2.)

Yerkes Observatory, 1907 *January* 23.

OBSERVATIONS OF COMETS,*

MADE AT THE GOODSELL OBSERVATORY OF CARLETON COLLEGE WITH THE 16-INCH TELESCOPE AND FILAR-MICROMETER,

By H. C. WILSON.

[Communicated by Wm. W. Payne, Director.]

1906 Northfield M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
				Comet *e* 1906 (Kopff).						
	h m s			m s	′ ″	h m s	° ′ ″			s	″
Nov. 11 9 47 30	1	21 , 8	−0 13.30	+2 47.6	22 26 26.80	+ 5 31 53.4	8.868	0.741	+2.44	+20.0
12 9 41 47	1	17 , 8	−0 5.80	−2 41.6	22 26 34.30	+ 5 27 24.2	8.845	0.742	+2.44	+20.0
18 8 51 28	3	12 , 8	+0 12.45	+0 59.2	22 27 51.68	+ 4 57 17.0	8.295	0.746	+2.37	+19.8
				Comet *f* 1906 (Thiele).						
Nov. 12 14 9 17	5	9 , 4	+3 28.50	+8 29.9	9 25 32.89	+15 3 55.8	*n*9.564	0.703	+1.82	−12.1
18 14 16 7	7	9 , 6	+1 27.49	+7 12.4	9 54 44.81	+22 52 41.8	*n*9.583	0.636	+1.80	−16.4
				Comet *g* 1906 (Metcalf).						
Nov. 19 8 6 24	8	9 , 6	−0 29.28	−5 22.1	4 3 30.68	− 3 25 1.2	*n*9.566	0.800	+3.06	+ 5.2
23 9 53 51	10	12 , 8	−0 0.61	+4 25.8	4 2 29.57	− 4 8 20.2	*n*9.322	0.814	+3.09	+ 4.9

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	22 26 37.66	+ 5 29 45.8	10m. Mic.comp.with No.2	7	9 53 15.52	+22 45 45.8	Lalande 19502
2	22 30 30.16	+ 5 28 3.3	Munich, 31226	8	4 3 56.90	− 3 19 44.3	10.5 mag. Micrometer comparison with No.9
3	22 27 36.86	+ 4 55 58.0	11m. Mic.comp.with No.4	9	4 2 18.08	− 3 15 55.0	Lalande 7681
4	22 30 49.43	+ 4 53 42.8	Göttingen 6256	10	4 2 27.09	− 4 12 50.7	12m. Mic.comp.with No.11
5	9 22 2.57	+14 55 38.0	DM +14.2051. Micrometer comparison with No. 6.	11	4 3 52.22	− 4 16 3.2	Lalande 7716
6	9 21 38.97	+14 54 8.1	Schjellerup 3473				

Comet *e* was exceedingly faint and difficult to observe. Comet *f* was bright, with a hazy nucleus, so that the measures are good. Comet *g* was faint, round, 1′ in diameter, with a rather sharp central condensation.

* *From Supplement to No. 591.*

CONTENTS.

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FOUNDED BY B. A. GOULD

No. 593.

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PARABOLIC ELEMENTS FOR COMET 1819 II,

By HENRY A. PECK.

In his review of the "Definitive Orbit of Comet 1819 II," Professor KREUTZ has expressed a preference for elements based upon the first solution of the differential equations. I have carried this work to completion, and am led to the result that a parabola represents the entire series of observations nearly as well as either one of the ellipses given. Starting from the first solution, as given in Nos. 584-5 of the *Astronomical Journal*, we have

$$\begin{aligned} \partial T &= 0.001351 \\ \partial \Omega &= +35''.26 \\ \partial i &= +12.10 \\ \partial \omega &= -17.17 \\ \partial q &= +0.0001170 \\ \partial e &= -0.000120 \end{aligned}$$

Corresponding to which are the elements:

$$T = 1819 \text{ June } 27.71682 \text{ G.M.T.}$$

$$\left.\begin{aligned} \Omega &= 273^\circ\ 42'\ \ 7''.3 \\ i &= \ \ 80\ \ 44\ \ 50.1 \\ \omega &= \ \ 13\ \ 25\ \ 56.8 \end{aligned}\right\} 1819.0$$

$$\log q = 9.533382$$
$$\log e = 0.999880$$

EQUATORIAL COORDINATES.

$$\begin{aligned} x &= [9.237944]\, r \sin(\ 35^\circ\ 21'\ \ 8''.8 + v) \\ y &= [9.996718]\, r \sin(260\ \ 42\ \ 15.6 + v) \\ z &= [9.996737]\, r \sin(349\ \ 50\ \ \ 3.3 + v) \end{aligned}$$

And computing the position of the comet for the last normal date, the residuals are

$$\Delta\alpha \cos\delta = -15''.5 \quad ; \quad \Delta\delta = +1.1$$

while substituting in the last normal equation they are $-14''.9$, $+1.1$ respectively.

The probable error of the correction to the eccentricity is larger than the quantity itself. To test this further the equations were again solved, the other unknown quantities being stated in terms of the eccentricity, with the result that

$$\begin{aligned} \partial k &= +14''.44 \ +2.0997 \frac{\partial e}{2} \\ k\sqrt{2}\,\partial T &= +13.39 \ +0.5352 \frac{\partial e}{2} \\ \partial q &= +28.63 \ +0.3619 \frac{\partial e}{2} \\ \partial \lambda &= -37.65 \ -0.5237 \frac{\partial e}{2} \\ \partial \nu &= +21.36 \ +0.1447 \frac{\partial e}{2} \end{aligned}$$

Substituting these values in the observation equations there results

$+\ 1''.26\ +0.034\partial e$	$+\ 1''.66\ +0.000\partial e$
$-\ 0.90\ -0.000$	$-\ 3.80\ -0.005$
$-\ 0.98\ -0.017$	$+\ 0.54\ +0.007$
$-\ 2.25\ -0.020$	$+\ 1.86\ +0.015$
$+\ 2.58\ -0.016$	$+\ 0.66\ +0.014$
$+\ 1.04\ -0.003$	$+\ 0.36\ +0.003$
$+\ 5.48\ +0.014$	$+14.55\ -0.012$
$-\ 1.02\ +0.051$	$-19.24\ -0.044$
$+\ 9.74\ +0.075$	$+\ 1.16\ -0.060$
$-11.98\ +0.110$	$-\ 0.70\ -0.074$

If values for ∂e, ranging from 0 to $-50''$, are substituted, the following table may be formed:

∂e	pvv
$-50''$	14282
40	13980
30	13953
20	13968
10	14153
0	14437

from which may be seen that although there is an incli-

nation towards an elliptic form, yet a parabola is not out of the range of possibilities.

Assuming $\partial e = 0$, there results the following corrections to the elements:

$$\begin{aligned} \partial T &= +0.00267 \\ \partial \Omega &= +42''.12 \\ \partial i &= +12.04 \\ \partial \omega &= +\ 7.67 \\ \partial q &= +0.0001388 \end{aligned}$$

Adding these corrections to the original elements, and computing the probable errors, we have as the final parabolic elements:

$$T = 1819 \text{ June } 27.71814 \pm 0.00100 \text{ Gr. M.T.}$$

$$\left.\begin{aligned} \Omega &= 273^\circ\ 42'\ 14''.1 \pm\ 3''.0 \\ i &= \ \ 80\ 44\ 50.0 \pm\ 4.5 \\ \omega &= \ \ 13\ 26\ 21.7 \pm 17.1 \end{aligned}\right\} 1819.0$$

$$\log q = 9.533409 \pm 0.000032$$

Syracuse University, 1907 *Jan.* 29.

EQUATORIAL COORDINATES.

$$\begin{aligned} x &= [9.237973]\, r \sin(\ 35^\circ\ 22'\ 10''.4 + v) \\ y &= [9.996716]\, r \sin(260\ 42\ 41.2 + v) \\ z &= [9.996737]\, r \sin(349\ 50\ 28.5 + v) \end{aligned}$$

As a check on the numerical part of the work the comparison of the residuals obtained by direct computation from the elements, and by substitution in the differential equations, is appended.

$\Delta\alpha \cos\delta$ Elements	$\Delta\alpha \cos\delta$ Equations	$\Delta\delta$ Elements	$\Delta\delta$ Equations
"	"	"	"
+ 1.3	+ 1.3	+ 1.6	+ 1.7
− 0.9	− 0.9	− 3.2	− 3.8
− 1.0	− 1.0	+ 1.0	+ 0.5
− 2.1	− 2.2	+ 2.0	+ 1.9
+ 2.7	+ 2.6	+ 0.2	+ 0.7
+ 1.0	+ 1.0	+ 0.5	+ 0.4
+ 5.8	+ 5.5	+14.2	+14.5
− 1.1	− 1.0	−18.7	−19.2
+ 9.7	+ 9.7	+ 1.3	+ 1.2
−12.3	−12.0	− 0.9	− 0.7

MAXIMA OF LONG-PERIOD VARIABLES,

BY IDA WHITESIDE.

The maxima of the following long-period variables were determined by the single-light curves, deduced from observations made with a four-inch Dolland telescope. The predicted times of maximum are those given in CHANDLER'S "Ephemerides of Long-Period Variables." The last column gives the authority for the magnitudes of the comparison stars used.

Star	Date of Maximum	Predicted Date	Magn.	No. of Obsns.	Time covered by Observations	Comparison Stars
7468 *T Aquarii*	Nov. 17, 1906	Dec. 23, 1906	7.60	8	Oct. 12, '06–Dec. 18, '06	Harvard
5194 *V Bootis*	Near Aug. 9, "	May 24, "	7.80	25	Mar. 12, " –Oct. 1, "	Harvard
5190 *R Camelopardalis*	Oct. 20, "	Nov. 2, "	8.10	10	Sept. 15, " –Dec. 4, "	Harvard
432 *S Cassiopeiae*	Sept. 21, "	Oct. 31, "	7.40	18	Aug. 13, " –Jan. 26, '07	Harvard
243 *U Cassiopeiae*	Sept. 15, "	Aug. 6, "	8.60	14	Aug. 9, " –Nov. 14, '06	Harvard
294 *W Cassiopeiae*	July 19, "	Aug. 4, "	8.90	16	June 29, " –Oct. 23, '06	B.D.
845 *R Ceti*	Nov. 18, "	Dec. 12, "	9.00	7	Oct. 12, " –Jan. 11, '07	Harvard
2942 *RT Cygni*	Nov. 30, "	Dec. 17, "	7.30	8	Oct. 12, " –Dec. 18, '06	B.D.
7299 *U Cygni*	Dec. 21, "	Oct. 28, "	6.40	17	Aug. 13, " –Jan. 10, '07	Harvard
7192 *Z Cygni*	Sept. 22, "	Oct. 13, "	9.25	11	Aug. 13, " –Nov. 1, '06	Harvard
7261 *R Delphini*	Nov. 7, "	Dec. 1, "	8.35	9	Oct. 12, " –Jan. 11, '07	Harvard
5955 *R Draconis*	After Nov. 24, "	Nov. 7, "	7.30 or +	10	Sept. 15, " –Nov. 24, '06	Harvard
5768 *RR Herculis*	Sept. 24, "	Nov. 18, "	8.40	8	Sept. 14, " –Nov. 14, "	B.D.
6512 *T Herculis*	Aug. 9, "	Aug. 1, "	7.80	13	June 29, " –Sept. 28, "	Harvard
5887 *V Ophiuchi*	On or bef. July 13 "	Sept. 3, "	7.90 or +	10	July 13, " –Oct. 13, "	B.D.
6682 *X Ophiuchi*	Aug. 8, "	Aug. 8, "	6.15	22	June 29, " –Dec. 18, "	Harvard
6207 *Z Ophiuchi*	On or aft. Sept. 8 "	Sept. 8, "	8.05	15	July 13, " –Nov. 24, "	B.D.
8290 *R Pegasi*	Sept. 15, "	Sept. 29, "	7.65	19	July 18, " –Dec. 18, "	Harvard
8373 *S Pegasi*	Sept. 14, "	Aug. 27, "	8.50	19	July 11, " –Dec. 11, "	Harvard
5677 *R Serpentis*	Near July 8, "	July 8, "	Abt. 7.65	16	June 29, " –Oct. 23, "	Hagen
5501 *S Serpentis*	Aug. 29, "	Oct. 11, "	9.15	6	Aug. 13, " –Oct. 12, "	Harvard
906 *R Trianguli*	Oct. 8, "	Nov. 12, "	6.00	12	Sept. 15, " –Jan. 11, '07	Harvard
5601 *S Ursae minoris*	Sept. 10, "	Sept. 19, "	8.00	19	July 13, " –Dec. 4, '06	B.D.

South Cambridge, N. Y.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENNA., WITH A $4\frac{1}{2}$-INCH REFRACTOR.

By A. W. QUIMBY.

1906	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Det.
July 1	6	..	7	58	4	fair
2	9	1	8	45	2	poor
3	6	2	10	89	1	good
4	4	..	8	54	3	poor
5	6	..	10	60	2	fair
6	6	..	10	32	4	fair
7	6	..	8	36	4	fair
8	6	..	8	23	2	poor
* 9	11	..	3	6	1	poor
*10	6	..	3	13	1	poor
*11	6	1	4	15	2	poor
12	6	2	6	19	2	fair
13	6	1	5	21	2	fair
14	6	..	5	25	3	fair
15	6	..	5	26	3	fair
16	6	1	6	30	2	fair
17	6	..	6	32	3	fair
18	6	1	7	38	2	fair
19	6	..	6	42	1	fair
20	5	1	7	47	2	fair
21	6	..	6	32	2	poor
22	6	1	7	46	4	fair
23	6	1	8	30	4	fair
24	6	1	7	29	3	fair
25	6	..	5	27	2	fair
26	6	1	6	20	3	fair
27	6	1	5	15	2	poor
28	7	..	5	15	1	poor
29	6	..	5	22	1	poor
30	8	..	5	31	1	fair
31	6	..	4	36	3	fair
Aug. 1	10	..	2	30	2	fair
2	1	..	2	22	..	poor
3	7	..	2	25	4	fair
4	6	..	2	12	2	fair
5	6	1	3	7	2	fair
6	7	..	2	4	2	fair
7	6	1	2	2	2	fair
8	6	..	1	2	2	fair
9	7	2	3	6	2	fair
10	6	..	3	6	4	fair
11	11	..	3	12	2	fair
12	6	1	4	12	1	fair
13	5	..	3	4	2	fair
14	5	..	3	6	2	fair
15	6	1	4	13	1	fair
16	5	..	3	3	1	fair
17	6	1	4	4	2	fair
18	7	1	2	3	1	poor
19	7	..	2	10	2	fair
20	7	..	1	5	..	poor
21	9	1	2	10	..	poor
22	7	..	2	5	..	v.poor
23	8	3	5	16	..	fair
24	6	..	5	12	..	poor
26	4	..	4	30	1	fair
27	7	..	4	30	..	poor

1906	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Det.
Aug. 28	9	1	5	35	..	poor
29	1	..	4	26	..	poor
30	4	2	6	34	2	fair
31	4	..	6	26	2	fair
Sept. 1	5	1	6	22	2	fair
2	6	..	4	14	3	fair
3	5	1	4	15	3	poor
4	5	1	5	16	3	fair
5	6	..	4	7	1	fair
6	7	1	4	16	1	fair
7	7	2	6	35	1	fair
8	7	..	5	26	2	fair
9	7	..	2	11	1	poor
10	7	2	4	9	3	poor
11	7	2	5	11	2	fair
12	4	1	4	30	3	fair
13	8	..	4	22	3	fair
14	9	..	4	22	2	fair
15	7	1	4	31	4	fair
16	7	1	4	40	3	fair
17	5	..	4	37	3	fair
18	7	..	4	36	3	fair
19	7	..	3	23	2	fair
20	12	..	2	14	1	poor
21	7	1	3	20	3	fair
22	7	..	3	9	2	poor
23	7	1	4	14	2	fair
24	7	..	3	13	2	fair
25	7	..	3	7	2	fair
26	7	1	3	5	1	poor
27	7	..	1	4	1	poor
28	7	..	1	4	1	fair
29	7	2	3	8	1	fair
30	9	..	1	5	..	poor
Oct. 1	7	..	1	9	..	fair
2	7	..	1	3	1	poor
3	7	..	1	7	1	poor
4	8	..	1	12	1	poor
5	12	..	1	7	1	poor
6	4	..	0	0	0	poor
7	7	..	0	0	0	poor
8	7	1	1	5	..	fair
9	7	1	2	3	1	poor
10	8	..	2	4	1	fair
11	7	..	1	4	2	fair
12	8	..	1	1	2	fair
13	8	..	..	..	2	fair
14	8	1	1	4	2	fair
15	7	..	1	1	1	fair
16	8	..	..	..	..	poor
17	3	..	..	..	..	poor
18	8	..	..	..	..	poor
20	9	..	..	..	..	poor
22	4	..	..	..	..	poor
23	4	1	1	1	1	fair
25	4	..	..	..	..	fair

1906	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Det.
Oct. 26	8	..	..	..	2	fair
27	11	..	..	..	..	fair
28	8	..	..	..	..	fair
29	8	..	..	..	2	fair
30	8	..	..	..	..	fair
Nov. *1	8	1	1	5	..	fair
*2	8	..	1	5	..	fair
3	8	..	1	9	..	fair
4	8	..	1	8	1	poor
5	8	..	1	10	1	poor
6	8	..	1	7	0	poor
7	8	..	1	4	..	poor
8	8	1	2	5	2	poor
9	8	..	1	12	2	poor
10	8	1	2	14	3	fair
11	5	..	2	14	2	poor
12	8	2	4	25	2	fair
13	8	..	4	30	..	fair
14	8	1	5	11	3	fair
16	8	..	4	9	..	poor
17	8	1	4	6	3	fair
18	8	..	2	8	3	poor
19	8	..	2	4	2	poor
22	8	..	1	2	..	poor
23	8	1	2	6	1	fair
24	8	..	2	8	1	fair
25	8	3	5	12	1	fair
26	8	..	5	15	1	fair
27	8	..	5	22	2	poor
28	3	..	4	24	1	fair
29	8	..	4	22	1	poor
30	8	..	3	24	..	poor
Dec. 1	10	..	3	16	..	poor
2	8	2	5	14	2	fair
3	3	..	5	12	3	fair
4	8	..	4	7	2	poor
*5	9	1	3	6	2	poor
6	2	..	2	4	2	poor
7	8	..	2	4	2	poor
8	8	..	2	3	1	poor
9	8	2	3	4	..	poor
11	8	1	2	9	1	fair
12	8	2	4	11	2	fair
13	8	..	4	20	3	fair
15	8	..	4	36	3	fair
18	8	1	4	50	1	fair
19	8	1	5	42	1	fair
21	9	2	6	80	3	fair
22	9	..	5	30	3	poor
23	3	..	5	27	3	fair
24	8	..	4	16	3	poor
25	3	..	2	11	2	poor
26	8	..	2	9	..	poor
27	8	1	3	26	1	fair
*28	1	..	2	6	..	poor
*29	1	..	1	3	..	poor

* $2\frac{1}{2}$-inch Refractor.

ON THE VARIABLE STAR *RY CASSIOPEIAE*,

By NAOZO ICHINOHE.

This star (28.1906) was discovered by Mme. CERASKI, and announced by Prof. CERASKI in *A.N.* 4077 as a variable, probably having short period. The stars designated by *a*, *b* and *d* are *B.D.* +57°2823, 2828 and 2824, respectively, and their *B.D.* magnitudes are all of them $9^{M}.5$. The star *e* is *B.D.* +57°2825, and its magnitude is $9^{M}.1$ in *B.D.*, but $8^{M}.9$ in *A.G.C.* Now, if I provisionally adopt the latter, then my observations with the value of my step give me the following magnitudes for these stars.

		Steps	Mag.
a	*B.D.* +57°2823	7.4	9.6
b	2821	2.2	10.3
c		0.0	10.5
d	2824	5.8	9.8
e	2825	13.4	8.9

The star *c* is not included in *B.D.*, but this plays a very important role in the observations of this variable.

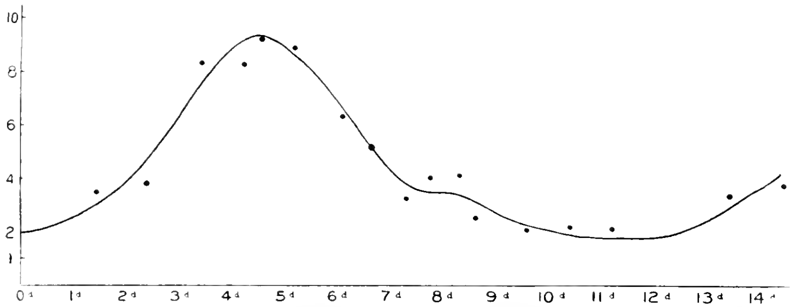
I began to pay attention to this star on October 13, 1906, and observed it as frequently as possible. After a few weeks I found that the period of light-change must be twelve days, nearly. I have now observed the change over four complete periods since the first observation, and fifty-eight estimates obtained in this interval were combined properly, and the following provisional period and light-curve were calculated:

1906 Nov. 21.9 + 12.07 E for maximum.
1906 Nov. 17.2 + 12.07 E for minimum.

t	*B*	*t*	*B*	*t*	*B*
d	s	d	s	d	s
1.52	3.6	5.33	8.9	8.41	4.3
2.51	3.9	6.22	6.4	8.69	2.6
3.59	8.4	6.73	5.3	9.70	2.2
4.38	8.3	7.49	3.3	10.53	2.3
4.69	9.2	7.85	4.2	11.28	2.2

In this table *t* denotes time elapsed after minimum, expressed in days and its decimals, and *B* denotes the brightness, expressed in steps. I plotted these and drew a mean curve through those points, as in the figure. The curve shows that the maximum brightness is nearly $9^{M}.4$, and the minimum $10^{M}.6$, so that the range of change is about 1.2; but I believe that if I observe the star further, the maximum brightness will be increased a little.

LIGHT-CURVE OF *RY Cassiopeiae*.



The general character of this curve is of δ *Cephei* type. The light increases pretty rapidly after it passes the minimum, and reaches its maximum in five days, and then it begins to decrease, rapidly at first, but slower than the rise. At 7.5 days after minimum its magnitude becomes about $10^{M}.4$, and then the decrease of light stops a little while; or becomes very slow, and the star remains at the minimum about three days.

Yerkes Observatory, 1906 *December* 10.

ON A MISSING DM. STAR IN *TAURUS* (15°629),

$\alpha = 4^{h}\,19^{m}\,53^{s}.5$; $\delta = +15°\,28'.4$ (1855).

By PAUL S. YENDELL.

On the evenings of February 22 and March 9 of the current year, while observing 1575 *W Tauri*, I noted the absence of the star DM. 15°629, rated in that catalogue as 9.5 magnitude.

In the winter of 1893–94, on eleven evenings, from December 4 to January 13, the absence of this star was noted, and I have no memory of ever having seen it since, though no further note was made of its being missing until the present season. These observations were all made with my $4\frac{1}{4}$-inch refractor, and the limit of visibility was at the time estimated as below the eleventh magnitude.

From 1896 until the present season, no observations were made in this region.

The observations of February 22 and March 9 were made with an aperture of 6.4 inches, and a power of 50. On March 9, within an hour of missing this star, I glimpsed *U Geminorum*, and held its comparison-star l (KNOTT, $13^{M}.7$) steadily, so that even in the glare of θ^2 *Tauri*, I should probably have seen the star had it been as bright as 11.5 magnitude.

No notice of the absence of this star has ever come to my knowledge. It is not included in CHANDLER's list of Missing *Durchmusterung* Stars (Third Catalogue, p. 172), nor is it mentioned in the errata to the *Durchmusterung*. It must presumably have been as bright as $9^{M}.5$ so late as 1865.

I propose to keep as continuous a watch of the region in the future as possible, in the hope of finding the star again visible.

Dorchester, 1906 *March* 11.

THE PARALLAX OF 61 *CYGNI*,

BY F. L. CHASE.

BERGSTRAND of Upsala, in 1905, in a compressed review of the subject, divided all the parallax determinations of 61 *Cygni* into three classes, covering three distinct periods, namely, 1838–'53, 1853–'80, and 1880–1905. The results obtained prior to BESSEL's heliometer determination are regarded as of no scientific value. The values derived range as follows:

Class I	$\pi = +0''.35$ to $+0''.40$	
Class II	$\pi = +0.43$ to $+0.56$	
Class III	$\pi = +0.21$ to $+0.29$	

to which should be added the photographic determinations, namely:

Class IV $\pi = +0''.29$ to $+0''.43$

Furthermore, WILSING and DAVIS, KAPTEYN and PETERS, all found somewhat different values for the two components, the former two finding larger values for 61 *Cygni*, while the latter two found just the opposite.

The wide disparity of the various results made it seem worth while to add the evidence of an extended investigation with the Yale Heliometer. Accordingly, two pairs of comparison stars were selected, whose position-angles from 61 *Cygni* are respectively about 278° and 101°, and distances 4690″ and 5060″, with magnitudes 6.5 and 8.4 for the first pair, and position-angles 205° and 23°, distances 5990″ and 5840″, and magnitudes 7.5 and 8.4 for the second pair, and in October, 1904, the investigation was begun. I secured twelve nights' observations for the first series at each of the five epochs of maximum parallactic displacement, and from ten to twelve for each of the five corresponding epochs of the second series — in all 230 complete sets.

The method of observation and treatment was the same as set forth in our previously published parallax work. The differences of the two distances for each component of 61 *Cygni* furnished 60 equations of condition for each component in the first series, and 55 for each in the second series, from which to find the parallax and proper motion.

The normals derived from these equations of condition, the values $[nn]$ and $[vv]$, the probable error of one equation, found from the solution, and finally the value of π in seconds of arc, together with its probable error and weight, are as follows:

FIRST SERIES.

		61₁ *Cygni*	61₂ *Cygni*
$+60.00x + 24.40y - 8.73z$	=	$+0''.481$	$= +0''.414$
$+24.40 \quad +178.44 \quad -4.67$	=	$+4.308$	$= +4.079$
$-8.73 \quad -4.67 \quad +30.59$	=	-0.082	$= -0.029$
$[nn]$	=	11.8559	= 10.8939
$[vv]$	=	1.4212	= 1.5203
Prob. Error 1 Equation	=	$\pm0''.135$	$= \pm0''.139$
π	=	$+0.039$	$= +0.295$
Wt. 168.5 ; Prob. Error	=	±0.010	$= \pm0.011$

SECOND SERIES.

		61₁ *Cygni*	61₂ *Cygni*
$+55.00x - 10.83y - 0.04z$	=	$-0''.267$	$= -0''.235$
$-10.83 \quad +175.58 \quad -0.10$	=	$+3.852$	$= +3.873$
$-0.04 \quad -0.10 \quad +25.47$	=	-0.131	$= -0.075$
$[nn]$	=	9.8273	= 10.6905
$[vv]$	=	1.3173	= 2.1263
Prob. Error 1 Equation	=	$\pm0''.136$	$= \pm0''.175$
π	=	$+0.279$	$= +0.281$
Wt. 173.4 ; Prob. Error	=	±0.010	$= \pm0.013$

The accordance of the four values indicates no systematic error for this piece of work, so that we may adopt for the probable errors of the work those derived from the solution. The weighted mean for the four results gives for a final value,

$$\pi = +0''.291 \quad +0''.005 \quad \text{Wt. } 683.9$$

A comparison of different catalogs shows no appreciable proper motion for any of the comparison stars. If we adopt KAPTEYN's values for the parallax of stars equal to them in brightness, the value of the absolute parallax would be 0″.009 larger.

In conclusion, it might be noted that the more recent heliometer determinations of Dr. PETERS, in which the reversing prism eye-piece was used, and the best photographic investigations, which also are presumably fairly free from systematic error, give results not much at variance with that I have derived, and the close agreement of my values for the two components does not confirm DAVIS's inference that the two stars are not physically connected.

Yale University Observatory, 1907 *April* 17.

OBSERVATIONS OF DOUBLE STARS,

MADE AT THE MORRISON OBSERVATORY,

BY HERBERT R. MORGAN.

Star	α (1880)	δ (1880)	Epoch 1906.+	p	s	n
	h m	° ′		°	″	
O. Arg. N 21	0 2.8	+58 58	0.058	144.6	23.61	3
H. 1001	0 3.0	+44 3	.065	77.0	15.75	2
H. 618	0 7.4	− 0 47	.071	248.8	5.01	1
Σ103	1 10.6	− 2 10	.071	246.8	5.54	1
H. 647	1 56.3	+ 7 6	.071	33.8	26.82	1
Σ213 AB	2 1.3	+50 30	.070	321.8	1.97	2
AC	. . .	. . .	.071	63.6	7.14	1
H. 204	2 38.8	+49 37	.068	144.3	3.03	1
Anon.	4 29.0	+ 1 0	.071	179. Est.	6. Est.	1[1]
W.^{1} IV 647	4 31.7	+42 6	.171	110.8	2.48	2
Σ582	4 37.0	+42 13	.266	23.8	5.58	2
W.^{1} IV 1215	4 56.1	+13 11	.068	81.9	4.21	1
W.^{2} V 269 AB	5 11.7	+36 5	.210	329.0	2.80	3
W.^{2} VII 118	7 6.0	+15 23	.295	158.6	2.36	3
DM. +50.1495	7 49.4	+50 35	.295	103.7	3.10	3
DM. −1.1949	7 59.8	− 1 25	.301	179.5	6.71	1
H. 778 (?)	8 6.0	− 1 31	.301	280.0	3.28	1
H. 177	11 3.4	− 2 46	.364	128.0	4.68	1
Σ1518 BC	11 8.3	+ 5 55	.367	351.1	3.24	3
DM. +12.2550	12 25.1	+ 2 46	.365	283. Est.	1.74	2
DM. +24.2711	14 11.1	+24 2	.372	79.9	. .	1
Anon.	. . .	. . .	.355	49.7	4.23	1[2]
DM. +4.3055	15 39.4	+ 4 55	.347	145.5	2.02	3
DM. +36.2640	15 40.2	+35 59	.369	43.4	4.49	3
S.D. −16.4169	15 45.5	−16 52	.464	273.0	2.29	3
DM. +5.3094	15 47.0	+ 5 2	.355	342. Est.	10.8	1
S.D. −17.4630	16 39.3	−17 8	.460	85.4	3.65	3[3]
S.D. −15.4651	17 35.3	−15 40	.546	273.6	4.31	3
H. N. 125	18 21.6	−25 7	.583	281.0	2.92	3
Anon.	19 8.0	+24 28	.635	324.4	5.01	1[4]
Ho. 445	19 8.0	+24 27	.506	242.7	4.71	2[4]
Ll. 38205 A.B.	19 55.5	− 0 32	.583	295.7	2.30	3
AB + C	. . .	. . .	.618	5.3	27.95	1
Skinner	20 39.8	−17 8	.760	295.1	3.70	3
Cin. X, 975	22 13.0	−24 15	.828	357.0	10.36	2
Anon.	22 18.0	+ 0 38	.778	9.8	5.97	2[5]
Harvard	22 22.9	+ 0 40	.772	182.2	3.02	3
Σ2933	22 36.8	+10 22	.790	215.8	3.60	2
Cin. X, 1009	22 36.2	−32 16	.856	34.0	2.78	1
H. 1825	22 47.8	+12 58	.828	220.3	1.95	4
Cin. X, 1022	22 59.3	− 4 54	.835	211.9	. .	1
Anon.	23 18.0	+45 5	.813	182.3	10.25	1[6]
Anon.	. . .	. . .	.813	5. Est.	14. Est.	1[6]
Cin. X, 1078	23 47.0	−22 7	0.856	193.6	45.74	1

NOTES. [1] A pair s.f.; [2] Looking for DM. +24.2711; [3] A pair n.p.; [4] Looking for Mädler 7; [5] Not in DM.; [6] Looking for H. 1825.
The list is from Yerkes II, and Cin. X.

OBSERVATIONS OF COMETS,

MADE AT THE MORRISON OBSERVATORY,

By HERBERT R. MORGAN.

1906 Glasgow M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ	Red. to App. Pl.
				Comet *d* 1906 (Finlay).				
			m s	′ ″	h m s	° ′ ″		s ″
Aug. 21 15 52 1	1	*d* 8 , 8	+0 28.69	−4 32.6	4 36 23.29	+12 19 46.2	*n*9.490 0.638	+1.06 + 0.3
24 15 25 40	2	15 , 6	−0 52.27	+1 30.5	4 56 10.52	+13 48 9.1	*n*9.557 0.639	+1.02 − 0.8
27 15 38 23	3	15 , 8	−0 43.53	+5 49.3	5 14 31.30	+15 3 37.7	*n*9.549 0.623	+1.04 − 1.7
28 14 59 46	4	18 , 8	−0 24.58	+3 8.2	5 20 7.78	+15 25 19.5	*n*9.606 0.645	+1.04 − 1.9
29 15 3 59	5	*d* 8 , 8	−0 6.45	+3 51.5	5 25 44.81	+15 46 44.4	*n*9.604 0.640	+1.05 − 2.1
Sept. 17 15 15 25	6	18 , 8	+1 3.11	+1 35.1	6 48 39.86	+19 35 0.5	*n*9.610 0.602	+1.14 − 5.3
19 15 25 45	7	*d* 8 , 8	−0 10.96	+1 43.3	6 55 25.62	+19 46 20.7	*n*9.595 0.589	+1.15 − 5.6
21 16 6 7	8	6 , 6	+2 14.83	−1 29.6	7 1 58.33	+19 56 20.6	*n*9.526 0.550	+1.18 − 5.9
24 15 18 6	9	12 , 8	−1 0.06	−5 53.4	7 11 1.95	+20 8 21.7	*n*9.601 0.588	+1.20 − 6.5
26 16 16 42	10	15 , 8	+1 56.53	−1 57.0	7 16 55.72	+20 15 19.5	*n*9.494 0.532	+1.24 − 6.6
				Comet *e* 1906 (Kopff).				
Aug. 24 14 32 56	11	*d* 4 , 4	−0 4.39	+0 4.5	22 47 51.96	+10 17 53.6	9.352 0.643	+2.45 +15.6
26 14 53 8	12	12 , 6	−1 32.42	+2 58.9	22 46 20.46	+10 12 8.9	9.440 0.647	+2.52 +16.3
27 14 26 13	13	*d* 7 , 6	−0 10.09	+6 49.9	22 45 35.37	+10 8 53.0	9.376 0.648	+2.56 +16.6
28 14 22 6	14	15 , 8	+0 34.87	+2 54.8	22 44 49.40	+10 5 21.4	9.378 0.648	+2.57 +16.6
29 14 16 16	14	*d* 8 , 8	−0 10.88	−1 0.1	22 44 3.66	+10 1 26.5	9.374 0.649	+2.58 +16.6
Sept. 7 9 6 30	15	*d* 8 , 6	+0 13.38	+0 52.0	22 37 35.80	+ 9 17 49.8	*n*9.441 0.664	+2.60 +17.8
10 10 29 46	17	*d* 8 , 5	+0 5.32	−6 22.5	22 35 30.79	+ 8 59 16.9	*n*8.985 0.649	+2.61 +18.3
13 10 56 0	18	*d* 8 , 8	−0 40.31	−6 50.3	22 33 37.16	+ 8 39 40.2	*n*8.244 0.651	+2.61 +18.5
				Comet *f* 1906 (Thiele).				
Nov. 12 14 30 36	19	12 , 8	−5 29.43	+3 59.1	9 25 36.08	+15 4 53.2	*n*9.573 0.632	+1.75 −12.3
14 15 36 21	20	12 , 8	−4 29.81	+2 0.5	9 35 0.89	+17 38 46.3	*n*9.452 0.560	+1.76 −13.6
17 15 51 58	21	12 , 7	+5 15.72	−0 23.5	9 49 52.13	+21 36 24.5	*n*9.413 0.482	+1.86 −15.7
21 16 29 13	22	11 , 6	−4 12.72	−0 39.3	10 11 42.92	+27 9 19.1	*n*9.339 0.327	+1.86 −18.5
26 16 32 30	23	21 , 8	−0 6.63	+4 4.5	10 42 23.06	+34 8 52.1	*n*9.401 0.073	+1.66 −21.5
27 17 28 33	24	14 , 8	−1 17.55	+0 12.7	10 49 16.84	+35 34 12.8	*n*9.126 9.819	+1.61 −22.1

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	4 35 53.54	+12 24 18.5	A.G. Leipzig	13	22 45 42.90	+10 1 46.5	Munich II
2	4 57 1.77	+13 46 39.4	A.G. Leipzig	14	22 44 11.96	+10 2 10.0	Gl.+Armg.
3	5 15 13.79	+14 57 50.1	Lick, Pub. VI	15	22 37 19.82	+ 9 16 40.0	Comp. with *16
4	5 20 31.32	+15 22 13.2	Lick, Pub. VI	16	22 34 7.84	+ 9 17 12.1	Glasgow
5	5 25 50.21	+15 42 55.0	A.G. Berlin	17	22 35 22.86	+ 9 5 21.1	Glasgow
6	6 47 35.61	+19 33 30.7	A.G. Berlin	18	22 33 44.86	+ 8 46 12.0	Glasgow
7	6 55 35.43	+19 44 43.0	A.G. Berlin	19	9 31 3.76	+15 1 6.4	A.G. Berlin
8	6 59 42.32	+19 57 56.1	A.G. Berlin	20	9 39 28.94	+17 36 59.4	Vienna IX
9	7 12 0.81	+20 14 21.6	A.G. Berlin	21	9 44 34.55	+21 37 3.7	Hedrick
10	7 14 57.95	+20 17 23.1	A.G. Berlin	22	10 15 53.78	+27 10 16.9	A.G. Cambridge
11	22 47 53.90	+10 17 33.5	10M, comp. with *12	23	10 42 28.03	+34 5 9.1	A.G. Leiden
12	22 47 50.36	+10 8 53.7	A.G. Leipzig	24	10 50 32.78	+35 34 22.2	A.G. Lund

Note. Comet *e* generally very faint and hard to measure.

COMET *b* 1907 (*MELLISH, April 14*).

A despatch of Prof. Comstock, *via* Harvard College Observatory, announces the discovery of a comet by Mellish, in the position,

1907 April 14.679 Gr. $\alpha = 6^h\ 40^m$, $\delta = +8°$

with a daily motion 3° east and 7° north.

Despatches from Lick and Washington observatories give observations by AITKEN and RICE, as follows:

	α	δ	
	h m s	° ′ ″	
1907 April 16.5759 Gr.	7 2 53.5	+18 32 44	Rice
April 16.6078	7 3 58.4	+19 3 4	Aitken

A further despatch, dated April 19, communicates elements and ephemeris computed by LAMSON and FREDERICK of the Washington Observatory, but the contents of the telegram are unfortunately indecipherable.

ELEMENTS AND EPHEMERIS FOR (1906 *TE*) *MARIANNA*,

By WILLIAM B. VARNUM, DUDLEY OBSERVATORY.

The following positions of (1906 *TE*) were communicated to me by Rev. J. H. METCALF, the discoverer:

1906	G.M.T.	R.A.	log $p\Delta$	Decl.	log $p\Delta$	C—O	
	h m s	h m s		° ′ ″		s	″
Feb. 16	14 43 40	10 23 51.56	9.444n	+10 24 50.7	0.612	−0.20	+1.9
Feb. 17	14 3 15	10 23 0.74	9.514n	+10 26 43.1	0.584	+0.06	−1.6
Feb. 22	13 2 54	10 18 41.64	8.958	+10 36 0.8	0.725	−0.04	−6.4
Feb. 23	16 1 0	10 17 42.86	8.960n	+10 37 39.1	0.278	+0.02	+0.5
Mar. 14	15 1 30	10 2 26.33	8.534n	+11 6 20.7	0.639	−0.11	−3.9
Mar. 17	14 2 30	10 0 24.70	9.072n	+11 8 58.9	0.634	−0.19	+3.4

With the exception of that for Feb. 22, which is by WOLF, at Heidelberg, the above are revised positions by METCALF, of Taunton, Mass., resulting from positions of the comparison stars determined according to the well-known methods employed at this Observatory.

Using four places, Feb. 16, Feb. 23, March 14 and March 17, the following elliptic elements have been computed:

$$
\begin{aligned}
\text{Epoch} &= 169^\circ\ 19'\ 54''.6 \\
\omega &= 41\ 36\ 47.8 \\
\Omega &= 333\ 7\ 48.6 \\
i &= 15\ 54\ 48.2
\end{aligned}
\left.\vphantom{\begin{aligned}a\\b\\c\end{aligned}}\right\}\ \text{Ecliptic and Mean Ecliptic of 1907.0}
$$

$$
\begin{aligned}
\phi &= 16\ 16\ 0.1 \\
\log a &= 0.490980 \\
\log \mu &= 2.813537 \\
\mu &= 650''.9343
\end{aligned}
$$

The columns (C—O) were obtained by comparing the observed places with an ephemeris computed with the above elements.

For the next opposition, which should occur 1907 April 8, the following ephemeris has been computed:

ELEMENTS AND EPHEMERIS FOR (1906 *TE*) *Marianna*.

1907 G.M.T.	R.A. 1907.0	Decl. 1907.0	Aberr. T.
	h m s	° ′ ″	m s
Mar. 14.5	13 0 14.6	−21 33 38	25 36
18.5	12 57 25.9	−21 32 28	25 22
22.5	12 54 27.8	−21 29 7	25 10
26.5	12 51 22.5	−21 23 39	25 1
30.5	12 48 12.3	−21 16 10	24 53
April 3.5	12 44 59.4	−21 6 47	24 49
7.5	12 41 46.2	−20 55 39	24 46
11.5	12 38 35.2	−20 42 57	24 46
15.5	12 35 28.8	−20 28 56	24 48
19.5	12 32 29.4	−20 13 49	24 53
23.5	12 29 39.1	−19 57 55	25 0
27.5	12 26 59.7	−19 41 30	25 9
May 1.5	12 24 32.8	−19 24 48	25 21
5.5	12 22 19.7	−19 8 7	25 34
9.5	12 20 21.5	−18 51 41	25 49

$+1^m = -9'$. Magn. = 12.6.

CONTENTS.

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DIRECT COMPUTATION OF THE EXPRESSIONS FOR THE COORDINATES IN ELLIPTIC MOTION,

BY F. R. MOULTON.

1. *Introduction.* In the usual solution of the problem of elliptic motion the coordinates are found by the intermediate use of KEPLER's equation, which is often treated by LAGRANGE's method of solving implicit functions of the type to which it belongs, or by BESSEL's functions. In order to make the discussion complete by either method a number of theoretical questions must be considered, and the practical details involve considerable labor.

In this paper it will be shown how the solutions expressed as power series in the eccentricity may be obtained by direct integration of the differential equations. While a beginner might find some difficulties in understanding the complete demonstration of the validity of the processes, he would have, on the other hand, the great advantage of employing in relatively simple problems methods which are used in the more difficult parts of celestial mechanics. It is even doubtful if the difficulties which this method presents are any more serious than those encountered in the standard methods, while it shows the close relation between the problem when there are only two bodies, and that when there are more than two.

2. *Solution of* KEPLER's *Equation.* The method which will be employed in the two-body problem will be illustrated first in the treatment of KEPLER's equation. In the usual notation we have

$$n(t-T) = M = E - e\sin E, \tag{1}$$

where the eccentric anomaly, E, is to be determined in terms of e and M. Some of the properties of the solution will be noted.

(*a*) For any M lying between $\kappa\pi$ and $(\kappa+1)\pi$ there is but one value of E, which also lies between $\kappa\pi$ and $(\kappa+1)\pi$, satisfying (1). For, if we write

$$F(E) = E - e\sin E - M$$

with such a value of M, then $F(\kappa\pi) < 0$ and $F(\kappa+1)\pi > 0$. Moreover, $F'(E) = 1 - e\cos E > 0$. Consequently, since the function is finite and continuous in the interval, it passes through zero once, and but once.

(*b*) If for $M = M_0$ the solution of (1) is $E = E_0$, then for $M = M_0 + 2\pi$ the solution is $E = E_0 + 2\pi$, whatever M_0 may be. Consequently $E - M$ is a periodic function of M.

(*c*) If for $M = M_0$ the solution of (1) is $E = E_0$, then for $M = -M_0$ the solution is $E = -E_0$. Therefore, E is an odd function of M.

(*d*) For all finite values of M only finite real values of E will satisfy (1), and E is a continuous function of M. It follows from (*b*), (*c*) and (*d*) that $E - M$ can be expressed as a FOURIER sine series in M.

The differential equation from which (1) is derived is

$$\frac{dE}{dM} = \frac{1}{1 - e\cos E}$$

Since $e < 1$ the right member of this equation can be expanded as a converging power series in e for all values of E. Thus,

$$\frac{dE}{dM} = 1 + \cos E\, e + \tfrac{1}{2}[1+\cos 2E]e^2 + \tfrac{1}{4}[3\cos E + \cos 3E]e^3 + \tfrac{1}{8}[3 + 4\cos 2E + \cos 4E]e^4 \ .\ .\ .\ . \tag{2}$$

CAUCHY has shown* that a differential equation of the type of (2) can be integrated so as to express the dependent variable as a power series in the parameter, which is e in the present problem. For a preassigned range of values of the independent variable, the series converge for all values of the parameter whose moduli are sufficiently small. The method of CAUCHY gives simply a limit for the modulus of the parameter which is in general much smaller than the true radius of convergence. However, in the present case the true radius of convergence for any special value of M,

* *Comptes Rendus*, Vol. XV (1842), pp. 14, 44 and especially 141; Collected Works, first series, Vol. 7, p. 62; also PICARD's *Traité d'Analyse*, Vol. III, p. 157.

and in particular the smallest true radius of convergence as M takes all values from 0 to 2π, has been found by LAPLACE* and HERMITE.† Or, it can be found by an application of the method employed by the writer ‡ in solving the closely related problem of finding the true radius of convergence when E is expressed as a power series in M. The method has, indeed, been applied by LEVI-CIVITA § and CHARLIER ‖ to the solution of the problem for all values of M. All the methods show that in order that the series may converge, the modulus of e must be less than 0.6627..... LEVI-CIVITA proved also the interesting result that if η is defined by the equation

$$\eta = \frac{e\,\epsilon^{\sqrt{1-e^2}}}{1+\sqrt{1-e^2}}$$

where ϵ is the naperian base, and if the expansion is made in powers of η, then whatever the value of M the series converge for all the values of η corresponding to all the values of e from 0 up to 1. At present we shall use only the fact that for any interval in M (naturally $0\ldots 2\pi$, since the solution is periodic with the period 2π) the series converge for the modulus of e sufficiently small.

According to CAUCHY's thorem the solution of (2) can be expressed in the form

$$E = \sum_{i=0}^{\infty} E_i(M)\, e^i \tag{3}$$

where the E_i are functions of M to be determined so that (2) shall be identically satisfied in M and e, and so that E shall have the prescribed initial value. The solution of (1) for $M=0$ is $E=0$ whatever e may be. Therefore

$$0 = \sum_{i=0}^{\infty} E_i(0)\, e^i$$

for all e sufficiently small, from which it follows that each E_i must separately vanish at $M = 0$.

Substituting (3) in (2), and arranging in powers of e, we have

$$\begin{aligned}\sum_{i=0}^{\infty}\frac{dE_i}{dM}e^i = {} & 1+[\cos E_0]\,e+[\tfrac{1}{2}+\tfrac{1}{2}\cos 2E_0-E_1\sin E_0]\,e^2 \\ & +[-\tfrac{1}{2}E_1^2\cos E_0-E_2\sin E_0-E_1\sin 2E_0+\tfrac{3}{4}\cos E_0 \\ & \qquad +\tfrac{1}{4}\cos 3E_0]\,e^8 \\ & +[-E_1E_2\cos E_0-E_3\sin E_0+\tfrac{1}{6}E_1^8\sin E_0-E_1^2\cos 2E_0 \\ & \quad -E_2\sin 2E_0-\tfrac{3}{4}E_1\sin E_0-\tfrac{3}{4}E_1\sin 3E_0 \\ & \quad +\tfrac{3}{8}+\tfrac{1}{2}\cos 2E_0+\tfrac{1}{8}\cos 4E_0]\,e^4+\ldots\end{aligned}$$

Since the solution must satisfy the differential equation identically in e, the coefficients of corresponding powers of e in the left and right members must be equal. Therefore

$$\begin{aligned}\frac{dE_0}{dM} &= 1 \\ \frac{dE_1}{dM} &= \cos E_0 \\ \frac{dE_2}{dM} &= \tfrac{1}{2}+\tfrac{1}{2}\cos 2E_0-E_1\sin E_0 \\ \frac{dE_3}{dM} &= -\tfrac{1}{2}E_1^2\cos E_0-E_2\sin E_0-E_1\sin 2E_0+\tfrac{3}{4}\cos E_0+\tfrac{1}{4}\cos 3E_0 \\ \frac{dE_4}{dM} &= -E_1E_2\cos E_0-E_3\sin E_0+\tfrac{1}{6}E_1^3\sin E_0-E_1^2\cos 2E_0 \\ &\quad -E_2\sin 2E_0-\tfrac{3}{4}E_1\sin E_0-\tfrac{3}{4}E_1\sin 3E_0+\tfrac{3}{8} \\ &\quad +\tfrac{1}{2}\cos 2E_0+\tfrac{1}{8}\cos 4E_0 \\ & \ldots\ldots\ldots\end{aligned}$$

These equations may be integrated in the order in which they are written, giving, when the condition that $E_i = 0$ at $M = 0$ is applied at each step, the results

$$\begin{aligned}E_0 &= M \\ \frac{dE_1}{dM} &= \cos M, \text{ whence } E_1 = \sin M \\ \frac{dE_2}{dM} &= \cos 2M, \text{ whence } E_2 = \tfrac{1}{2}\sin 2M \\ \frac{dE_3}{dM} &= -\tfrac{1}{8}\cos M+\tfrac{9}{8}\cos 3M, \text{ whence } E_3=\tfrac{1}{8}(-\sin M+3\sin 3M) \\ \frac{dE_4}{dM} &= -\tfrac{1}{3}\cos 2M+\tfrac{4}{3}\cos 4M, \text{ whence} \\ & \qquad E_4 = \tfrac{1}{6}(-\sin 2M+2\sin 4M) \\ & \ldots\ldots\ldots\end{aligned}$$

Hence E becomes

$$\begin{aligned}E = M+[\sin M]\,e+\tfrac{1}{2}[\sin 2M]\,e^2+\tfrac{1}{8}[-\sin M+3\sin 3M]\,e^8 \\ +\tfrac{1}{6}[-\sin 2M+2\sin 4M]\,e^4+\ldots\end{aligned}$$

agreeing with well known results.

It can be established by induction from the equations involved in the process (a) that the expression for each $\frac{dE_i}{dM}$ involves only cosines of multiples (zero excluded) of M, (b) that the sum of its coefficients (for $M = 0$) is unity, (c) that when i is even only even multiples of M occur, and when i is odd only odd multiples of M occur, and (d) that the highest multiple of M is i. Hence, when $E_1, \ldots, E_{i-1}$ have been found, the equation determining E_i will have the form

$$\frac{dE_i}{dM} = A_1^{(i)}\cos M+A_2^{(i)}\cos 2M+\ldots+A_i^{(i)}\cos iM, \text{ where}$$

$A_1^{(i)}+A_2^{(i)}+\ldots+A_i^{(i)}=1$; if i is odd $A_{2j}^{(i)}=0$, $j=0,\ldots,\frac{i-1}{2}$

and if i is even $A_{2j+1}^{(i)}=0$, $j=0,\ldots,\frac{i-2}{2}$

The solution of this equation satisfying the condition that $E_i=0$ at $M=0$ is

$$E_i = A_1^{(i)}\sin M+\tfrac{1}{2}A_2^{(i)}\sin 2M+\ldots+\frac{1}{i}A_i^{(i)}\sin iM$$

Hence it is only necessary to compute the $A_j^{(i)}$ in order to obtain the E_i, and therefore E.

* *Mécanique Céleste*, Vol. 5, Supplement.

† *Cours a la Fac. des Sci. de Paris*, 3d edition, (1886), p. 167.

‡ *Astronomical Journal*, Nos. 537–538, May, 1903.

§ *Rendiconti della R. Accademia dei Lincei*, Vol. XIII, March, 1904.

‖ *Meddelanden från Lunds Ast. Obs.*, No. 22, 1904.

3. *Computation of Polar Coordinates.* It will be shown in this section how the explicit expressions for the polar coordinates may be obtained directly from the differential equations. When the problem is treated in this way the consideration of KEPLER's equation is superfluous.

The differential equations of motion are in polar coordinates

$$(4)\quad \begin{cases} \dfrac{d^2r}{dt^2} - r\left(\dfrac{dv}{dt}\right)^2 + \dfrac{k^2(1+m)}{r^2} = 0 \\ \dfrac{d}{dt}\left(r^2\dfrac{dv}{dt}\right) = 0 \end{cases}$$

The integral of the second equation is

$$(5)\quad r^2\frac{dv}{dt} = c$$

The constants of integration are easily determined in terms of the elements without expressing the coordinates explicitly in terms of the time. It is found that in elliptic orbits

$$c = k\sqrt{(1+m)\,a\,(1-e^2)}$$
$$k^2(1+m) = n^2a^3$$

where

a = the major semi-axis of the orbit,
e = the eccentricity of the orbit,
n = the mean angular velocity in the orbit,
$M = n(t-T)$ = the mean anomaly.

By means of (5) and these relations the first equation of (4) becomes

$$(6)\quad \frac{d^2r}{dM^2} - \frac{a^4(1-e^2)}{r^3} + \frac{a^3}{r^2} = 0$$

When $e = 0$ a solution of this equation is $r = a$. Hence, to find the deviation from circular motion in an elliptic orbit it is convenient to put

$$r = a\,(1+\rho e)$$

Then (6) becomes

$$\frac{d^2\rho}{dM^2} + \frac{(\rho+e)}{(1+\rho e)^3} = 0$$

whence

$$(7)\quad \frac{d^2\rho}{dM^2} + \rho = -(1-3\rho^2)\,e + \rho\,(3-6\rho^2)\,e^2 - \rho^2(6-10\rho^2)\,e^3 \ldots$$

Since in an elliptic orbit r varies between $a(1-e)$ and $a(1+e)$ it follows that ρ varies between -1 and $+1$, and that the right member of (7) converges for all these values of ρ.

Equation (7) is to be integrated as a power series in e of the form

$$(8)\quad \rho = \sum_{i=0}^{\infty} \rho_i\, e^i$$

The process is valid as is shown by CAUCHY's theorem.

Substituting (8) in (7), and arranging according to powers of e, we have

$$(9)\quad \sum_{i=0}^{\infty}\frac{d^2\rho_i}{dM^2}e^i + \sum_{i=0}^{\infty}\rho_i e^i = -[1-3\rho_0^2]\,e + 3\rho_0[1+2\rho_1-2\rho_0^2]\,e^2 + [6\rho_0\rho_2+3\rho_1(1+\rho_1-6\rho_0^2) - \rho_0^2(6-10\rho_0^2)]\,e^3 + \ldots$$

Before solving equation (9) we shall recall some of the properties the solution must possess. Since the body moves in a closed curve so that the law of areas is fulfilled the motion is periodic. If at $M = 0$ the body is at an apse, we shall have $r = a(1-e)$, whence $\rho = -1$, and

$$\frac{dr}{dM} = ae\frac{d\rho}{dM} = 0$$

Since these conditions are true whatever e may be, and since the series (8) converges for e sufficiently small, it follows that at $M = 0$

$$(10)\quad \begin{cases} \rho_0 = -1 \\ \rho_i = 0,\ i = 1, \ldots \infty \\ \dfrac{d\rho_i}{dM} = 0,\ i = 0, \ldots \infty \end{cases}$$

Equating coefficients of corresponding powers of e in the left and right members of (9), we have

$$\frac{d^2\rho_0}{dM^2} + \rho_0 = 0 \qquad (a)$$

$$\frac{d^2\rho_1}{dM^2} + \rho_1 = -1 + 3\rho_0^2 \qquad (b)$$

$$\frac{d^2\rho_2}{dM^2} + \rho_2 = 3\rho_0(1+2\rho_1-2\rho_0^2) \qquad (c)$$

$$\frac{d^2\rho_3}{dM^2} + \rho_3 = 6\rho_0\rho_2 + 3\rho_1(1+\rho_1-6\rho_0^2) - \rho_0^2(6-10\rho_0^2) \qquad (d)$$

. . . .

The only solution of (a) satisfying conditions (10) is

$$\rho_0 = -\cos M$$

Then equation (b) becomes

$$\frac{d^2\rho_1}{dM^2} + \rho_1 = \tfrac{1}{2} + \tfrac{3}{2}\cos 2M$$

The only solution of this equation satisfying the conditions (10) is

$$\rho_1 = \tfrac{1}{2}(1-\cos 2M)$$

Then equations (c) and (d) can be solved in order in a similar manner. These equations and their solutions are easily found to be

$$\frac{d^2\rho_2}{dM^2} + \rho_2 = 3\cos 3M$$
$$\rho_2 = \tfrac{3}{8}(\cos M - \cos 3M)$$
$$\frac{d^2\rho_3}{dM^2} + \rho_3 = -\cos 2M + 5\cos 4M$$
$$\rho_3 = \tfrac{1}{3}\cos(2M - \cos 4M)$$

Substituting these values for the ρ_i in (8), and then back in the expression for r, we find

(11)
$$r = a\{1-\cos M \,.\, e+\tfrac{1}{2}(1-\cos 2M)e^2+\tfrac{3}{8}(\cos M-\cos 3M)e^3 + \tfrac{1}{3}(\cos 2M-\cos 4M)e^4+\ldots\}$$

Making use of the value of r, and the relation between t and M, equation (5) becomes

$$\frac{dv}{dM} = \frac{a^2}{r^2}\sqrt{1-e^2} \tag{12}$$

Substituting for r its value as found in (11) and expanding as a series in e, we find

$$\frac{dv}{dM} = 1+[2\cos M]e+[\tfrac{5}{2}\cos 2M]e^2+[-\tfrac{1}{4}\cos M+\tfrac{13}{4}\cos 3M]e^3 +[-\tfrac{11}{12}\cos 2M+\tfrac{103}{24}\cos 4M]e^4+\ldots$$

Integrating and making $v = 0$ at $M = 0$, we get

(13)
$$v = M+[2\sin M]e+[\tfrac{5}{4}\sin 2M]e^2+[-\tfrac{1}{4}\sin M+\tfrac{13}{12}\sin 3M]e^3 +[-\tfrac{11}{24}\sin 2M+\tfrac{103}{96}\sin 4M]e^4+\ldots$$

4. *Computation of the Rectangular Coordinates.* After the polar coordinates have been computed the rectangular coordinates may easily be found from the equations

$$x = r\cos v \quad , \quad y = r\sin v$$

But they can also be found directly from the differential equations in rectangular coordinates,

$$\frac{d^2x}{dt^2}+k^2(1+m)\frac{x}{r^3} = 0$$
$$\frac{d^2y}{dt^2}+k^2(1+m)\frac{y}{r^3} = 0$$

Passing to the independent variable M these equations become

$$\frac{d^2x}{dM^2}+\frac{a^3x}{r^3} = 0$$
$$\frac{d^2y}{dM^2}+\frac{a^3y}{r^3} = 0$$

These equations admit the circular solution,

$$x = a\cos M \quad , \quad y = a\sin M$$

When the orbit is an ellipse with the same major axis the coordinates may be conveniently represented in the form

$$\begin{aligned} x &= a\cos M + a\,\xi\, e \\ y &= a\sin M + a\,\eta\, e \end{aligned} \tag{14}$$

Substituting the expressions (14) in the differential equations, we get upon expanding as power series in e,

$$(15)\quad \left\{\begin{aligned} &\frac{d^2\xi}{dM^2}-\tfrac{1}{2}[(1+3\cos 2M)\xi+3\sin 2M\,\eta]+[\tfrac{3}{8}(3\cos M+5\cos 3M)\xi^2 \\ &\qquad +\tfrac{3}{8}(\cos M-5\cos 3M)\eta^2+\tfrac{3}{4}\sin M\xi\eta+\tfrac{15}{4}\sin 3M\xi\eta]e+\ldots = 0 \\ &\frac{d^2\eta}{dM^2}-\tfrac{1}{2}[3\sin 2M\,\xi+(1-3\cos 2M)\eta]+[\tfrac{3}{8}(\sin M+5\sin 3M)\xi^2 \\ &\qquad +\tfrac{3}{8}(3\sin M-5\sin 3M)\eta^2+\tfrac{3}{4}\cos M\xi\eta-\tfrac{15}{4}\cos 3M\xi\eta]e+\ldots = 0 \end{aligned}\right.$$

We shall express the solutions of these equations as power series in e of the form,

$$(16)\quad \left\{\begin{aligned} \xi &= \sum_{i=0}^{\infty}\xi_i e^i \\ \eta &= \sum_{i=0}^{\infty}\eta_i e^i \end{aligned}\right.$$

Suppose that at $M = 0$ the body is at its perihelion, that is, that

$$x = a(1-e), \quad \frac{dx}{dM} = 0, \quad y=0, \quad \text{while } \frac{dy}{dM}$$

has the value defined by the areas equation

$$x\frac{dy}{dM}-y\frac{dx}{dM} = a^2\sqrt{1-e^2}$$

Substituting the values of x and y in this equation it is found that, at $M = 0$,

$$\frac{dy}{dM} = a\sqrt{\frac{1+e}{1-e}}$$

Then it is found from equations (14) that, at $M = 0$,

$$\xi = -1, \quad \frac{d\xi}{dM} = 0, \quad \eta = 0, \quad \frac{d\eta}{dM} = \frac{\sqrt{1+e}-\sqrt{1-e}}{e\sqrt{1-e}}$$

With these initial conditions the motion is, as before, periodic with the period 2π, the ξ is a cosine series, and the η is a sine series.

Substituting the expressions (16) back in (15), expanding as power series in e, we find

$$\sum_{i=0}^{\infty}\frac{d^2\xi_i}{dM^2}e^i-\tfrac{1}{2}\left[(1+3\cos 2M)\sum_{i=0}^{\infty}\xi_i e^i+3\sin 2M\sum_{i=0}^{\infty}\eta_i e^i\right]$$
$$= [-\tfrac{3}{8}(3\cos M+5\cos 3M)\xi_0{}^2-\tfrac{3}{8}(\cos M-5\cos 3M)\eta_0{}^2 -\tfrac{3}{4}\sin M\xi_0\eta_0-\tfrac{15}{4}\sin 3M\xi_0\eta_0]e+\ldots$$

$$\sum_{i=0}^{\infty}\frac{d^2\eta_i}{dM^2}e^i-\tfrac{1}{2}\left[3\sin 2M\sum_{i=0}^{\infty}\xi_i e^i+(1-3\cos 2M)\sum_{i=0}^{\infty}\eta_i e^i\right]$$
$$= [-\tfrac{3}{8}(\sin M+5\sin 3M)\xi_0{}^2-\tfrac{3}{8}(3\sin M-5\sin 3M)\eta_0{}^2 -\tfrac{3}{4}\cos M\xi_0\eta_0+\tfrac{15}{4}\cos 3M\xi_0\eta_0]e+\ldots$$

The equations determining ξ_0 and η_0 are

$$\frac{d^2\xi_0}{dM^2} - \tfrac{1}{2}(1+3\cos 2M)\,\xi_0 - \tfrac{3}{2}\sin 2M\;\eta_0 = 0$$

$$\frac{d^2\eta_0}{dM^2} - \tfrac{3}{2}\sin 2M\,\xi_0 - \tfrac{1}{2}(1+3\cos 2M)\;\eta_0 = 0$$

The general solutions of these linear equations with periodic coefficients can be found without difficulty, and the initial conditions imposed. But the general solutions will not have the period 2π, as is known from the general theory of equations of this type, as well as from the fact that the period does not remain unchanged except for special displacements from a circular orbit. It is known from the properties of the orbit that the displacement may be made so that the orbit will be elliptic with the same period. Therefore, since the condition has been imposed that the body is starting from its perihelion, the equations have particular solutions of the form

$$\begin{aligned} \xi_0 &= A_0^{(0)} + A_1^{(0)}\cos M + A_2^{(0)}\cos 2M + \dots \\ \eta_0 &= \qquad\quad B_1^{(0)}\sin M + B_2^{(0)}\sin 2M + \dots \end{aligned}$$

Substituting in the differential equations, and imposing the conditions that they shall be identically satisfied in M, we find

$$A_0^{(0)} = -3A_2^{(0)}\;,\quad A_1^{(0)} = 0\;,\quad B_1^{(0)} = 0,$$
$$B_2^{(0)} = A_2^{(0)}\;,\quad A_i^{(0)} = B_i^{(0)} = 0\;(i>2)$$

From the condition that $\xi_0 = -1$ at $M = 0$ it is found that $A_2 = \frac{1}{2}$. Therefore

$$(17)\qquad \begin{aligned} \xi_0 &= -\tfrac{3}{2} + \tfrac{1}{2}\cos 2M \\ \eta_0 &= \tfrac{1}{2}\sin 2M \end{aligned}$$

The equations which define ξ_1 and η_1 are

$$\begin{aligned} \frac{d^2\xi_1}{dM^2} - \tfrac{1}{2}(1+3\cos 2M)\,\xi_1 - \tfrac{3}{2}\sin 2M\,\eta_1 &= -\tfrac{3}{8}(3\cos M + 5\cos 3M)\,\xi_0^2 \\ &\quad - \tfrac{3}{8}(\cos M - 5\cos 3M)\,\eta_0^2 \\ &\quad - \tfrac{3}{4}(\sin M + 5\sin 3M)\,\xi_0\eta_0 \end{aligned}$$

$$\begin{aligned} \frac{d^2\eta_1}{dM^2} - \tfrac{3}{2}\sin 2M\,\xi_1 - \tfrac{1}{2}(1-3\cos 2M)\,\eta_1 &= -\tfrac{3}{8}(\sin M + 5\sin 3M)\,\xi_0^2 \\ &\quad - \tfrac{3}{8}(3\sin M - 5\sin 3M)\,\eta_0^2 \\ &\quad - \tfrac{3}{4}(\cos M - 5\cos 3M)\,\xi_0\eta_0 \end{aligned}$$

Computing the right members of these equations by means of (17), we obtain

$$\frac{d^2\xi_1}{dM^2} - \tfrac{1}{2}(1+3\cos 2M)\xi_1 - \tfrac{3}{2}\sin 2M\,\eta_1 = +\tfrac{3}{4}\cos M - \tfrac{15}{4}\cos 3M$$

$$\frac{d^2\eta_1}{dM^2} - \tfrac{3}{2}\sin 2M\,\xi_1 - \tfrac{1}{2}(1-3\cos 2M)\eta_1 = +\tfrac{9}{4}\sin M - \tfrac{15}{4}\sin 3M$$

From the properties of the solution which have been given it follows that the solutions of these equations having the period 2π in M, and such that, at $M = 0$,

$$\xi_1 = \eta_1 = \frac{d\xi_1}{dM} = \frac{d\eta_1}{dM} = 0$$

will have the form

$$\begin{aligned} \xi_1 &= A_0^{(1)} + A_1^{(1)}\cos M + A_2^{(1)}\cos 2M + \dots \\ \eta_1 &= B_1^{(1)}\sin M + B_2^{(1)}\sin 2M + \dots \end{aligned}$$

Substituting these expressions in the differential equations, and imposing the conditions that they shall be identically satisfied in M, as well as that, at $M = 0$,

$$\xi_1 = \frac{d\eta_1}{dM} = 0$$

we find

$$A_0^{(1)} = A_2^{(1)} = 0\;,\quad A_1^{(1)} = -\tfrac{3}{8}\;,\quad A_3^{(1)} = +\tfrac{3}{8}\;,\quad A_i^{(1)} = 0\;(i>3)$$
$$B_2^{(1)} = 0\quad B_1^{(1)} = -\tfrac{5}{8}\;,\quad B_3^{(1)} = +\tfrac{3}{8}\;,\quad B_i^{(1)} = 0\;(i>3)$$

Hence

$$\begin{aligned} \xi_1 &= -\tfrac{3}{8}\cos M + \tfrac{3}{8}\cos 3M \\ \eta_1 &= -\tfrac{5}{8}\sin M + \tfrac{3}{8}\sin 3M \end{aligned} \qquad (18)$$

Substituting (17) and (18) back into (16) and (14), we obtain

$$\begin{aligned} x &= a\{\cos M + (-\tfrac{3}{2} + \tfrac{1}{2}\cos 2M)\,e + \tfrac{3}{8}(-\cos M + \cos 3M)e^2 + \dots\} \\ y &= a\{\sin M + \tfrac{1}{2}\sin 2M\,e + \tfrac{1}{8}(-5\sin M + 3\sin 3M)e^2 + \dots\} \end{aligned} \qquad (19)$$

While this process may be extended as far as it may be desired to carry the computations, it is not to be recommended as being convenient in the case of the rectangular coordinates.

When one looks over the ordinary method of treating the two-body problem by solving the implicit relations among the coordinates and time which arise, he will see that it fails if amost any imaginable disturbing accelerations are added. On the other hand, the methods employed in this paper may be used step by step after the addition of almost any disturbing forces, while at least in computing the polar coordinates they are convenient. But their chief value lies in the fact that the two-body problem is considered as a special simple case in the general problems of celestial mechanics, and one whose realm of validity can be completely determined.

The University of Chicago, 1907 *April* 10.

PARALLAX OF α *GEMINORUM* AND σ *DRACONIS*,

By MASON F. SMITH.

Considerable interest attaches to a determination of the parallax of *α Geminorum* as a bright binary of appreciable proper motion, for which no reliable result has been obtained. The only published values we are aware of are those of JOHNSON, +0″.20 and FLINT, −0″.17. So I have. in the past two years, taken it up with the Yale Helio-

meter, securing two series with different pairs of comparison stars of 20 observations each, the approximate Distances, Position-Angles and Magnitudes being:

	Distances	Position-Angles	Magnitudes
Series I	4302″ and 4055″	274°.1 and 83°.4	8.0 and 7.0
Series II	4197 and 3354	229.8 and 53.5	8.0 and 7.5

The measures were made and reduced according to the methods followed in all our parallax work, pointing on the brighter component α_2. Six, eight and six sets were secured at three successive parallax maximum epochs, leading to the following Normals and Solutions:

Series I.

Normals:
$$\begin{aligned} +20.00x + 7.83y + 5.79z &= +0.007^{\text{R}} \\ +7.83 \quad +69.12 \quad +3.47 &= +0.090 \\ +5.79 \quad +3.47 \quad +4.59 &= -0.012 \end{aligned}$$

$[nn] = 0.00270 \quad [vv] = 0.00254$

Solution:
$$\begin{aligned} x &= +0.0014^{\text{R}} \pm 0.0022^{\text{R}} \quad \text{Wt. } 12.5 \\ y &= +0.0014 \pm 0.0010 \quad 65.6 \\ z &= -0.0055 \pm 0.0046 \quad 2.9 \end{aligned}$$

Prob. Error 1 Eq. $= \pm 0''.099$

$$\pi = +0''.018 \pm 0''.012$$

Series II.

Normals:
$$\begin{aligned} +20.00x + 5.88y + 5.80z &= +0.056^{\text{R}} \\ +5.88 \quad +33.61 \quad +2.59 &= +0.075 \\ +5.80 \quad +2.59 \quad +4.77 &= -0.019 \end{aligned}$$

$[nn] = 0.00369 \quad [vv] = 0.00300$

Solution:
$$\begin{aligned} x &= +0.0056^{\text{R}} \pm 0.0024^{\text{R}} \quad \text{Wt. } 12.7 \\ y &= +0.0022 \pm 0.0015 \quad 31.6 \\ z &= -0.0121 \pm 0.0048 \quad 3.1 \end{aligned}$$

Prob. Error 1 Eq. $= \pm 0''.108$

$$\pi = +0''.028 \pm 0''.019$$

The mean result by weights of both pairs is:

$$\pi = +0''.022 \pm 0''.010$$

and the agreement of the two values does not imply any systematic error. According to Kapteyn, the parallax of the comparison stars is $+0''.008$.

For *σ Draconis*, a corroboration of the large values derived by Brünnow, $+0''.24$, and by Peter, $+0''.17$, seemed worth while to secure. Accordingly a series was undertaken with a pair of comparison stars for which the data are:

Distances	Position-Angles	Magnitudes
6272″ and 4371″	271°.5 and 81°.3	8.1 and 8.0

At three successive maximum parallax epochs six, nine and six sets were secured, furnishing the following Normals and Solution:

Normals:
$$\begin{aligned} +21.00x - 5.32y + 6.29z &= -0.127^{\text{R}} \\ -5.32 \quad +74.96 \quad -0.86 &= +1.436 \\ +6.92 \quad -0.86 \quad +4.57 &= -0.025 \end{aligned}$$

$[nn] = 0.03283 \quad [vv] = 0.00529$

Solution:
$$\begin{aligned} x &= -0.0012^{\text{R}} \pm 0.0031^{\text{R}} \quad \text{Wt. } 12.1 \\ y &= +0.0191 \pm 0.0013 \quad 73.4 \\ z &= -0.0000 \pm 0.0066 \quad 2.7 \end{aligned}$$

Prob. Error 1 Eq. $= \pm 0''.139$

$$\pi = +0''.243 \pm 0''.016$$

Kapteyn's parallax of the comparison stars would be $+0''.008$. It would therefore seem that a parallax of about $0''.2$ is well assured for this star.

Yale University Observatory.

OBSERVATIONS OF COMET *a* 1907 (*Giacobini*),

MADE WITH THE 16-INCH REFRACTOR AT THE GOODSELL OBSERVATORY, NORTHFIELD, MINN.

By H. C. WILSON.

[Communicated by the Director, Wm. W. Payne.]

1907 Northfield M.T.	⁎	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log pΔ		Red. to App. Pl.	
h m s			m s	′ ″	h m s	° ′ ″			s	″
Mar. 12 10 36 14	1	6 , 4	−1 9.60	+0 53.1	6 55 16.43	−15 15 26.6	9.487	0.850	+0.27	−15.7
15 10 5 55	2	12 , 6	+0 39.47	+0 11.9	6 48 0.55	−12 36 14.8	9.459	0.845	+0.14	−15.3
15 10 6 38	3	6 , 2	+3 2.95	−2 14.9	6 48 0.46	−12 36 14.6	9.461	0.845	+0.14	−15.3
Apr. 1 8 37 57	4	12 , 6	+0 50.56	−1 8.6	6 21 47.07	− 0 25 31.1	9.459	0.790	−0.34	−11.9
5 8 18 24	5	10 , 6	−0 44.02	+2 41.7	6 18 21.10	+ 1 46 45.7	9.499	0.778	−0.43	−11.2
8 9 3 18	7	21 , 6	−0 0.31	+0 55.3	6 16 14.73	+ 3 19 2.5	9.551	0.763	−0.50	−10.5

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	6 56 25.76	−15 16 4.0	Bonn VI, 201	5	6 19 5.55	+1 44 15.2	Microm. comp. with 6
2	6 47 20.94	−12 36 11.4	Microm. comp. with 3	6	6 20 29.37	+1 33 11.4	Göttin. 1316, Schj. 2187
3	6 44 57.39	−12 33 44.4	Lalande 13179	7	6 16 15.57	+3 18 17.7	B.D. VI, +3°1205
4	6 20 56.85	− 0 24 10.6	Schjellerup 2194				

NOTES ON SOME LONG-PERIOD VARIABLE STARS.

By A. STANLEY WILLIAMS.

The following notes give the chief results derived from my last year's observations of long-period variable stars, and are in continuation of those which have previously been published in the *Astronomical Journal*. The observations were all made with a 6½-inch reflector, the usual magnifying power being 73. The dates of maxima and minima have all been derived from single curves.

RU Andromedae.

R.A. = $1^h\,32^m\,47^s$; Decl. = +38° 9′.5 (1900).

Observations made on 16 nights between, July 24 and Oct. 23, give 1906 Aug. 1 (J.D. 2417424) for the date of maximum ($9^M.9$). This date may not be very exact, there being only two observations a little before the maximum brightness. The above date is 12 days earlier than that computed from the elements published in the *A.J.*, No. 586, p. 80.

562. *Y Andromedae.*

A well observed maximum occurred on 1906 Sept. 23 (J.D. 2417477). Magnitude = 8.1. Observations were secured on 27 nights, between July 27 and Dec. 7. The decline was slower than the rise, with a not very pronounced hump in the light-curve about Oct. 23. The above date of maximum is 38 days later than the date given by Dr. Hartwig in the *V.J.S. Ephemerides*.

RV Andromedae.

R.A = $2^h\,4^m\,34^s$; Decl. = +48° 27′.6 (1900).

A very sharply defined and well observed maximum occurred on 1906 Sept. 1 (J.D. 2417455), and a less sharply defined minimum on Nov. 3 of the same year (J.D. 2417518). The brightness at maximum was $8^M.2$, and at minimum $10^M.6$. Twenty-two observations, between 1906 July 24 and 1907 Jan. 17. The rise after minimum was slow, with a standstill about the end of December, so that the light-curve about this time somewhat resembles that at the end of the preceding year (See *A.J.*, No. 586, p. 80).

1205. *Y Persei.*

Observations were made on 18 nights, between 1906 Aug. 19, and 1907 Jan. 17, and these give 1906 Sept. 26 (J.D. 2417480) for the date of maximum. This date is 36 days earlier than that computed from Dr. Graff's elements,* but the decline after maximum is only very slight, and there are indications of a secondary maximum in the early part of December. In the case of a light-curve of this form it may well be questioned, it seems to me, whether the slightly brighter, though unsymmetrically situated, first maximum should be adopted as being the actual date of maximum. Rather, as Professor H. H. Turner is recently reported to have said (*The Observatory*, April 1907, p. 157), we should take to calculating the areas of the curves and the centres of gravity. The brightness of *Y Persei* at the recent maximum was $8^M.6$. The strong color of this star renders the observations difficult and uncertain, particularly when it is bright.

RU Persei.

R.A. = $3^h\,23^m\,57^s$; Decl. = +39° 18′.9 (1900).

This variable passed a faint ($10^M.0$) and not very sharply defined maximum on 1906 Oct. 29 (J.D. 2417513). Twenty observations were made between Aug. 19 and 1907 Jan. 17, but the results are somewhat discordant. On the last mentioned date the star was nearly at its minimum brightness.

6827. *RT Lyrae.*

A well defined maximum occurred on 1906 Aug. 20 (J.D. 2417443), the same date as that computed from the elements of Dr. Graff.† The brightness at maximum was $10^M.0$. Thirteen observations, between July 24 and Nov. 11.

6895. *RU Lyrae.*

Thirteen observations, between July 24 and Nov. 11, give 1906 Aug. 25 (J.D. 2417448) for the date of maximum. This date is 12 days earlier than that computed from the elements published in the *Monthly Notices*, Vol. 66, p. 432, but the maximum was an unusually flat one, for this star, the decline in particular being abnormally slow. The maximum was also a faint one, the star not rising above $11^M.0$.

* *Mitteilungen der Hamburger Sternwarte*, No. 8, p. 24.
† *Mitteilungen der Hamburger Sternwarte*, No. 8, p. 44.

7019. *TY Cygni.*

A fairly well defined maximum occurred 1906 Sept. 19 (J.D. 2417473), from 17 observations, between July 24 and Nov. 11. This date is 8 days later than that calculated from the elements in the *Monthly Notices*, Vol. 66, p. 433. The maximum brightness was only $9^{M}.4$, which is the faintest maximum hitherto observed.

7571*a*. *TW Cygni.*

Six observations, between March 29 and June 17, indicate 1906 April 26 (J.D. 2417327) as the date of maximum ($9^{M}.0$). This date, which is not very exactly determined, owing to the paucity of observations, is 17 days earlier than the date computed from the elements published in the *Monthly Notices*, Vol. 66, p. 433. The existence of a somewhat large periodic irregularity is thus confirmed.

RV Pegasi.

R.A. = $22^h\,21^m\,2^s$; Decl. = $+29°\,57'.9$ (1900).

According to observations made on 28 nights, between July 24 and Dec. 7, a maximum occurred on 1906 Sept. 22 (J.D. 2417476), if regard be had chiefly to the increasing and decreasing phases. The light-curve shows, however, an unsymmetrical hump, indicating for the actual instant of greatest observed brightness (about $9\frac{1}{2}^{M}$) a date 6 days later than that already mentioned. The former date appears to be the preferable of the two,* for throughout the time when the star was bright the observations are somewhat discordant; and although these discordances may be due merely to errors of observation, yet they have every appearance of being real, and due to real minor fluctuations in brightness of the variable. The hump above mentioned would seem therefore to be probably due merely to a temporary minor fluctuation in brightness. The increase in brightness of the star was very rapid. On July 24 it was only just visible in a $6\frac{1}{2}$-inch reflector (13^{M}), whilst by the middle of August it had already risen to 10^{M}.

The previous year the maximum occurred, according to Prof. A. J. Nijland,† on August 12, and according to the writer's observations,‡ on Aug. 5, 1905. Taking the mean of the two dates, and comparing it with the recent maximum, we get the following approximate elements of variation,

$$\text{Maximum} = \left\{\begin{array}{c}\text{1905 August 8.5}\\ \text{J.D. 2417066.5}\end{array}\right\} + 409^{d}.5\ E$$

* See *Astr. Nach.*, No. 4116, col. 183.

† *Astronomical Journal*, No. 586, p. 81.

‡ Since the foregoing was written, Prof. Nijland has published the results of his recent observations of long-period variables in the *A. N.*, No. 4164, and gives 1906 Sept. 13, as the date of maximum of *RV Pegasi*; curve flat.

20 *Hove Park Villas, Hove*, 1907 *April* 10.

THE PROPER MOTION OF THE STAR B.D. −21°1377,

By J. G. Porter.

This faint star appears to have the unusually large motion in declination of $-0''.73$ per year.

In Schönfeld's *Durchmusterung* it is indicated as having been observed in the Washington Zones, but I fail to find it there.

The star first occurs in the Cincinnati Zone Catalogue, and two years later it was observed at Northfield. Two recent observations, by Mr. Elliott Smith, with the meridian circle of this Observatory, establish beyond doubt the fact of motion.

The observations reduced to 1900 are as follows:

	Epoch	R.A. h m s	Obs.	Decl. ° ′ ″
Cincinnati Z.	1886.07	6 6 21.95	5	−21 49 17.5
Northfield	1888.12	21.82	2	21.0
Cin. M.C.	1907.14	6 6 21.83	2	−21 49 33.0

Cincinnati, 1907 *March* 5.

CORRIGENDA.

Page 141, col. 1, l. 29 and p. 142, col. 1, l. 7, *for* Peters *put* Peter.
" 141, col. 1, l. 31, *for* 61 *Cygni* *put* 61_2 *Cygni.*
" 141, col. 2, l. 30, *for* +0.039 *put* 0.309.
" 141, col. 2, bottom line, *for* $+0''.005$ *put* $\pm0''.005$.

CONTENTS.

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THE

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LATITUDE DETERMINATIONS AT THE FLOWER OBSERVATORY DURING THE YEARS 1904, 1905 AND 1906,

BY C. L. DOOLITTLE.

The latitude series begun at this place in 1896 was regarded as closed Dec. 28, 1903.

On January 9, 1904, work was begun on a more extended program, embracing two series of observations carried on simultaneously.

The Zenith Telescope had been thoroughly overhauled, and the 4-inch objective replaced by one of $5\frac{1}{2}$ inches. An accident had ruined the micrometer screw, which was accordingly replaced by a new one. For practical purposes this was therefore a new instrument.

A Reflex Zenith Tube of 8 inches aperture, alike in principle with the Greenwich instrument, was installed during the summer of 1904. Means for this were furnished by Mr. Joseph Wharton. It is therefore known as the Wharton instrument.

Some experimental work was done during the autumn, and a number of changes were made which proved necessary or desirable.

Regular work was begun Dec. 13, 1904. Both instruments are used by myself. On reasonably favorable nights stars of the 8th magnitude, or fainter, can be readily observed with either.

A program is therefore possible which closely approaches the ideal one in excluding pairs of large zenith distance, and those requiring large micrometer corrections. In the present case very few of the latter much exceed 5′.

The groups — four in number — as heretofore, are distributed as follows in right-ascension:

I	$5^h 42^m - 7^h 58^m$
II	12 24 – 15 19
III	16 44 – 19 19
IV	20 52 – 23 22

As suitable pairs for the Zenith Telescope are much more numerous than available zenith stars for the Wharton Instrument, the latter are first selected — 8 to 10 for each group — and the Zenith Telescope pairs are then fitted into the intervals. With the possibility of employing stars as faint as the 8th magnitude this generally offers but little difficulty.

The actual arrangement is as follows:

α represents a zenith telescope pair.

β represents a zenith star.

Group	I	$\beta, \alpha, \beta, \alpha, \beta, \beta, \alpha, \beta, \alpha, \alpha, \alpha, \alpha, \alpha, \alpha, \alpha, \beta, \beta, \beta.$
	II	$\alpha, \alpha, \alpha, \alpha, \alpha, \alpha, \beta, \alpha, \alpha, \beta, \beta, \beta, \beta, \alpha, \alpha, \beta, \beta, \beta.$
	III	$\alpha, \beta, \beta, \alpha, \alpha, \alpha, \alpha, \beta, \beta, \alpha, \beta, \alpha, \beta, \beta, \alpha, \alpha, \beta, \alpha, \beta, \alpha.$
	IV	$\beta, \beta, \beta, \beta, \beta, \beta, \alpha, \alpha, \alpha, \alpha, \beta, \alpha, \alpha, \alpha, \alpha, \beta, \beta, \alpha, \alpha, \beta.$

Each group also contains one pair, and one zenith star suitably chosen for investigating the temperatnre coefficient.

During the year 1904, before the Wharton Instrument was ready for use, work was carried on with the Zenith Telescope. The zenith stars were included, and were reduced as a separate series. At first each group included seven of these, but afterwards the number was increased to eight or ten.

There has been some delay in completing this work, owing to want of accurate places for a considerable number of the stars employed.

This would be of little importance for present purposes except for the possibility of large proper motions in some of them. Mr. TUCKER has, however, very kindly observed all of the weaker ones — 78 stars in all — and some positions have been courteously furnished in advance of publication by the Director of the U.S. Naval Observatory.

The observations have all been made by myself. Most of the computation has been carried out by Miss EDITH D. KAST.

The results are as follows:

1904		Z. Pairs			Z. Stars			
	I	No.	II	No.	I	No.	II	No.
	"		"		"		"	
Jan. 9	1.95	6	..	..	1.84	3	..	..
14	2.07	9	..	..	1.92	6	..	..
15	1.93	10	..	..	1.83	7	..	..
17	2.14	10	..	..	2.08	6	..	..
18	2.18	10	..	..	2.01	6	..	..
19	2.12	5	..	..	2.11	4	..	..
24	1.92	10	2.16	5	2.14	7	2.09	4
25	2.09	10	2.03	10	1.87	7	1.78	7
26	2.07	1	2.08	9	2.20	2	1.84	7
27	2.17	9	1.93	7	2.13	7	1.72	6
30	2.15	10	..	..	1.99	4	..	..
Feb. 1	2.11	10	1.98	10	2.00	7	2.00	7
2	..	..	..	..	2.00	1	..	..
3	1.99	8	2.08	10	1.98	3	2.11	6
4	1.97	10	..	..	1.94	7	..	..
6	1.99	9	..	..	2.03	6	..	..
8	2.17	10	2.22	9	2.04	7	2.04	7
9	2.23	10	1.98	4	1.89	7	..	..
11	2.22	9	2.21	10	1.99	7	2.25	5
12	..	..	2.17	10	..	..	1.97	7

1904		Z. Pairs			Z. Stars			
	I	No.	II	No.	I	No.	II	No.
	"		"		"		"	
Feb. 13	2.14	4	..	..	2.08	3	..	..
15	2.13	10	2.33	10	2.04	7	2.06	6
16	2.17	10	2.03	8	1.98	6	1.76	6
20	1.96	10	2.21	8	1.95	7	2.05	3
22	2.16	9	2.04	9	1.93	7	1.86	4
24	2.04	10	..	..	1.86	7	..	..
25	2.06	10	2.12	9	1.97	7	2.20	4
27	..	..	2.09	5	..	..	..	..
Mar. 1	2.17	9	2.09	4	1.86	7	..	..
4	2.26	10	2.15	9	2.15	7	1.92	7
5	2.17	7	..	..	1.91	3	..	..
8	1.83	1	..	..	2.00	2	..	..
9	2.18	10	2.22	10	1.91	7	1.99	7
13	2.20	10	2.19	9	1.81	7	2.07	7
15	2.04	6	..	..	1.99	3	..	..
16	2.04	10	2.17	10	1.98	7	2.03	7
18	..	..	2.08	10	..	..	2.01	7
19	1.97	4	..	..	1.74	2	..	..
20	2.19	9	2.15	8	1.97	6	1.94	7

1904		Z. Pairs			Z. Stars			
	II	No.	III	No.	II	No.	III	No.
	"		"		"		"	
April 5	2.21	10	..	..	1.95	7	..	..
11	2.06	3	..	..	..	..	..	..
13	2.16	7	..	..	2.28	4	..	..
14	2.22	10	..	..	2.10	7	..	..
16	2.18	9	..	..	2.02	7	..	..
18	2.15	10	..	..	1.94	7	..	..
19	2.10	10	..	..	2.14	7	..	..
May 2	2.33	10	2.27	7	2.07	7	2.21	7
3	2.16	9	..	..	1.78	2	..	..
4	2.23	10	2.16	8	2.20	6	2.24	6
5	2.13	10	2.00	10	2.13	7	2.08	7
6	2.19	10	..	..	2.02	7	..	..
7	2.17	10	2.25	9	2.13	7	2.20	7
8	2.20	9	..	..	2.18	6	..	..
13	2.31	10	2.41	10	2.31	6	2.26	6
15	..	..	2.10	10	..	..	2.10	6

1904		Z. Pairs			Z. Stars			
	II	No.	III	No.	II	No.	III	No.
	"		"		"		"	
May 19	2.02	2	..	..	..	..	..	..
20	2.31	9	2.18	6	2.23	5	2.27	5
21	2.08	7	2.25	10	2.25	7	2.07	7
22	1.98	8	..	..	1.66	1	..	..
24	2.22	8	..	..	2.25	2	..	..
25	2.36	3	..	..	..	..	..	..
27	2.29	10	2.01	10	2.25	7	2.02	7
28	2.24	10	2.24	10	2.20	7	2.11	7
29	2.15	9	2.27	3	1.86	5	2.07	5
June 3	2.27	7	2.27	9	2.36	5	2.25	4
6	2.36	6	..	..	2.03	2	..	..
8	2.28	7	2.27	6	2.16	7	2.29	5
10	2.62	6	2.37	10	2.41	7	2.27	7
11	2.13	6	2.53	10	2.24	6	2.36	7
12	..	..	2.22	6	..	..	2.29	5

1904		Z. Pairs			Z. Stars			
	III	No.	IV	No.	III	No.	IV	No.
	"		"		"		"	
June 13	2.14	8	..	..	2.22	5	..	..
14	2.20	10	..	..	2.25	6	..	..
15	2.01	3	..	..	2.04	3	..	..
18	2.24	10	..	..	2.10	6	..	..
22	2.35	9	..	..	2.42	6	..	..
23	2.10	9	..	..	2.13	7	2.25	1
24	2.18	10	2.08	6	2.20	6	2.30	4
25	2.04	4	2.10	8	2.11	4	2.16	5
27	2.22	6	..	..	2.33	6	2.50	1
July 2	2.15	10	2.30	8	2.16	7	2.19	5
3	2.17	10	1.91	1	2.19	6	2.20	4
4	2.20	8	..	..	2.43	4	..	..
8	2.20	10	.	..	2.31	7	2.48	2
9	2.15	5	..	..	2.51	3	..	..
11	2.23	9	..	..	2.40	5	2.30	2
13	2.29	10	2.24	9	2.52	6	2.33	6
14	2.23	8	2.28	6	1.99	5	2.21	4

1904		Z. Pairs			Z. Stars			
	III	No.	IV	No.	III	No.	IV	No.
	"		"		"		"	
July 15	2.06	6	..	..	2.16	2	..	..
16	2.16	10	2.33	10	2.12	7	2.25	7
17	..	..	..	..	2.36	2	..	..
18	2.25	8	2.11	10	2.31	6	2.26	6
20	2.15	9	2.20	10	2.13	6	2.20	7
24	2.25	5	2.15	6	2.38	4	2.19	4
29	2.43	9	2.40	9	2.28	7	2.36	6
31	2.04	10	..	..	2.06	7	2.28	2
Aug. 3	2.22	9	..	..	2.14	5	..	..
4	2.03	6	..	..	2.35	6	..	..
6	2.06	2	2.32	9	..	..	2.34	6
11	2.32	8	2.33	10	..	..	2.48	7
13	..	..	..	..	2.02	1	..	..
14	2.16	6	2.35	10	2.14	1	2.22	6
15	2.50	10	2.31	10	2.29	7	2.51	7
18	2.36	10	2.36	10	2.52	5	2.17	7
20	2.21	3	..	..	..	..	..	..

	Z. Pairs				Z. Stars			
1904	III	No.	IV	No.	III	No.	IV	No.
	″		″		″		″	
Aug. 21	2.38	10	2.31	10	2.53	5	2.50	6
23	2.33	10	2.42	10	2.52	3	2.16	7
24	2.26	10	2.20	10	2.10	5	2.25	7
25	2.25	9	2.24	10	2.20	5	2.19	7
26	2.36	9	2.26	10	2.48	4	2.35	7
27	2.32	4	2.26	10	..	..	2.01	7
28	2.06	9	2.08	9	2.08	4	2.25	7
29	..	..	2.28	10	..	..	2.24	6

	Z. Pairs				Z. Stars			
1904	III	No.	IV	No.	III	No.	IV	No.
	″		″		″		″	
Aug. 30	..	..	2.28	10	..	..	2.22	6
Sept. 4	..	..	2.46	7	..	..	2.18	4
5	..	..	2.30	8	..	..	2.24	3
7	..	..	2.28	9	..	..	2.33	7
8	..	..	2.14	2	..	..	2.05	3
10	..	..	2.27	10	..	..	2.27	6
11	..	..	2.31	7	..	..	2.22	6

	Z. Pairs				Z. Stars			
1904	IV	No.	I	No.	IV	No.	I	No.
	″		″		″		″	
Sept. 26	2.28	6	..	..	2.27	3	..	..
28	..	..	..	..	2.08	2	..	..
30	2.20	8	..	..	2.18	7	..	.
Oct. 1	2.27	9	..	..	2.16	7	..	..
3	2.21	10	..	..	2.13	7	..	..
4	2.27	10	.	..	2.00	7	..	..
7	2.14	10	..	..	2.13	7	..	..
9	2.26	9	..	..	2.20	6	..	..
10	2.08	9	..	..	2.22	7	..	..
14	2.31	9	2.19	10	2.09	7	2.20	4
15	2.17	10	2.17	10	2.04	6	2.10	4
16	2.20	10	2.31	10	2.22	7	2.13	5
17	2.17	10	2.24	9	2.00	7	2.26	5
18	2.25	8	2.32	10	2.09 ·	7	2.19	5
19	2.19	6	..	..	2.11	4	..	..
21	2.08	10	2.10	9	2.24	6	2.15	7
22	2.18	9	2.06	9	2.05	7	2.22	6
23	..	..	2.27	10	..	..	2.15	5
24	2.23	10	2.12	8	2.23	6	2.09	6
25	2.20	2	..	..	2.07	3	..	..
26	..	..	2.24	10	..	..	1.97	7

	Z. Pairs				Z. Stars			
1904	IV	No.	I	No.	IV	No.	I	No.
	″		″		″		″	
Oct. 27	2.21	10	2.06	10	2.19	7	2.15	7
28	2.23	8	2.08	10	1.98	7	2.30	6
29	2.22	10	2.16	9	2.17	7	2.11	7
30	2.16	10	2.15	10	2.10	7	2.22	7
31	2.12	8	2.23	7	2.11	6	2.08	5
Nov. 3	1.97	9	..	..	2.19	5	..	..
7	2.18	10	2.00	10	1.90	6	2.08	6
11	1.94	10	1.93	10	2.05	6	1.96	7
14	2.09	10	2.05	10	2.19	6	1.88	6
15	1.96	10	2.23	8	2.10	7	1.94	6
16	..	..	2.15	10	..	..	1.91	7
17	2.23	10	2.12	10	2.00	7	2.16	7
19	2.08	5	2.07	10	2.15	6	1.96	6
20	1.80	2	..	..	..	..	..	..
21	2.05	10	2.15	10	..	..	..	..
23	2.09	9	..	..	..	..	..	..
24	2.18	6	..	..	..	..	..	..
27	2.00	9	2.10	10	..	..	..	..
28	2.14	10	..	..	..	..	..	..
30	..	..	2.11	10	..	..	..	.
Dec. 4	..	..	2.03	9	..	..	..	..

	Z. Tel.				Wharton			
1904	I	No.	II	No.	I	No.	II	No.
	″		″		″		″	
Dec. 13	..	..	..	..	2.15	7	..	..
16	2.10	10	..	..	2.22	5	..	..
18	2.01	8	..	..	2.29	3	..	..
19	2.05	10	..	..	2.05	8	..	..
21	2.03	10	..	..	1.76	8	..	..
31	1.98	9	..	..	2.32	5	..	..
1905 Jan. 1	2.00	10	..	..	2.03	8	..	..
4	2.11	10	..	..	1.93	8	..	..
8	2.03	10	..	..	..	..	..	..
15	1.90	10	..	..	2.02	8	..	..
16	1.92	10	..	..	2.22	8	..	..
17	2.02	7	..	..	..	..	..	..
18	..	..	..	..	2.36	1	..	..
22	2.20	10	1.68	3	2.20	7	2.16	6
26	1.82	10	1.99	4	1.87	8	2.35	1
28	1.98	10	2.09	10	2.12	7	2.08	7
30	2.02	10	2.00	8	1.94	8	2.22	3
31	2.09	10	2.02	3	2.01	8	..	..
Feb. 2	2.01	9	1.94	9	2.12	8	2.14	7
4	1.97	10	2.11	9	2.11	8	2.12	8
6	..	..	2.08	10	..	..	2.03	7
7	1.95	10	2.14	5	1.85	7	..	..
10	1.82	10	2.02	10	1.79	8	1.96	8

	Z. Tel.				Wharton			
1905	I	No.	II	No.	I	No.	II	No.
	″		″		″		″	
Feb. 14	1.98	10	..	..	2.24	8	..	..
15	2.04	10	1.90	9	2.01	8	2.05	8
17	2.07	2	..	..	..	..	..	..
18	1.76	10	2.06	10	2.16	8	1.94	8
21	2.00	10	..	..	2.10	7	..	..
23	1.92	8	1.99	10	1.95	3	2.06	8
24	1.93	10	2.03	10	1.87	8	1.95	8
26	2.04	10	1.77	10	2.03	8	2.02	8
27	1.97	10	..	..	2.25	8	..	..
28	1.95	9	..	..	2.01	5	..	..
Mar. 1	..	..	2.01	10	..	..	1.95	8
2	1.90	10	1.92	10	1.92	7	1.94	7
3	2.12	7	..	..	2.05	4	..	..
4	1.93	8	2.03	10	2.02	8	2.06	8
6	1.99	10	..	..	2.19	8	..	..
10	1.91	9	2.01	10	2.28	7	1.87	8
11	1.94	5	..	..	2.07	5	..	..
13	2.07	10	2.10	9	2.07	8	2.32	7
14	2.28	4	2.00	10	2.13	4	2.30	8
15	2.10	10	2.01	5	2.09	8	2.12	2
16	2.11	10	..	..	2.11	3	..	..
18	1.95	9	1.96	5	..	..	..	..

1905		Z. Tel.				Wharton			
		II	No.	III	No.	II	No.	III	No.
		″		″		″		″	
April	16	2.07	6	1.9	..	..	..	..	..
	17	1.92	10	..	..	2.13	8	..	..
	18	2.16	10	..	..	2.13	8	..	..
	19	2.03	10	..	..	2.00	7	..	..
	22	1.97	10	..	..	2.25	8	..	..
	23	1.92	10	..	..	2.07	8	..	..
	24	2.08	10	..	..	2.25	8	..	..
	25	2.05	10	..	..	2.25	8	..	..
	30	2.11	10	2.12	11	2.16	8	2.21	8
May	1	2.22	10	2.09	9	2.19	8	2.13	7
	2	2.06	9	2.05	7	2.33	8	2.30	4
	3	2.12	9	..	..	2.37	5	..	..
	7	2.18	10	2.14	11	2.11	8	2.24	8
	8	2.03	10	2.18	4	2.26	7	2.36	2
	12	2.18	4	..	..	2.08	2	..	..

1905		Z. Tel.				Wharton			
		II	No.	III	No.	II	No.	III	No.
		″		″		″		″	
May	18	2.31	2	2.10	11	2.21	7	2.18	9
	19	2.16	10	2.24	8	2.20	8	2.24	8
	20	2.13	10	2.13	11	2.20	8	2.29	9
	22	2.26	7	..	..	2.35	3	..	..
	23	2.07	10	2.23	11	2.16	8	2.26	9
	24	2.08	10	2.19	11	2.28	8	2.15	8
	25	2.24	9	2.15	9	2.30	8	2.26	7
	27	2.34	2	..	..	2.35	1	..	..
	28	2.28	7	2.16	9	2.35	8	2.32	6
	29	2.10	8	2.28	3	2.27	3	2.04	1
June	1	2.05	10	2.23	10	2.27	8	2.33	9
	3	2.23	10	2.20	11	2.34	8	2.30	9
	5	2.10	4	..	..	1.99	2	..	..
	8	2.24	7	2.14	11	2.28	8	2.38	8
	9	2.18	7	2.17	9	2.28	8	2.23	7

1905		Z. Tel.				Wharton			
		III	No.	IV	No.	III	No.	IV	No.
		″		″		″		″	
June	13	2.15	10	..	..	2.39	7	..	..
	14	2.19	9	..	..	2.22	3	..	..
	17	2.11	10	..	..	2.40	5	..	..
	25	2.22	11	..	..	2.32	6	..	..
	28	2.21	11	2.16	8	2.20	9	..	..
	29	2.07	11	..	..	2.23	8	..	..
	30	2.22	6	..	..	2.50	4	..	..
July	3	2.14	11	2.02	7	2.19	8	2.36	3
	9	2.23	11	2.01	4	2.27	9	2.34	4
	11	2.21	11	2.12	10	2.30	9	2.29	9
	12	1.98	11	..	..	2.33	9	2.11	2
	15	2.35	11	2.22	10	2.20	9	2.19	9
	16	2.31	10	2.17	10	2.24	8	2.10	8
	17	2.25	11	2.27	10	2.33	9	2.43	9
	18	..	..	2.38	7	..	..	2.45	8
	19	2.19	5	2.33	10	2.26	4	2.49	8
	20	2.32	11	2.31	10	2.28	9	2.38	7
	24	2.24	11	..	..	2.41	9	2.30	5

1905		Z. Tel.				Wharton			
		III	No.	IV	No.	III	No.	IV	No.
		″		″		″		″	
July	25	2.25	11	2.26	10	2.39	9	2.48	9
	26	2.25	11	2.18	10	2.54	9	2.36	9
	27	2.19	11	2.09	10	2.72	7	2.52	7
	30	2.34	3	2.19	4	2.09	3	2.44	6
Aug.	2	2.22	11	2.19	4	2.53	9	2.31	9
	3	2.26	10	..	..	2.26	8	..	..
	5	2.27	9	..	..	2.36	7	..	..
	11	2.15	11	..	..	2.31	9	..	..
	15	2.28	4	..	..	2.27	2	..	..
	17	2.26	10	2.31	10	2.31	8	2.32	9
	18	2.29	10	2.34	9	2.36	8	2.50	8
	19	2.27	10	2.38	10	2.38	8	2.40	9
	21	2.31	10	2.21	10	2.36	5	2.29	7
	22	..	..	..	..	2.21	3	..	..
	23	2.26	9	2.41	10	2.41	5	2.40	9
	26	2.30	10	2.24	10	2.41	7	2.21	8
	27	2.26	3	2.32	6	2.71	5	..	..

1905		Z. Tel.				Wharton			
		IV	No.	I	No.	IV	No.	I	No.
		″		″		″		″	
Aug.	28	..	..	..	..	2.11	1	..	..
	29	2.28	9	..	..	2.49	8	..	..
Sept.	4	2.38	5	..	..	1.99	3	..	..
	5	2.21	10	..	..	2.34	9	..	..
	6	2.26	10	..	..	2.33	6	..	..
	8	2.23	10	..	..	2.48	9	..	..
	9	2.28	10	..	..	..	..	..	..
	18	2.44	10	..	..	..	..	..	..
	21	2.17	10	..	..	2.56	10	..	..
	22	2.15	10	..	..	2.21	9	..	..
	23	2.27	10	..	..	2.23	10	..	..
	24	2.22	10	..	..	2.26	10	..	..
	25	2.05	10	..	..	2.14	10	..	..
	26	2.24	10	..	..	2.40	10	..	..
	27	2.11	10	..	..	2.67	10	..	..
	28	2.22	10	..	..	2.43	10	..	..
	29	2.27	10	..	..	2.44	9	..	..
	30	2.08	10	..	..	2.42	10	..	..

1905		Z. Tel.				Wharton			
		IV	No.	I	No.	IV	No.	I	No.
		″		″		″		″	
Oct.	1	2.18	9	..	..	2.27	8	..	..
	4	2.14	10	..	..	2.47	9	..	..
	5	2.16	10	..	..	2.35	9	..	..
	6	2.36	10	..	..	2.35	8	..	..
	7	2.29	10	2.32	4	2.61	10	..	..
	8	2.18	10	2.32	4	2.55	10	2.50	4
	9	2.22	10	2.26	6	2.69	10	2.39	5
	13	2.30	10	2.16	8	2.32	10	2.47	5
	16	2.24	10	2.18	9	2.24	10	2.30	4
	17	2.32	10	..	..	2.28	10	..	..
	18	2.31	1	..	..	2.56	2	..	..
	21	2.25	10	2.23	9	2.47	10	2.44	7
	22	2.03	10	2.20	10	2.49	7	2.50	8
	23	2.15	9	..	..	2.54	10	..	..
	26	2.39	7	..	..	2.48	10	..	..
	28	2.18	10	2.25	10	2.52	10	2.39	8
	29	2.36	6	2.21	10	2.53	4	2.54	8
	30	2.24	10	2.26	10	2.44	10	2.42	7

1905	Z. Tel. IV	No.	I	No.	Wharton IV	No.	I	No.
Oct. 31	2.31	6	..	..	2.43	8	..	..
Nov. 1	2.20	10	2.19	10	2.41	10	2.22	8
2	2.26	10	..	..	2.71	10	..	..
3	..	..	2.18	9	..	..	2.50	8
4	2.06	6	2.17	10	2.44	5	2.38	8
6	2.09	1	2.12	9	2.66	1	2.62	7
8	..	..	1.94	2	..	..	..	..
9	2.30	6	2.16	9	2.36	6	2.52	8
10	2.20	3	2.08	10	2.65	3	2.53	8
11	2.17	10	2.16	9	2.44	10	2.48	8
12	2.10	10	2.08	10	2.53	9	2.36	8

1905	Z. Tel. IV	No.	I	No.	Wharton IV	No.	I	No.
Nov. 14	2.15	10	2.21	10	2.40	10	2.39	8
16	..	..	2.23	9	..	..	2.52	7
17	2.10	10	..	..	2.38	10	2.80	1
18	..	..	..	..	2.19	1	..	..
19	..	..	2.26	10	..	..	2.48	8
20	2.07	10	2.18	10	2.26	10	2.56	8
21	2.06	10	2.20	9	2.30	9	2.38	8
22	2.21	10	2.22	10	2.58	7	2.30	8
23	2.04	10	2.16	10	2.26	7	2.27	8
25	2.15	10	..	..	2.19	8	..	..
27	2.07	10	..	..	2.16	8	..	..

1905	Z. Tel. I	No.	II	No.	Wharton I	No.	II	No.
Dec. 17	2.17	10	..	..	2.18	8	..	..
23	1.88	1	..	..	2.14	2	..	..
24	2.17	10	..	..	2.08	8	..	..
25	1.94	10	..	..	2.24	8	..	..
26	2.04	8	..	..	2.19	6	..	..
27	2.14	2	..	..	2.39	2	..	..
30	2.14	10	..	..	2.11	8	..	..
1906 31	2.10	10	..	..	2.24	7	..	..
Jan. 1	2.06	10	..	..	2.04	8	..	..
5	2.07	1	..	..	2.04	1	..	..
6	2.08	10	..	..	2.24	8	..	..
9	2.07	10	..	..	2.15	8	..	..
10	2.14	10	..	..	2.07	8	..	..
24	1.97	10	..	..	2.22	8	..	..
25	2.01	9	..	..	2.20	8	..	..
29	1.94	10	1.90	10	2.08	8	2.10	8
30	2.08	8	..	..	2.25	8	..	..
31	1.99	10	2.07	10	2.19	8	2.15	8
Feb. 2	2.13	6	2.06	8	2.04	7	1.90	1
3	..	..	2.07	8	..	..	2.28	8
4	1.98	10	..	..	2.23	6	..	..
5	2.08	10	2.14	9	2.38	8	2.23	8
6	2.02	2	..	..	2.16	3	..	..
7	1.77	3	2.17	4	2.54	1	..	..
9	1.93	4	1.93	10	1.99	3	2.08	8

1905	Z. Tel. I	No.	II	No.	Wharton I	No.	II	No.
Feb. 10	2.05	10	2.17	9	2.01	8	2.06	8
11	2.08	10	2.01	9	2.19	8	2.17	7
13	1.99	10	..	..	2.05	8	..	..
15	1.97	10	1.99	10	2.09	8	1.99	8
16	2.02	10	2.00	10	2.19	7	2.26	8
17	2.03	10	2.04	1	2.05	8	..	..
19	2.00	10	2.02	10	2.15	7	2.08	8
22	2.01	10	2.10	10	2.15	8	2.23	8
23	2.04	10	2.09	10	2.12	8	2.22	7
24	1.93	10	2.06	6	2.17	8	2.28	1
26	1.99	10	..	..	2.16	8	..	..
28	2.15	10	1.89	10	1.98	6	2.04	7
Mar. 1	2.09	4	..	..	2.15	7	..	..
5	2.05	10	2.01	10	2.17	8	2.22	8
6	1.99	8	..	..	2.33	4	..	..
9	..	..	1.96	10	..	..	2.11	8
10	2.03	10	2.08	10	2.10	8	2.22	8
12	..	..	..	..	2.33	1	..	..
17	2.07	9	2.09	10	2.05	7	2.10	8
18	2.04	10	2.16	2	2.21	7	..	..
20	2.06	9	1.99	10	2.12	7	2.19	8
21	..	..	2.17	6	..	..	2.42	1
22	2.06	8	2.01	10	2.15	5	2.10	7
23	..	..	2.11	8	..	..	2.19	6

1906	Z. Tel. II	No.	III	No.	Wharton II	No.	III	No.
April 22	1.94	2	..	..	..	..	..	..
24	1.90	10	..	..	2.05	8	..	..
25	2.04	10	1.90	10	2.09	8	2.04	7
26	1.96	10	2.01	4	2.19	8	2.19	6
27	1.91	8	2.05	11	2.12	8	2.24	9
28	2.10	10	2.03	11	2.03	8	2.15	7
30	1.80	2	..	..	..	..	..	..
May 3	1.84	2	1.99	11	2.24	6	2.01	9
4	1.95	10	1.94	8	2.04	8	2.28	6
8	1.95	9	..	..	2.07	8	..	..
10	1.86	6	2.07	11	2.11	8	2.04	9
11	2.16	10	2.01	10	2.17	8	2.22	8
12	1.98	9	1.96	11	2.31	8	2.24	8
15	2.07	10	2.05	10	2.20	8	2.19	9
16	2.10	5	..	..	2.22	3	..	..
19	2.16	10	2.11	11	2.30	8	2.26	9
20	2.08	10	2.11	11	2.24	8	2.15	9

1906	Z. Tel. II	No.	III	No.	Wharton II	No.	III	No.
May 21	1.96	8	..	..	2.17	6	..	..
22	1.94	10	1.97	11	2.11	8	2.24	9
23	..	..	1.96	10	..	..	2.27	8
24	2.13	10	2.07	11	2.23	8	2.22	9
25	1.99	9	2.07	10	2.16	8	2.04	8
26	2.04	9	..	..	2.23	3	..	..
29	2.02	10	2.01	11	2.24	8	2.26	9
30	2.22	4	..	..	..	..	..	..
June 2	2.09	9	2.09	11	2.24	8	2.25	9
3	..	..	2.05	10	..	..	2.22	9
4	2.00	10	..	..	2.38	8	..	..
11	..	..	2.14	11	..	..	2.27	6
12	..	..	2.13	8	..	..	2.30	5
14	..	..	2.01	11	..	..	2.28	8
19	..	..	1.97	4	..	..	2.37	3
24	..	..	2.19	11	..	..	2.22	9
25	..	..	..	..	..	..	2.53	1

1906		Z. Tel. III	No.	Z. Tel. IV	No.	Wharton III	No.	Wharton IV	No.
		"		"		"		"	
July	5	2.02	11	1.98	10	2.35	7	2.23	7
	6	1.99	11	2.18	6	2.30	9	2.32	8
	7	2.11	10	..	..	2.26	9	..	..
	9	2.17	11	1.99	10	2.27	9	2.37	10
	12	2.03	10	2.29	10	2.63	2	2.34	10
	13	2.22	11	2.23	10	2.39	8	2.47	10
	14	2.03	10	..	..	2.30	9	..	..
	15	2.12	5	2.06	9	2.19	1	2.22	10
	18	2.16	10	..	..	2.47	7	..	..
	19	2.04	10	..	..	2.50	6	2.62	3
	20	2.05	8	..	..	2.26	4	..	..
	21	2.15	11	..	..	2.34	9	2.41	3
	24	2.37	4	..	..	2.31	2	..	..
	25	2.24	11	..	..	2.40	8	2.56	2
	26	2.20	11	2.22	10	2.42	8	2.46	10
	28	2.14	5	..	..	2.62	2	..	..
	30	..	..	2.12	7	..	..	2.36	4
Aug.	4	2.17	5	2.20	10	2.51	6	2.36	10
	5	2.26	11	2.23	9	2.41	9	2.35	9
	6	2.26	4	2.16	6	2.70	3	2.37	9
	7	..	..	2.13	6	..	..	2.29	8
	9	2.23	11	2.16	3	2.42	9	2.54	7
	11	2.29	10	2.18	3	2.39	8	..	..
	12	2.23	4	..	..	2.56	5	..	..
	13	2.42	5	2.29	2	2.21	3	2.42	6
	14	2.18	11	2.23	10	2.41	9	2.38	9
	15	2.10	8	2.14	8	..	..	2.50	9
Aug.	17	2.24	7	..	..	2.33	6	..	..
	18	2.19	4	2.02	2	2.18	3	2.44	6
	19	2.16	6	2.20	8	2.48	7	2.50	7
	20	2.35	3	..	..	2.24	1	..	..
	22	2.08	10	..	..	2.41	7	..	..
	23	2.24	8	..	..	2.64	5	..	..
Sept.	5	..	..	2.07	10	..	..	2.38	9
	6	..	..	2.11	10	..	..	2.38	10
	7	..	..	2.18	8	..	..	2.36	10
	8	..	..	..	..	..	..	2.30	5
	9	..	..	2.14	10	..	..	2.33	9
	10	..	..	2.27	10	..	..	2.54	10
	15	..	..	2.33	9	..	..	2.33	9
	16	..	..	2.14	5	..	..	2.23	5
	17	..	..	2.24	10	..	..	2.56	8
	18	..	..	2.23	6	..	..	2.28	8
	19	..	..	2.08	9	..	..	2.44	9
	21	..	..	2.21	5	..	..	2.30	2
	23	..	..	2.22	10	..	..	2.47	8
	24	..	..	2.23	10	..	..	2.34	10
	25	..	..	2.31	10	..	..	2.23	10
Oct.	6	..	..	2.13	7	..	..	2.45	7
	7	..	..	2.14	10	..	..	2.26	8
	8	..	..	..	..	..	..	2.29	5
	11	..	..	2.21	10	..	..	2.33	10
	12	..	..	2.29	10	..	..	2.23	9

1906		Z. Tel. IV	No.	Z. Tel. I	No.	Wharton IV	No.	Wharton I	No.
		"		"		"		"	
Oct.	13	2.30	10	2.26	8	2.38	10	2.21	5
	14	2.14	10	2.13	7	2.30	10	2.27	5
	15	..	..	2.41	1	..	..	2.36	3
	16	2.31	4	..	..	2.39	6	..	..
	23	2.08	6	..	..	2.40	5	..	..
	25	2.23	10	2.27	10	2.49	10	2.24	8
	26	2.25	10	..	..	2.46	10	..	..
	27	..	..	..	..	2.34	2	..	..
	28	..	..	2.13	9	..	..	2.33	7
	29	2.21	9	2.20	10	2.55	10	2.40	7
	30	2.39	5	..	..	2.42	8	..	..
	31	..	..	2.34	10	..	..	2.50	8
Nov.	1	2.26	10	2.25	10	2.33	10	2.29	8
	2	2.15	10	2.36	10	2.48	10	2.62	8
	3	2.08	6	2.21	10	2.36	9	2.49	8
	4	2.10	10	2.25	10	2.34	10	2.39	8
	5	2.35	3	2.23	10	2.40	2	2.26	8
Nov.	6	2.25	10	2.15	10	2.41	10	2.41	8
	7	2.17	10	2.18	10	2.36	10	2.35	8
	8	2.25	9	2.31	10	2.41	10	2.49	8
	13	..	..	2.18	8	..	..	2.44	8
	14	2.37	9	..	..	2.48	10	..	..
	16	2.18	10	2.32	10	2.40	10	2.45	8
	22	2.15	10	2.29	6	2.30	8	2.19	8
	23	2.20	9	..	..	2.36	8	..	..
	24	2.21	10	2.22	10	2.27	8	2.32	8
	25	2.24	10	2.22	10	2.42	8	2.40	8
	26	..	..	..	..	2.46	2	..	..
	28	2.20	4	2.17	10	2.39	6	2.44	8
	29	2.26	10	2.14	10	2.26	7	2.42	8
Dec.	1	..	..	2.25	10	..	..	2.46	7
	2	2.23	9	..	..	2.35	6	..	..
	3	..	..	2.39	8	..	..	2.62	8

$\varphi = 39° 58' +$

Weighted Mean Date	Pairs	No.	Z. Stars	No.
	"		"	
1904 Jan. 16	2.071	50	1.966	32
29	2.041	128	1.968	88
Feb 16	2.141	171	1.994	107
Mar. 12	2.152	136	1.971	93
Apr. 14	2.164	59	2.063	39
May 10	2.201	166	2.163	111
June 3	2.261	141	2.203	96
17	2.202	49	2.222	26
1904 July 6	2.204	154	2.250	116
28	2.231	120	2.270	81
Aug. 22	2.292	189	2.284	108
Sept. 5	2.296	63	2.232	41
Oct. 4	2.208	71	2.145	53
19	2.202	179	2.124	114
Nov. 4	2.099	179	2.088	124
21	2.106	121	2.048	33
		1982		1259

$\varphi = 39° 58' +$

	Z. Tel.	No.	Wharton	No.	W—T
1904 Dec. 31	2.021	104	2.069	69	+048
1905 Feb. 1	2.004	160	2.044	116	040
21	1.960	148	2.043	103	083
Mar. 10	2.012	151	2.084	110	072
Apr. 21	2.018	76	2.159	55	+141
May 9	2.126	156	2.226	124	+100
30	2.174	175	2.280	137	106
June 24	2.162	68	2.315	42	153
July 13	2.202	149	2.295	125	093
27	2.229	136	2.412	122	+183
Aug. 20	2.289	142	2.356	110	+067
Sept. 13	2.258	104	2.341	75	083
30	2.186	99	2.395	93	209
Oct. 15	2.224	140	2.451	122	227
Nov. 2	2.214	164	2.472	147	+258
Nov. 18	2.154	187	2.382	161	+228
1905 Dec. 26	2.098	61	2.183	49	085
1906 Jan. 12	2.063	60	2.151	49	088
Feb. 5	2.032	160	2.158	124	126
21	2.018	171	2.127	130	+109
Mar. 15	2.046	140	2.160	101	+114
May 4	1.994	210	2.144	175	150
26	2.045	195	2.220	152	175
June 16	2.100	45	2.278	32	178
July 12	2.100	162	2.345	129	+245
Aug. 8	2.198	233	2.420	209	+222
Sept. 11	2.195	78	2.385	83	190
Oct. 1	2.210	81	2.338	78	128
24	2.235	119	2.384	114	149
Nov. 6	2.249	165	2.409	153	160
25	2.231	146	2.383	126	+152
		4185		3415	

Total, ... 10841

For the Constant of Aberration we find:

Zenith Telescope					
1904	20.545	±008	Zenith Stars	20.544	±012
1905	20.492	±008	Wharton	20.450	±010
1906	20.514	±008	"	20.473	±010

The probable error of a single determination of latitude is found to be as follows:

Latitude Pairs

1904	±0.112
1905	± .116
1906	±0.120

Zenith Stars

Zenith Telescope	
1904	±0″.133
Wharton Instrument	
1905	±0″.135
1906	±0 .132

It is doubtful whether the first year's results from the Wharton Instrument are entitled to very much confidence. Various sources of annoyance were more or less in evidence, particularly during the early part of the year. The most troublesome were as follows:

(*a*) Unsatisfactory illumination. It is necessary to adjust the small mirror which reflects the light into the field every evening. The result is a different effect from one evening to another. Experience has overcome this difficulty in part. It is proposed to try a different plan, which it is hoped may be more satisfactory.

(*b*) Difficulty in keeping the instrument in focus. This trouble has practically disappeared as the instrumental peculiarities became better understood.

(*c*) A source of much annoyance, particularly on humid nights, is the condensation of moisture on the various optical surfaces. The under side of the reflecting prism in front of the objective has been very troublesome, as it can not be reached for the purpose of removing the moisture. Mr. Brashear has suggested a means of overcoming this difficulty, which it is proposed to try.

One of the questions on which it was hoped that some light might be thrown by this investigation, relates to apparent fluctuations of a systematic character which occasionally effect the work of a whole evening. The agreement or disagreement of the result given by both instruments should show whether or not these changes from day to day are real.

A comparison of residuals, found by subtracting the daily latitudes from the mean values given above, resulted as follows. Values from less than four observations are excluded.

There are 409 cases available for comparison.

No. of cases where one residual or the other is zero,		51
Like signs for both series,		182
Unlike,		176
Both residuals greater than 0″.20	like signs, 1	unlike, 0
Between 0″.20 and 0″.10	14	8
" .10 .05	66	59
" .05 .01	101	109

The large residuals seem to show systematic deviations to a limited extent. No such evidence, however, is shown by those smaller than 0″.05.

1906 *April* 16.

ELEMENTS AND EPHEMERIS OF COMET *c* 1907 (*Giacobini, June 1*).

(From *A.N.* 4183.)

Elements by Dr. E. Strömgren.

T = 1907 May 31.2079 Berlin

$$\left.\begin{aligned}\omega &= 39°\,35'.12\\ \Omega &= 160\;52.25\\ i &= 14\;50.98\end{aligned}\right\}1907.0 \qquad \log q = 0.0942$$

Ephemeris for Berlin Midnight, by M. Ebell.

1907	α	δ	log Δ	Br.
	h m s	° ′		
June 29	12 17 18	+17 31.2	0.0115	0.80
July 3	12 34 20	16 5.0	0.0194	0.74
7	12 51 1	14 34.2	0.0286	0.69
11	13 7 15	+13 0.2	0.0391	0.63

OBSERVATIONS OF COMETS AND MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL OF THE VASSAR COLLEGE OBSERVATORY,

BY MARY W. WHITNEY AND CAROLINE E. FURNESS.

1906 Greenwich M.T.	⁎	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.
				Comet *b* 1906.					
Apr. 13 15h 53m 8s	1	*11, 11	+0m 7.20s	+5′ 37.8″	11h 21m 7.76s	+ 2° 28′ 52.5″	9.073	0.746	+1.12s − 8.8″[1]
16 13 54 40	1	*12, 10	−0 20.70	+6 12.7	11 20 39.84	+ 2 29 27.4	n8.929	0.746	+1.10 − 8.8[1]
17 15 50 24	1	10, 8	−0 29.62	+6 19.8	11 20 30.91	+ 2 29 34.6	9.156	0.746	+1.09 − 8.7[1]
18 15 4 23	1	10, 10	−0 36.12	+6 17.7	11 20 24.41	+ 2 29 32.6	8.810	0.745	+1.09 − 8.6[2]
				(65) *Cybele.*					
Apr. 27 15 3 49	2	10, 10	−0 47.75	+6 14.8	15 3 57.36	−12 30 54.3	n9.452	0.834	+1.58 − 1.9[2]
28 15 4 47	3	10, 8	+1 49.96	+5 24.9	15 3 15.96	−12 27 11.2	n9.438	0.836	+1.60 − 2.1[1]
				(42) *Isis.*					
Sept. 14 16 1 44	4	10, 10	−0 31.43	+3 40.1	23 30 33.18	−22 5 47.6	n9.026	0.895	+2.59 +17.2[2]
15 14 34 2	4	10, 10	−1 18.56	+0 20.7	23 29 46.06	−22 9 7.1	n9.422	0.875	+2.60 +17.1[2]
25 13 41 34	5	*10, 10	−0 14.88	+9 15.2	23 21 54.00	−22 25 47.7	n9.438	0.874	+2.63 +16.0[2]
27 13 53 6	6	10, 8	+0 46.11	+1 32.5	23 20 30.04	−22 25 0.7	n9.375	0.881	+2.63 +15.8[2]
				(47) *Aglaia.*					
Oct. 8 14 49 24	7	10, 8	−0 47.40	−3 28.3	0 48 12.47	+ 6 42 40.3	n9.301	0.710	+2.80 +16.2[2]
16 13 26 52	8	10, 10	+0 41.11	−1 27.9	0 41 31.91	+ 6 15 57.8	n9.238	0.712	+2.81 +16.8[2]
				Comet *g* 1906 (Thiele)					
Nov.†16 22 34 8	9	–, 4	. . . . .	+4 39.8	. . . . .	+20 17 45.9	. .	0.506	+1.80 −15.0[1]
16 22 47 19	9	* 3, –	+0 16.10	. . . .	9 44 57.96	. . . . . .	n8.351	. .	+1.80 −15.0[1]
22 20 7 3	10	15, 10	+0 42.38	−3 47.7	10 16 52.99	+28 24 28.4	n9.550	0.466	+1.77 −19.2[2]
24 19 38 36	11	10, 10	+0 55.66	−2 59.4	10 28 49.43	+31 11 12.6	n9.615	0.470	+1.73 −20.4[2]
26 21 13 49	12	*12, 12	+0 23.94	+0 45.9	10 41 58.52	+34 3 39.7	n9.432	0.219	+1.67 −21.6[2]

Observer, [1] MARY W. WHITNEY. Observer, [2] CAROLINE E. FURNESS.

Mean Places of Comparison-Stars for the beginning of the year.

⁎	α	δ	Authority	⁎	α	δ	Authority
1	11h 20m 59.44s	+ 2° 23′ 23.5″	Albany, A.G. 4271	7	0h 48m 57.07s	+ 6° 45′ 52.4″	Leipzig II, A.G. 306
2	15 4 43.53	−12 37 7.2	Paris 1875, 18742	8	0 40 47.99	+ 6 17 8.9	Leipzig II, A.G. 249
3	15 1 24.40	−12 32 34.0	Radcliffe 1890, 3903	9	9 44 40.06	+20 13 21.1	Berlin B, A.G. 3853
4	23 31 2.02	−22 9 44.9	Cincinnati 1890, 1955	10	10 16 8.84	+28 28 35.3	Camb.(Eng.), A.G. 5314
5	23 22 6.25	−22 35 18.9	Oe. Arg. 17945	11	10 27 52.04	+31 14 32.4	Leiden, A.G. 4217
6	23 19 41.30	−22 29 49.0	Arg. Gen. 31711	12	10 41 32.91	+34 3 15.4	Leiden, A.G. 4275

* $\Delta\alpha$ observed directly. † Daylight prevented completion of observation.

COMET *d* 1907.

Various telegraphic dispatches, received from Harvard College Observatory, communicate the information given below. The comet was found on June 9, at Princeton, by Mr. ZACCHEUS DANIEL, who obtained the approximate position on that date given below.

June 9.804	23h 48m 33s	−1° 8′ –″	Daniel
13.9679	23 59 14.1	−0 10 16	Aitken
14.8206	0 2 10.0	+0 1 43	Hammond
14.9679	0 2 35.2	+0 3 18	Aitken
15.8236	0 5 4.3	+0 16 27	Hammond

Dr. R. T. CRAWFORD communicated elements computed by EINARSON and GLANCY, Students' Observatory, at Berkeley, California, from observations on June 13, 15 and 19, as follows:

$$T = 1907 \text{ Sept. } 3.76 \text{ Gr.}$$

$$\omega = 294^\circ 47'$$
$$\Omega = 143\ \ 2$$
$$i = 9\ \ 9 \qquad q = 0.506.$$

The portion of the despatch giving ephemeris from these elements was indecipherable, and is omitted.

CONTENTS.

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DEFINITIVE ORBIT OF THE COMET OF 1796,

BY HENRY A. PECK.

In 1796, while OLBERS was testing the Hoffmann Comet Finder, which he subsequently used for a series of years in time determinations, he discovered a small nebulous mass one minute in diameter, and decided it to be a comet. He made a series of measures with the five-foot Dolland telescope, extending from March 31 to April 14 inclusive, when the comet became too faint to measure in the bright moonlight. On the twenty-third of the month he sought it again, but was unable to find it. This was the second comet of that long series that OLBERS observed. He reduced his observations, and published the apparent position in *Bode's Jahrbuch* for 1799, neglecting to preserve the original observations except in one instance. He records that April first $8^h\ 55^m$ true time, the comet occulted a seventh magnitude star that follows 53 *Virgo* to the south. There is no star so bright as the seventh magnitude that answers OLBERS's description. There can be no doubt, however, that B.D. -15.3621 is the star in question. I have only been able to find two records of this object, one by ARGELANDER, and the other is the mean of three observations in the Washington Zones. The mean position adopted, referred to the equinox and ecliptic of 1796, is

$$\alpha = 13^h\ 3^m\ 18^s.5 \quad ; \quad \delta = -15°\ 28'\ 6''$$

Aside from this one observation, there are no means at present to correct the positions as given in the *Jahrbuch*.

The next day OLBERS sent word of his discovery to SCHROETER, at Lilienthal, and SCHROETER observed the comet on the 2d, 4th, 5th, 7th, 9th and the 11th of April. In general his observations are differences in right-ascension with reference to δ and γ *Corvi*, and 31, ζ and β *Crateris*. The first two and last are well known time stars whose positions have been taken from the *Jahrbuch* list. The positions of 31 and φ *Crateris* were kindly furnished me by the Dudley Observatory at Albany, from their forthcoming fundamental catalog. I have been unable to use either of SCHROETER's observations made upon the 4th, and his right-ascension of the 9th is a half degree in error. With one exception, his declinations have been neglected, as for the most part they are only approximations.

As a basis for the ephemeris the following positions of the Sun were computed from NEWCOMB's Tables of the Sun.

	True Longitude ° ′ ″	Log R	Latitude ″	Sid. Time h m s	X	Y	Z	Equation of time to be subtracted from app. time m s
March 29	9 31 28.8	9.999871	+0.42	0 30 23.1	+0.985920	+0.151742	+0.065872	
30	10 30 37.2	0.000000	.31	34 19.6	.983218	.167327	.072636	
31	11 29 43.8	128	.20	38 16.2	.980226	.182864	.079380	
April 1	12 28 48.7	256	+0.08	42 12.8	.976947	.198348	.086101	
2	13 27 51.8	384	−0.04	46 9.4	.973375	.213775	.092798	3 22
3	14 26 53.0	511	.15	50 5.9	.969514	.229139	.099467	4
4	15 25 52.4	639	.24	54 2.3	.965370	.244436	.106107	2 47
5	16 24 50.1	765	.30	0 57 58.9	.960937	.259662	.112717	30
6	17 23 45.8	891	.35	1 1 55.4	.956220	.274815	.119292	12
7	18 22 39.6	1016	.36	5 52.0	.951220	.289886	.125834	1 54
8	19 21 31.4	1139	.35	9 48.5	.945938	.304871	.132338	38
9	20 20 21.2	1262	.29	13 45.1	.940380	.319763	.138805	21
10	21 19 8.9	1383	.22	17 41.7	.934542	.334561	.145229	4
11	22 17 54.5	1503	−0.11	21 38.2	.928428	.349260	.151611	0 48
12	23 16 37.9	1623	+0.01	25 34.8	.922043	.363857	.157947	
13	24 15 19.0	1742	.16	29 31.3	.915390	.378345	.164237	
14	25 13 57.8	1860	.31	33 27.4	.908461	.392720	.170478	
15	26 12 34.6	1977	.46	37 24.4	.901276	.406981	.176669	
16	27 11 8.9	0.002094	+0.57	1 41 21.0	+0.893828	+0.421119	+0.182808	

The mean value of the obliquity of the ecliptic is 23° 27′ 56″.9.

The constants for the reduction from mean to apparent position are

		f	G	$\log g$	H	$\log h$	i
		″	° ′		° ′		
March	29	−4.33	134 5	0.4293	259 38	1.2748	−8.03
	30	.24	133 29	.4249	258 34	50	8.01
	31	.17	132 51	.4204	257 30	53	7.98
April	1	4.08	13	.4185	256 26	56	.95
	2	3.98	131 42	.4125	255 22	59	.92
	3	.90	2	.4080	254 18	62	.88
	4	.81	130 21	.4036	253 15	65	.85
	5	.73	129 50	.4003	252 11	69	.81
	6	.64	18	.3970	251 8	73	.77
	7	.55	128 24	.3915	250 4	77	.73
	8	.46	127 32	.3885	249 1	81	.68
	9	.37	126 38	.3856	247 58	86	.63
	10	.27	125 52	.3813	246 56	90	.58
	11	.17	18	.3782	245 53	95	.53
	12	3.09	124 30	.3740	244 50	1.2800	.48
	13	2.99	123 33	.3714	243 48	05	.42
	14	.89	122 16	.3696	242 46	10	.36
	15	.80	121 26	.3656	241 44	15	.30
	16	−2.71	120 22	0.3608	240 42	1.2851	−7.24

As a preliminary, new elements were computed from approximate normal places as follows:

$$T = \text{April } 2.77367 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 184^\circ\ 17'\ 7'' \\ \Omega &= 17\ \ 2\ 43 \\ i &= 115\ \ 9\ 23 \end{aligned}\right\} 1796.0$$

$$\log q = +0.198611$$

EQUATORIAL COORDINATES.

$$x = [9.984149]\, r \sin(266^\circ\ 51'\ 35''+v)$$
$$y = [9.892623]\, r \sin(344\ \ 8\ 45+v)$$
$$z = [9.831605]\, r \sin(194\ 11\ 24+v)$$

and from which we have the following ephemeris:

		α app.	δ app.	$\log r$	$\log \Delta$	Ab.time units 5 places
		° ′ ″	° ′ ″			
Mar.	29	205 41 19	−13 2 26	0.1990	9.7799	348
	30	202 49 40	47 29	88	7749	344
	31	199 53 6	14 31 11	87	7711	341
Apr.	1	196 52 35	15 13 2	87	7686	339
	2	193 49 12	52 33	86	7675	338
	3	190 44 11	16 29 21	86	7675	338
	4	187 38 46	17 3 3	86	7690	339
	5	184 34 16	33 23	87	7718	341
	6	181 31 55	18 0 13	88	7759	344
	7	178 32 54	23 27	89	7810	348
	8	175 38 14	43 13	91	7874	354
	9	172 48 51	59 35	92	7947	360
	10	170 5 33	19 12 45	95	8028	366
	11	167 28 46	22 58	0.1997	8118	374
	12	164 58 52	30 31	0.2000	8214	382
	13	162 36 12	35 41	3	8316	392
	14	160 20 52	38 46	7	8422	401
	15	158 12 47	−19 40 4	0.2010	9.8532	412

The comparison of the observations with the ephemeris is as follows:

Date	Observer	α app.	π	δ app.	π	$\Delta\alpha \cos\delta$	$\Delta\delta$
		° ′ ″		° ′ ″	″	″	″
March 31.49669	Olbers	198 25 46	0	−14 51 59	+14	+111	+ 29
April 1.34112	Olbers	195 49 52	−7	15 28 0	+13	− 31	− 59
1.42485	Olbers	35 9	−3	31 57	14	+ 9	− 96
1.45086	Schroeter	30 22	−2	. . . .	..	+ 8	..
1.48733	Schroeter	22 22	−2	. . . .	..	− 70	..
3.40670	Olbers	189 29 45	−3	16 42 55	14	+ 55	+155
4.38513	Olbers	186 26 22	−4	17 14 42	14	− 73	+ 40
5.38952	Olbers	183 23 58	−3	42 7	14	+ 58	+143
5.42294	Schroeter	25 15	−3	. . . .	..	+ 44	..
7.40937	Olbers	177 19 12	3	18 34 25	14	− 91	−129
7.49458	Schroeter	7 29	3	. . .	..	− 84	..
9.33558	Olbers	171 52 53	3	19 6 19	13	− 36	−106
9.41639	Schroeter	. . . .	..	19 7 47	14	..	−126
11.43379	Schroeter	166 23 1	1	. . . .	..	+ 10	..
12.37656	Olbers	164 1 40	0	34 23	13	−149	− 88
14.34819	Olbers	159 36 54	0	−19 37 22	12	+ 82	+134

From this comparison arises the following table of Normal Places:

		Wt.		Wt.
April 2.0	$\Delta\alpha\cos\delta = +1''.3$	7	$\Delta\delta = +13''.8$	5
7.0	-21.8	5	-54.5	4
13.0	-19.0	3	$+23.0$	2

The Schönfeld equations are

$-0.3020\,\partial k$	$+0.0046\,k\sqrt{2}\,\partial T$	$-8.5314\,\partial q$	$+0.2585\,\partial\lambda$	$-8.2663\,\partial\nu$	$= +1''.3$
.3043	0.0036	+9.6351	0.2142	+8.9238	−21.8
.2475	9.9316	9.8995	+0.0792	9.1787	−19.0
.2562	9.9592	9.2851	−0.2983	+8.3061	+13.8
.2086	9.9088	9.4284	.3180	−9.0276	−54.5
−0.1017	+9.7972	+9.3829	−0.2889	−9.3884	+23.0

If now we let

$$x = 0.3043\,\partial k \qquad y = 0.0046\,k\sqrt{2}\,\partial T$$
$$z = 9.8995\,\partial q \qquad u = 0.3180\,\partial\lambda$$
$$w = 9.3884\,\partial\nu$$

and pass from logarithms to units of the third place of decimals we have the homogeneous equations

						Wt.
$-995x$	$+1000y$	$-43z$	$+872u$	$-75w$	$= +1''.3$	7
1000	998	+ 544	787	+ 343	−21.8	5
877	845	1000	+ 577	617	−19.0	3
895	901	243	− 956	+ 83	+13.8	5
802	802	338	1000	− 436	−54.5	4
− 627	+ 623	+ 304	− 935	−1000	+23.0	2

The weighted normal equations are

$+21.60x$	$-7.60z$	$-2.87u$	$-0.53w$	$= +234''$	$+21.56y$
− 7.60	+5.43	+ 0.53	+1.71	−159	− 7.51
− 2.87	+0.53	+19.73	+4.78	− 2	− 2.81
− 0.53	+1.71	+ 4.78	+4.57	− 19	− 0.48

and after solving and again substituting, the final values of the unknown quantities are

$$\begin{aligned}
\partial k &= -\,0''.17 &&+0.4949\,k\sqrt{2}\,\partial T\\
\partial q &= -40.68 &&+0.0339\\
\partial\lambda &= -\,0.06 &&+0.0001\\
\partial\nu &= +32.74 &&+0.0055
\end{aligned}$$

The indetermination with regard to x and y, and therefore with regard to the time of perihelion passage and the longitude of the perihelion, arises from the short arc described during the time of observation, and also from the fact that the observations were made in the immediate neighborhood of the perihelion. If the values already obtained are substituted in the original equations we have

$$\begin{aligned}
+\,1''.3 &= +0.023\,k\sqrt{2}\,\partial T\\
-\,6.4 & \quad 26\\
+\,8.7 & \quad 08\\
+19.5 & \quad 24\\
-41.1 & \quad 20\\
+39.5 & +0.011
\end{aligned}$$

These equations apparently result in $k\sqrt{2}\,\partial T = -40''.4$, but how valueless this is may be seen from the fact that $[pvv]$ formerly equalled 12285, and now is 12269. It is evident, therefore, that within certain limits the ratio between ∂k and ∂T must remain unknown.

Syracuse University, 1907 *May* 15.

The changes in the elements result in the following most probable system:

$$\begin{aligned}
T &= \text{April } 2.77367 \text{ G.M.T.}\\
\omega &= 184^\circ\ 17'\ \ 5''.4 + [9.6944]\,k\sqrt{2}\,\partial T\\
\Omega &= \ \ 17\ \ \ 2\ 39.6 - [6.4689]\\
i &= 115\ \ \ 8\ 50.4 - [7.7433]\\
\log q &= 0.198557 \quad - [2.6551]
\end{aligned}\left.\vphantom{\begin{aligned}a\\a\\a\end{aligned}}\right\}1796.0$$

where $k\sqrt{2}\,\partial T$ is to be expressed in seconds of arc.

In order to test the accuracy of the numerical work $k\sqrt{2}\,\partial T$ may be assumed to equal zero, and a comparison made between the residuals arising from the differential equations and those obtained directly from the elements. The results are

Equations	Elements	Equations	Elements
$+1''.3$	$+1''.2$	$+19''.5$	$+19''.8$
−6.4	− 5.4	−41.1	−40.7
+9.4	+11.0	+39.5	+39.0

In investigating the increasing difference between the right-ascension residuals, it was found that $\partial\lambda$ had been used as $-0''.6$ instead of $-0''.06$; but as this can not affect any of the elements by a second of arc, it was not thought necessary to make a correction on this account.

OBSERVATIONS OF THE SATELLITE OF *NEPTUNE* AT THE OPPOSITION OF 1906–7.

By E. E. BARNARD.

The season has been unusually bad for observations, and the satellite has often been very difficult. Under fair conditions, however, it is an easy object with the great telescope. At times it has been so bright that I have thought it might be variable. It might be well to examine the notes for the relative visibility of this object at various position-angles. On account of the increasing apparent eccentricity of the orbit, the satellite will also become more and more difficult in certain positions.

The measures follow:

Measures of the Satellites.

1906–7	Central Stand. Time (h m s)	P.A. (°)	Dist. (″)	Obs.	
Oct. 30	15 56 26	251.52	. .	5	
	16 0 10	. .	14.81	4	
	16 3 29	. .	14.98	4	
Nov. 13	13 15 0	104.10	. .	6	Ques. if the satel.
	13 22 17	. .	16.73	4	Excess. difficult,
	13 28 35	. .	16.67	4	and poor seeing.
Dec. 18	15 5 35	112.89	. .	5	Satellite bright.
	15 8 50	. .	15.85	4	
	15 10 55	. .	16.16	4	
Dec. 22	16 3 35	244.25	. .	6	Very faint.
	16 8 41	. .	14.73	4	Excessively bad.
	16 12 0	. .	14.66	4	seeing.
Jan. 8	10 45 44	282.20	. .	5	Seeing excessive-
	10 49 14	. .	16.48	4	ly bad.
	10 51 52	. .	16.36	4	
Feb. 5	9 23 43	19.87	. .	6	
	9 29 5	. .	11.20	5	
	9 32 46	. .	11.34	5	
Feb. 10	12 54 17	69.21	. .	6	Quite bright.
	12 59 7	. .	15.28	4	Seeing very bad.
	13 1 56	. .	14.97	4	
Feb. 12	12 31 37	288.51	. .	6	Seeing very bad.
	12 35 38	. .	15.70	4	Satellite faint.
	12 38 44	. .	15.81	4	
Feb. 17	10 53 42	351.42	. .	6	Difficult. Seeing
	10 57 55	. .	11.05	4	very poor.
	11 0 26	. .	10.94	4	

1907	Central Stand. Time (h m s)	P.A. (°)	Dist. (″)	Obs.	
Mar. 3	12 2 14	226.59	. .	5	Seeing good.
	12 6 40	. .	12.86	4	
	12 10 2	. .	12.92	4	
Mar. 5	10 1 33	96.44	. .	5	Bright and easy.
	10 4 55	. .	16.45	4	
	10 7 17	. .	16.60	4	
Mar. 10	11 6 48	136.50	. .	6	
	11 11 6	. .	12.75	4	
	11 13 29	. .	12.84	4	
Mar. 26	8 6 27	261.14	. .	7	Very difficult.
	8 11 15	. .	15.67	4	Nearly full moon
	8 15 22	. .	16.20	4	See'g excess. bad.
April 2	8 27 38	188.21	. .	6	Very faint. Sky
	8 32 4	. .	10.84	5	very thick.
	8 34 40	. .	10.88	4	
April 9	8 34 44	109.90	. .	7	Lost in clouds before dist. could be measured.
April 14	8 17 30	166.35	. .	6	Faint. 14ᴹ.2.
	8 21 37	. .	11.11	4	
	8 23 55	. .	11.15	4	
April 23	7 39 6	332.65	. .	5	Satellite easy
	7 42 53	. .	11.49	4	
	7 45 41	. .	11.50	4	

Measures of *Neptune* and a Star.

1907	Central Stand. Time (h m s)	P.A. (°)	Dist. (″)	Obs.	
Feb. 10	13 5 42	349.91	. .	5	8ᴹ star.
	13 10 5	. .	58.48	4	
	13 12 53	. .	58.49	4	
Feb. 12	12 46 58	71.22	. .	5	The same star
	12 51 32	. .	133.67	4	as on Feb. 10.
	12 54 54	. .	134.51	4	

Yerkes Observatory, Williams Bay, Wis., 1907 June 3.

DEFINITIVE ORBIT OF COMET 1822 III.

By HENRY A. PECK.

The third comet of 1822 was discovered by Pons May 31. On account of lack of instrumental equipment, however, he was unable to obtain proper observations. The position was first measured by Caturegli, at Bologna, on June 8, and his observations during the next four days, together with two by Gambart, at Marseilles, are the only known European observations. From this insufficient data von Heiligenstein published two orbits in the fourth volume of the *Astronomische Nachrichten*. On June 18, Lieutenant William Robertson, R.N., on board H.M.S. *Creole*, in the harbor of Rio de Janeiro, discovered the comet. Although it had that day passed its conjunction, its extreme southern latitude favored its observation, and for a number of days, assisted by Lieutenant Charles Drinkwater, R.N., Robertson was able to note its position with regard to certain bright stars by sextant measurements. These measurements, together with an approximate orbit deduced from them by Henderson, were published in 1831 in the *Philosophical Transactions of the Royal Society*. In *Nature*, XXII (July 1, 1880) is a note describing an orbit by Hind, obtained by combining the observations in both hemispheres, but giving no hint as to the method used. It is evident from what follows that Hind was too modest, as the final corrections to his orbit are very small when we consider the approximate nature of the material that must be used. The comet is of considerable interest on account of its close approach to the earth. The day of its conjunction its distance from the earth was less than 0.14, and its proper motion in right-ascension approximated a degree every hour.

As it becomes necessary in dealing with old comet material to first find the coordinates of the sun from modern tables, these were computed from Newcomb's tables, with the following results:

	☉ (1822.0)	Log R	S.T.G.M.N.	Equation of T. to be sub. from appt.
	° ′ ″		h m s	m s
June 6	75 8 32	0.006514	4 57 19	
7	76 5 53	566	5 1 16	1 40
8	77 3 13	617	5 12	29
9	78 0 33	666	9 9	19
10	57 52	713	13 6	1 7
11	79 55 11	759	17 2	0 54
12	80 52 30	803	20 59	42
13	81 49 49	845	24 55	0 29
14	82 47 7	886	28 52	
15	83 44 24	924	32 48	
16	84 41 40	959	36 45	
17	85 38 57	992	40 41	
18	86 36 14	7023	44 38	
19	87 33 30	0.007051	5 48 35	
June 20	88 30 45	0.007076	5 52 31	
21	89 28 0	098	5 56 28	
22	90 25 15	118	6 0 24	
23	91 22 30	135	4 21	
24	92 19 43	150	8 17	
25	93 16 56	163	12 14	
26	94 14 9	0.007174	6 16 10	

The value of the Mean Obliquity of the Ecliptic is 23° 27′ 45″.

From these data are derived the following Equatorial Coordinates of the Sun, referred to the mean equinox and ecliptic of 1822.0.

	X	Y	Z
June 6	+0.260296	+0.900050	+0.390652
7	243922	904016	392373
8	227478	907728	393985
9	210969	911186	395485
10	194400	914385	396874
11	177773	917326	398151
12	161093	920012	399316
13	144363	922438	400368
14	127595	924604	401309
15	110793	926506	402135
16	093960	928145	402846
17	077096	929523	403445
18	060205	930638	403928
19	043299	931490	404298
20	026385	932075	404552
21	+0.009462	932397	404692
22	−0.007466	932455	404716
23	024393	932250	404627
24	041305	931780	404423
25	058206	931048	404105
26	−0.075092	+0.930054	+0.403675

The constants for passing from the mean to apparent place are

	f	G	log g	H	log h	i
	″	° ′		° ′		″
June 6	+29.3	−27 5	1.155	193 38	1.309	−2.0
10	30.0	26 23	1.162	190 6	1.310	1.5
14	30.7	25 43	1.170	186 34	1.311	1.0
18	31.4	25 5	1.177	183 2	1.311	−0.4
22	32.1	24 31	1.185	179 34	1.311	+0.1
26	+32.8	−24 1	1.192	176 4	1.311	+0.7

Hind's elements were made the basis of the work. These elements are found in the English magazine, *Nature*, Vol. XXII, and are as follows:

$$T = \text{July } 15.84420 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 237^\circ\ 44'\ 54'' \\ \Omega &= \ \ 97\ \ 44\ \ 18 \\ i &= 143\ \ 42\ \ 30 \end{aligned}\right\} 1822.0$$

$$\log q = 9.92797$$

Equatorial Coordinates.

$$x = [9.908456]\, r \sin(228^\circ\ 10'\ 43.6'' + v)$$
$$y = [9.963362]\, r \sin(336\ 15\ 51.7 + v)$$
$$z = [9.849155]\, r \sin(271\ 41\ 27.7 + v)$$

In forming the ephemeris, only five-place logarithms were used. The greater share of the material is of such an approximate nature, that any further refinement in computation seemed to be superfluous. The ephemeris positions are

	α 1822.0	δ 1822.0	log r	log Δ	Ab. time Units of 5th place
	° ′	° ′			
June 7.5	346 22.99	− 6 58.09	. .	9.5885	224
8.0	347 1.60	7 45.81	0.0385	5678	213
8.5	44.13	8 38.39	. .	5462	203
9.0	348 31.30	9 36.55	0341	5237	193
9.5	349 23.98	10 41.18	. .	5002	183
10.0	350 23.21	11 53.29	0297	4755	173
10.5	351 30.27	13 14.10	. .	4497	163
11.0	352 46.81	14 45.02	0254	4227	153
11.5	354 14.97	16 27.72	. .	3945	143
12.0	355 57.44	18 24.19	0211	3652	134
12.5	357 57.77	20 36.74	. .	3348	125
13.0	360 20.83	23 8.14	0.0168	3034	116

	α 1822.0	δ 1822.0	log r	log Δ	Ab. time Units of 5th place
	° ′	° ′			
June 18.0	82 9.20	−51 46.35	9.9958	9.1372	79
18.5	93 35.13	50 4.35	. .	1589	83
19.0	102 57.03	47 40.27	9917	1857	89
19.5	110 23.66	44 59.80	. .	2157	95
20.0	116 16.25	42 19.01	9878	2473	102
20.5	120 56.11	39 46.57	. .	2795	110
21.0	124 40.80	37 26.52	9838	3114	118
21.5	127 43.58	35 20.00	. .	3426	127
22.0	130 14.38	33 26.57	9800	3728	136
22.5	132 20.50	31 44.82	. .	4019	146
23.0	134 7.09	30 13.70	9762	4298	155
23.5	135 38.28	28 52.14	. .	4565	165
24.0	136 56.96	27 39.00	9726	4820	175
24.5	138 5.54	26 33.00	. .	5063	185
25.0	139 5.79	25 33.21	9.9690	5296	195
25.5	139 59.00	−24 38.94	. .	9.5519	206

The observations of Caturegli, at Bologna, are given in the *Monatliche Correspondenz of* von Zach, Vol. VI. No particulars are given, and the measures are only approximate. These observations, with the dates reduced for aberration are:

	α app.	π	δ app.	π	O − C Δα cos δ	O − C Δδ
	° ′	′	° ′	′	′	′
June 8.59743	347 39	−0.21	− 8 49	+0.21	(−14.49)	+0.38
10.58142	351 43	30	13 28	29	+ 0.10	+0.26
11.58040	354 32	36	16 46	35	+ 0.66	−0.40
12.59113	358 25	−0.47	−21 5	+0.43	+ 1.83	−1.95

The observations of Gambart, at Marseilles, are also to be found at the same place in the *Monatliche Correspondenz*. They consist of measured differences in right-ascension and declination with regard to two stars. These stars and their coordinates are

ψ^3 *Aquarii*	α = 347° 25.50′	δ = −10° 34.93′
133 *Piazzi Hora*, XXIII	352 6.39	−14 2.72

And the observations are

	α app.	π	δ app.	π	O − C Δα cos δ	O − C Δδ
	° ′	′	° ′	′	′	′
June 9.61108	349 37.19	−0.25	−10 56.46	+0.36	−0.13	+0.22
10.60094	351 46.46	−0.31	13 32.11	+0.41	+0.72	−0.30

As before stated, the observations of Robertson are published in the *Philosophical Transactions of the Royal Society for 1831*. As they are of somewhat unusual form for comet observations, I have transcribed so much of them as have a bearing on the orbit.

Rio de Janerio, June 18, 1822, at $6^h\ 30^m$ P.M. Observed a bright circular nebula near *Canopus*. On directing the telescope to it, we find it to have the appearance of a comet. At $6^h\ 40^m$ mean time the following distances were taken with sextants:

	° ′ ″
From *Canopus*	3 6 20
Sirius	31 27 10
α Hydrae	58 9 20
α Crucis	47 58 50

June 19. The comet appeared fainter than last night. There was a thin haze in the sky. The following observations were taken at $6^h\ 40^m$ P.M.:

	° ′ ″
From *Canopus*	11 33 30
Sirius	30 3 37
α Hydrae	46 2 47
α Crucis	14 15 30

June 22. Fine, clear moonlight. Observed the comet without a telescope. It is still of a round shape, no tail or nucleus observed when looked at with a telescope. The following angular distances were taken at $7^h\ 0^m$ P.M.

		°	′	″
From	*Canopus*	33	35	0
	Sirius	33	12	0
	α Hydrae	25	9	45
	α Crucis	44	36	25

June 23. Clear weather. The following angular distances were taken at $6^h\ 34^m$ P.M.

		°	′	″
From	*Canopus*	37	29	20
	Sirius	35	15	45
	α Hydrae	21	38	50
	α Crucis	45	13	10

June 24. Clear weather. Moonlight. The following distances were taken at $6^h\ 30^m$ P.M.

		°	′	″
From	*α Hydrae*	18	57	25
	α Crucis	46	37	30

The places adopted for the stars referred to the mean equinox of 1822.0 were

	α	δ
	° ′	° ′
Canopus	95 0.02	−52 36.11
Sirius	99 19.60	−16 28.76
α Hydrae	139 42.60	− 7 53.50
α Crucis	184 11.68	−62 6.70

In the first place, the coordinates of the stars were reduced from the mean to the apparent position, and also corrected for refraction. The apparent positions of a comet moving in Hind's orbit were then computed for the dates of observation minus the aberration time, and these in turn corrected for parallax and refraction. The apparent distance between the star and comet was computed by the formula

$$\cos \varDelta = \sin \delta'' \sin \delta' + \cos \delta'' \cos \delta' \cos (\alpha' - \alpha'')$$

when α', δ', α'' and δ'' are the right-ascension and declination of the comet and star respectively. Comparing the computed and observed distances we have following:

	June 18	June 19	June 22	June 23	June 24
	′	′	′	′	′
Canopus	− 3.17	+3.00	−0.22	0.00	. .
Sirius	−11.38	−4.24	. .	+ 2.98	. .
α Hydrae	−14.72	+2.01	−1.57	+ 0.23	−0.58
α Crucis	− 8.50	−6.19	−0.27	(−24.75)	−1.19

The signs here are in the usual sense of O—C. The *Sirius* observation for the 22d is omitted on account of the great zenith distance of the star.

If the formula used for finding the distance between the star and comet be differentiated there result

$$\cos B\, \partial\delta' + \sin B\, (\cos \delta' \partial\alpha') = -\partial\varDelta$$

where B is the angle opposite $90 - \delta''$, and is given by the equation

$$\sin B = \cos \delta'' \sin (\alpha'' - \alpha') \operatorname{cosec} \varDelta$$

Solving equations of this form for the observations of each day by the Least-Square Method there result the following corrections to the ephemeris from Hind's elements.

	$\partial\alpha \cos\delta$	$\partial\delta$
	′	′
June 18.38994	+3.92	+9.15
19.39676	+1.97	+1.99
22.41015	+0.35	+0.82
23.39180	+2.07	+0.10
24.38882	+1.78	+0.40

Four normal places were formed by using the formula $a + bt = c$, with the result,

	$\partial\alpha \cos\delta$	$\partial\delta$
	′	′
June 9.5	+0.14	+0.21
12.0	+1.13	−1.04
19.0	+2.75	+4.85
23.5	+1.47	+0.42

The Schönfeld equations for these dates are

∂k	$k\sqrt{2}\,\partial T$	∂q	$\partial\lambda$	$\partial\nu$		
+8.9987	−0.1139	−0.1621	+9.9856	−0.1396	=	+9.1461
+9.8788	−0.3558	−0.1533	+0.0822	−0.1912		+0.0531
+0.6549	−0.5681	+0.7928	−0.4869	+0.4624		+0.4393
+9.8905	−8.3527	+0.4275	−0.2594	+0.1388		+0.1673
−0.3825	+0.1466	−0.5645	−0.0702	+0.2242		+9.3222
−0.5460	+0.3455	−0.7254	−0.1394	+0.2484		−0.0170
+9.6793	−0.4438	−0.6009	−0.1295	+0.1050		+0.6857
+9.3415	−0.0228	−0.2530	−0.2577	+0.1371		+9.6232

From these we have the Normal Equations.

∂k	$k\sqrt{2}\,\partial T$	∂q	$\partial\lambda$	$\partial\nu$		
+1.6026	−1.4958	+1.7334	−0.8814	+0.5428	=	+1.3007
−1.4958	+1.5589	−1.3393	+0.9212	−0.6180		−1.4603
+1.7334	−1.3393	+2.0439	−0.8319	+0.4456		+0.5855
−0.8814	+0.9212	−0.8319	+1.3709	−1.3659		−1.1965
+0.5428	−0.6180	+0.4456	−1.3659	+1.3813		+1.1242

and therefore the elimination equations,

$$\begin{aligned}
\partial k \quad -9.8932\, k\sqrt{2}\,\partial T \quad +0.1308\,\partial q \quad -9.2788\,\partial\lambda \quad +8.9402\,\partial\nu &= +9.6981 \\
k\sqrt{2}\,\partial T \quad +0.2424\,\partial q \quad +9.3095\,\partial\lambda \quad -9.0835\,\partial\nu &= -0.0527 \\
\partial q \quad -9.6052\,\partial\lambda \quad +9.5160\,\partial\nu &= -8.3403 \\
\partial\lambda \quad -0.0154\,\partial\nu &= -9.6375 \\
\partial\nu &= +9.8671
\end{aligned}$$

The solution of these gives

$$\partial k = -0.02 \qquad k\sqrt{2}\,\partial T = -0.88$$
$$q = -0.13 \qquad \partial\lambda = +0.33$$
$$\partial\nu = +0.74$$

and from these the corrections to Hind's elements are

$$\partial\omega = -37''.8 \qquad \partial\Omega = 45''.6$$
$$\partial i = -40.4 \qquad \partial T = -0.01052$$
$$\log \partial q = -\ 5.5777$$

Therefore the new elements are

$$T = \text{July } 15.83368 \text{ G.M.T}$$
$$\left.\begin{aligned}\omega &= 237^\circ\ 44'\ 16'' \\ \Omega &= 97\ 43\ 32 \\ i &= 143\ 41\ 50\end{aligned}\right\}\ 1822.0$$
$$\log q = 9.927950$$

Equatorial Coordinates.

$$x = [9.908388]\, r \sin(228^\circ\ 10'\ 57'' + v)$$
$$y = [9.963392]\, r \sin(336\ 16\ 6 + v)$$
$$z = [9.849194]\, r \sin(271\ 40\ 41 + v)$$

Syracuse University, 1907 *May* 29.

Computing the places of the comet for the normal dates, and comparing the residuals with those obtained from substitution in the Schönfeld equations, there results,

$\partial\alpha \cos\delta$		$\partial\delta$	
$-0''.50$	$-0''.32$	$+0''.06$	$+0''.25$
-0.28	-0.43	-0.70	-0.56
-0.74	-0.69	$+1.40$	$+2.23$
$+1.41$	$+1.21$	-1.14	-0.99

The divergence for the third place in declination is perhaps somewhat larger than one would expect, even with five-place logarithms. A duplicate computation of both the coefficients involved, as well as the ephemeris place, fails to make any change, however, and I have therefore allowed them to stand. The elements undoubtedly represent the observations within the limits of their uncertainties.

ELEMENTS OF PLANET 1907 *XZ*,

By ELEANOR A. LAMSON.

[Communicated by Rear-Admiral Asa Walker, U.S.N., Superintendent U.S. Naval Observatory.]

From observations of March 21, April 1, April 12 and April 24, 1907, made by Messrs. Hammond and Frederickson, at Washington, the following elements were computed:

1907 April 24.5 Greenwich Mean Time.

$$M = 7^\circ\ 31'\ 46''.4$$
$$\left.\begin{aligned}\omega &= 201\ 23\ 58.5 \\ \Omega &= 335\ 11\ 17.5 \\ i &= 11\ 45\ 15.4\end{aligned}\right\}\ 1907.0$$
$$\varphi = 8\ 58\ 51.8$$
$$\mu = 626''.309$$
$$\log a = 0.502146$$

The observations, made at Washington, give the following corrections to an ephemeris, computed from these elements:

1907		O—C α	O—C δ
March	21	$+0^s.14$	$+0''.2$
	22	+0.09	−2.3
	25	+0.06	−1.5
	28	+0.26	−2.2
April	1	+0.15	+0.8
	12	+0.14	−2.2
	16	+0.14	−0.9
	20	+0.18	−1.5
	24	+0.07	−2.8
May	18	−0.46	−1.2

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THE LUMINOSITY OF THE FIXED STARS.

By GEORGE C. COMSTOCK.

We may define the luminosity of a star as the amount of light delivered by it in unit time to a surface of unit area placed normally to the incident beam at unit distance from the star. In all strictness the above definition should be restricted to a specific wave length of light, and luminosity should be regarded as a function of the wave length which when integrated over the entire length of the spectrum furnishes the light integral of the star or its total luminosity; but for the present purposes it seems unnecessary to insist upon this refinement. With reference to luminosity as above defined we may establish between the quantities,

L = Luminosity
π = Parallax
m = Stellar Magnitude
ρ^2 = The Light Ratio = $\sqrt[5]{100}$

the following relation, viz.:

$$L\,\pi^2\rho^{2m} = c \tag{1}$$

in which the possible absorption of light in transmission from the stars is neglected, and where c is a constant whose value is to be determined.

Let π be expressed in seconds of arc, let the unit of distance be that for which $\pi = 1''$, let the unit of area above introduced be so chosen that for the sun $L = 1$, and let the sun's stellar magnitude be represented by S. In terms of these units we find from Eq. 1

$$\log c = 10.6288 + 0.4\,S$$

If we now put $S = -26.57$, a number which differs from the best determinations of the sun's stellar magnitude by less than their probable errors, we shall find $c = 1$, which may be regarded as a plausible value of the constant. In the following computations, however, I have employed $c = \sqrt{2}$, corresponding to $S = -26.20$.

We may now write Eq. 1 in the form,

$$\log L = 0.150 - 0.4m - 2\log\pi \tag{2}$$

a relation which in the following pages is applied to a statistical study of the luminosities of those fixed stars for which suitable values of the parallax are attainable. It is obvious that for this purpose we may use large numbers of parallaxes that individually are not of the highest degree of precision, trusting to at least a partial elimination of the errors inherent in the several values. That this elimination is, however, a very limited and imperfect one is readily shown from a consideration of the mean luminosity derived by taking the average of n individual results for L, each result based upon a slightly erroneous parallax whose error may be represented by h. We find by developing Eq. 1 that the mean value of the luminosity, L, is represented by

$$n\,L = \Sigma L - 2\,\Sigma L\,.\,\frac{h}{\pi} + 3\Sigma L\left(\frac{h}{\pi}\right)^2 \ldots \quad \text{etc.} \tag{3}$$

In this expression the effect of the term involving the first power of h may be supposed eliminated in the mean of a considerable number of observed values through the equally frequent occurrence of positive and negative errors, but the term in h^2 remains always positive, and will tend to make $\bar{L}$ too great, particularly when small values of π are employed. I have therefore decided to use for the present purpose no parallax less than 0''.03 in absolute amount, *i.e.*, after there has been applied to each observed relative parallax a correction ranging from +0''.006 to +0''.012 corresponding to the assumed parallax of the comparison stars. Table VI shows the material that I have been able to collect for this purpose, and while it is, doubtless, not exhaustive, I have not knowingly omitted from it any star for which there is an apparently good determination of

parallax coming within the limit above defined. With respect to that limit we may say that the present investigation relates to that part of space included within a sphere whose center is at the sun and whose radius is approximately 7,000,000 times the radius of the earth's orbit. In the table, m and π denote the adopted magnitude and parallax of the stars, m corresponding in general to the Harvard Meridian Photometry; L is the luminosity determined through Eq. 2; and the last column of the table indicates the authority for the adopted parallax. For stars contained in KAPTEYN's list, Publications of the Astronomical Laboratory at Groningen, No. 8, I have in general adopted the value there given, and the symbol K denotes such a parallax. In some cases these values have been supplemented by more recent determinations, and these, together with all other cases where a mean of two or more determinations are employed, are indicated by the symbol M. Parallaxes adopted from a single determination are designated as follows:

P. Pritchard's Photographic Parallaxes. Astronomical Observations *** Oxford No. IV, Part II.

Y. Transactions of the Astronomical Observatory of Yale University, Vol. II, Part I.

F. Flint's determinations of stellar parallax by meridian transits. In part contained in publications of the Washburn Observatory, Vol. XI, and in part hitherto unpublished.

R. Unpublished photographic results kindly furnished me in manuscript by Mr. HENRY NORRIS RUSSELL.

A few isolated results included in the table are specially designated, and for the sake of completeness there are included in the table a few results not available at the time the discussion of the luminosities was made. It may be noted that considerably more than half of the entire data is obtained from the source designated Y and F. While the material here presented certainly comprises only a minute portion of the stars included in the space above defined, it would be sufficient in quantity to furnish at least approximate ideas of the distribution of luminosity among those stars provided it were legitimate to assume that the 230 stars here employed have not been selected from the larger aggregate by any rule of choice that impairs their representative character. This condition unfortunately is not fulfilled. As is well known, stars observed for parallax are almost without exception those characterized by large proper motion or by unusual brilliancy, and while the former property is perhaps not prejudicial to the present purpose the latter certainly results in a biased choice of material, that must be reckoned with in any interpretation of its results, *e.g.*, there will be found among the available data an excessive proportion of intrinsically bright stars, the faint stars will be far less in number than is typical of their actual frequency, and the mean luminosity furnished by the data must be considerably greater than that which actually obtains. Any results of this kind which are to be derived from existing data must therefore be regarded as limits from which the values actually characteristic of the stellar system depart in the directions above assigned.

Although the considerations thus outlined constitute a serious limitation upon the conclusions to be drawn from the available material it still remains possible to obtain from it a considerable amount of information relative to the luminosity of the stars.

(A) With respect to the stars of greatest intrinsic brilliancy the data must be regarded as complete for the region under investigation. Thus, if there existed within a distance from the sun equal to 7,000,000 radii of the earth's orbit, a star whose total output of light exceeded that of the sun one thousand-fold, Eq. 2 shows that its stellar magnitude could not be less than 0.5, and as all stars of this magnitude have been carefully observed for parallax we may conclude that no such star exists in the region thus defined. It will be later shown, Table II, that the stars having observed parallaxes equal to or greater than 0″.10 present no case of a luminosity one hundred-fold greater than that of the sun, and, as above, we may show that the data are here exhaustive and that if any such star existed within a distance equal to 2,000,000 radii of the earth's orbit it would have been found and included in the data here discussed.

(B) In any investigation of the mean luminosity of the stars it seems desirable to consider separately stars at different distances from the earth, since the effects of the biased selection of data above noted increase rapidly with increasing distance. To present such a classified exhibit of mean luminosities, Table I has been formed as follows: The stars having been arranged in the order of their parallaxes a mean value of the luminosity, L, was formed for groups whose central value of π is shown in the first column of the table.

In order to smooth out irregularities due to the small amount of available data, the several groups have been allowed to overlap so that each individual value of L, save at the ends of the table, enters into several consecutive groups, thus producing in effect a series of adjusted numbers Notwithstanding some notable irregularities the increase in the mean value of L with increasing distance, which might *á priori* be expected from the character of the data, is here conspicuously shown and the sequence of the numbers inspires some confidence in the adequacy of the data employed:

TABLE I.

APPARENT RELATION OF LUMINOSITY TO DISTANCE. ADJUSTED MEAN VALUES.

π	Mean L	Median L	No. of Stars
″			
0.05	45.	1.6	88
.06	28.	1.6	85
.07	16.	1.5	84
.08	12.	1.9	71
.09	10.	2.1	62
.10	8.0	1.9	57
.11	5.1	1.4	51
.12	4.1	1.1	44
.13	3.2	0.8	36
.15	4.9	0.9	14
.18	1.2	0.4	12
.25	0.8	0.04	13
.34	3.5	0.06	12
.57	0.4	0.03	3

In the presence of such influences as are here manifest, the mean luminosity has little significance and a better presentation of the characteristics of the stellar system may be obtained through the median value of L, *i.e.*, that value which occupies the middle place when the individual values are arranged in order of magnitude (the mean of two values when the group consists of an even number of units). The median values of the several groups are shown in the third column of Table I and we find here an approximate uniformity of the median for all values of π not exceeding 0″.1. For greater values of π there is, however, a notable diminution and with respect to the character of the data we must regard the smaller numbers as furnishing by far the better representation of the average stellar brilliancies, if indeed, even the smallest numbers here shown are not still much too large. We may at least conclude that by comparison with the average star the sun is a body of exceptional brilliancy, instead of being below the average, as is sometimes affirmed.

(C) In any given part of space bright and faint stars are intermingled in a definite ratio, and if it be assumed that the relative frequency of the several degrees of luminosity is substantially the same over wide areas we may represent this frequency, y, by a so-called luminosity curve, $y = f(L)$. It is a major purpose of the present investigation to determine the general form of this curve, and in view of the considerations regarding the data above set forth, I have divided the available material into two parts, *viz.*, those stars whose parallax is equal to or greater than 0″.10 and those whose parallax is less than that limit, presuming that the prejudicial effects of a biased selection of stars will be less manifest in the former than in the latter category, and that the difference between the resulting curves may furnish some clew to the amount of their deviation from the truth. In each body of data thus formed the stars were arranged in the order of increasing luminosity and the number of stars included between assigned limits of L were counted, a star falling exactly upon the limit between two groups being counted as half a star in each group. The adopted limits together with the number of stars falling between them are shown in Tables II and III.

TABLE II.

DISTRIBUTION OF LUMINOSITIES $\pi > 0''.095$.

Limits of L	Mean L	n	y
0.00 – 0.01	0.008	8	4625
.01 – .05	.03	12	1735
.05 – .10	.07	8	925
.10 – .20	.14	13	750
.20 – .40	.29	12.5	360
.40 – .70	.57	6.5	125
.70 – 1.00	.86	8	154
1.0 – 2.0	1.4	8	46
2.0 – 5.0	3.0	9	17
5.0 – 10.0	7.7	9	10
10.0 – 50.0	28.2	7	1
50. –	. .	0	0

TABLE III.

DISTRIBUTION OF LUMINOSITIES $\pi < 0''.095$.

Limits of L	Mean L	n	y
0.0 – 0.2	0.13	13	283
0.2 – 0.3	.26	11	478
0.3 – 0.5	.38	12	261
0.5 – 1.0	.72	12.5	109
1.0 – 1.5	1.2	10.5	91
1.5 – 2.0	1.6	7	61
2.0 – 3.0	2.4	6	26
3.0 – 5.0	3.8	7	15
5.0 – 10.0	6.9	10	9
10. – 20.	15.1	11	5
20. – 50.	30.6	11	1.6
50. – 100.	67.	9.5	0.83
100. – 300.	193.	5.5	0.12
300. – 700.	477.	5.	0.05
700. – .		0.	0.00

There is also shown, under the heading y, a number proportional to the ordinate of the resulting frequency curve, obtained by dividing the number of stars actually enumerated in each group by the interval between the limits of the group and multiplying the quotient by a constant factor so chosen as to make $y = 100$ when $L = 1$. A comparison of the values of y contained in the two tables shows upon the whole a satisfactory agreement which may be regarded as a confirmation of the genuineness of the small parallaxes used in Table III and their adequacy for the present purpose. The discordances between the two tables for very small and very great values of L are of the kind anticipated, and with reference to their numerical amount I am constrained to regard the values of y given at the beginning of Table II as being in considerable measure too small, and similarly the values of y given at the end of Table III are appreciably too great. In utilizing the present data for

the construction of a luminosity curve I have, however, ignored these considerations, and resorting to graphical methods I have plotted y as a function of the tabular mean L, without distinction between the two tables, and have drawn through the plotted points a smooth curve from which I have read the ordinates whose values are given in Table IV under the heading adjusted y. These quantities are to be regarded as the principal result of the present investigation. They define an empirical curve whose relation to the true luminosity curve is as follows:

For $L = 1$ the two curves are in forced agreement; for values of L much less than unity the ordinates of the true curve are greater than those of the empirical curve; for values of L much greater than unity the ordinates of the true curve are less than those of the empirical curve. I find by trial that these adjusted values of y can be very approximately represented by the equation

$$y_1 = \frac{A}{L} \log \frac{B}{L} \tag{4}$$

where A and B are constants, which for the present data have the values $A = 100 \div 3$, $B = 1000$, when the logarithms are taken to the base 10. Values of y_1, computed from this formula, are shown in Table IV. Their agreement with the observed values is all that could be expected from the character of the data and at the beginning and end of the table they depart from the observed values toward the true curve.

The last column of Table IV contains values of y computed from the data given by KAPTEYN in *Publications of Astronomical Laboratory at Groningen*, No. 11, "On the Luminosity of the Fixed Stars." Solution V, p. 19.

TABLE IV.
THE LUMINOSITY CURVE.

L	Adjusted y	y_1	Kapteyn
0.01	. . .	16700	263
0.2	600	618	204
0.4	250	284	143
0.6	165	179	120
0.8	125	129	110
1.0	100	100	100
2.0	35	45	71
4.0	15	20	47
10.	7	7	22
20.	3	3	12
40.	0.8	1.1	6
100.	0.4	0.3	1.8
200	0.15	0.11	0.72
400.	0.05	0.03	0.28
1000.	.	0.	0.07

While the general character of the function represented by the numbers in the last two columns of this table may be described as not unlike, since both show a preponderance of faint stars, the numerical differences between them are very great, particularly at the beginning and end of the table. For values of L less than 0.1 KAPTEYN's function is practically constant or, as he suggests, possesses a maximum near $L = 0.01$, while Eq. 4 has in this region its steepest slope and contains no suggestion of a diminishing frequency as L approaches the limit 0. This characteristic of the curve above found is entirely consonant with what is believed on other grounds with regard to the wide diffusion of meteoric matter, whose individual particles may be regarded, for the present purposes, as stars of zero luminosity and whose total number must vastly exceed the number of visibly luminous bodies. For large values of the luminosity KAPTEYN's numbers are five to ten times greater than those here found and their ratio to the ordinates of the true curve must be exaggerated in even greater measure. I have elsewhere expressed my dissent from the theoretical basis assigned for KAPTEYN's luminosity curve, and in view of the comparison between that curve and the observed data here presented it would seem that, within the region under consideration, the value of the KAPTEYN curve, even as an empirical relation, can be maintained only by assuming the general worthlessness of the observed parallaxes. The general agreement between the results furnished by large and small parallaxes strongly opposes this assumption, and if the parallaxes are even roughly a measure of the stellar distances KAPTEYN's curve does not fairly represent the actual distribution of luminosities.

In view of their relatively small distances from the earth all the stars here considered may be regarded as lying in or near the plane of the galaxy, and it therefore appears improbable that their luminosity function should be appreciably dependent upon galactic latitude. I have, nevertheless, divided the data into two classes, those stars lying within 30° of the central line of the galaxy and those more remote from it, and by a process entirely similar to that above described I have obtained the ordinates and corresponding mean luminosities shown in Table V. Despite

TABLE V.
INFLUENCE OF GALACTIC LATITUDE.

Limits of L	Galactic Stars n	$\bar{L}$	y	Non-Galactic Stars n	$\bar{L}$	y
0.0 – 0.1	11	0.04	526	20	0.04	762
0.1 – 0.2	9	0.14	435	14	0.15	533
0.2 – 0.4	14	0.30	337	17	0.29	323
0.4 – 0.8	7	0.54	84	13	0.58	123
0.8 1.2	12.5	0.99	150	7	1.02	67
1.2 2.0	6.5	1.5	39	11	1.5	52
2.0 – 5.0	10	3.1	16	12	3.1	15
5.0 – 10.	10	7.1	10	9	7.4	7
10. – 40.	11	22.	1.8	14	20.	1.8
40. – 100.	10	57.	0.8	4	75.	0.3
100. – . . .	6	298.	. . . .	4	402.	. . . .

some irregularities the agreement between the values of y furnished by the galactic and non-galactic stars is probably as great as should be expected, and while the numbers thus found might possibly be construed as indicating in the galaxy a relative excess of bright stars and defect of faint ones, any such inference should be supported by more data than is here available.

We may summarize the results above obtained, as follows: Within a distance equal to 7,000,000 radii of the earth's orbit there exists no star whose luminosity exceeds that of the sun one thousand-fold. The average star, brighter than the ninth magnitude, is intrinsically much fainter than the sun, probably emitting less than one-tenth as much light. The faint stars are far more numerous than the bright ones and with diminishing luminosity their relative number rapidly increases, without present appearance of any limit to this increase. The law of this increase may be approximately represented by an equation of the form

$$y = \frac{A}{L} \log \frac{B}{L}$$

The law of distribution of luminosities is appreciably the same for galactic and extra-galactic stars.

TABLE VI.—Luminosities.

Star	R.A.	Decl.	m	π	L	Auth'y
	h m	°		″		
Ll. 47231	0 0	+45	8.3	0.14	0.035	Y
α Andromedae	0 3	+29	2.1	0.06	56.2	P
Br. 3212	1	+28	7.5	0.15	0.063	Y
β Cassiopeiae	4	+59	2.4	0.14	7.9	M
γ Pegasi	8	+15	3.0	0.08	13.8	F
Gr. 34	12	+43	7.9	0.31	0.010	M
3 *Tucanae*	15	−65	4.1	0.15	1.44	K
β Hydri	20	−78	2.7	0.14	6.03	K
P., 0^h, 130	32	−25	5.6	0.36	0.063	F
54 *Piscium*	34	+21	5.5	0.15	0.40	Y
Ll. 1198	40	+ 1	8.2	0.08	0.12	Y
Gr. 145	43	+70	7.8	0.03	1.20	Y
Mayer 20	43	+ 5	5.7	0.17	0.095	M
η Cassiopeiae	43	+57	3.8	0.23	0.81	M
Ll. 1799	0 57	+ 5	8.3	0.06	0.19	Y
μ Cassiopeiae	2	+54	5.4	0.11	0.81	M
Ll. 1964	2	+22	8.4	0.10	0.062	Y
Ll. 1966	3	+61	7.8	0.08	0.17	Y
β Andromedae	4	+35	2.2	0.06	51.3	M
θ Ceti	19	− 9	3.8	0.10	4.27	F
ω Andromedae	21	+45	4.8	0.09	2.09	F
Polaris	23	+90	2.1	0.08	31.6	M
α Eridani	34	−58	0.5	0.05	355.	K
Fed. 263	34	+66	7.4	0.04	0.98	Y
41 *H. Andromedae*	36	+42	5.2	0.12	0.81	Y
107 *Piscium*	37	+20	5.4	0.13	0.58	Y
τ Ceti	39	−16	3.7	0.36	0.36	K
P. I, 159	40	+63	6.1	0.06	1.41	Y
β Arietis	1 49	+20	2.8	0.06	29.5	F
α Arietis	2 1	+23	2.0	0.09	27.5	M
Br. 3227	8	+67	7.8	0.10	0.11	Y
Ll. 4141	10	+24	6.5	0.03	3.98	Y
δ Trianguli	11	+34	5.0	0.12	0.98	M
o Ceti	14	− 3	Var.	0.14	*	R
P. II, 123	31	+ 6	5.9	0.14	0.34	Y
Ll. 4855	2 33	+30	7.3	0.04	1.07	F
Ll. 5490–6	56	+61	6.7	0.06	0.81	Y
ρ Persei	2 59	+38	3.8	0.09	5.25	R
ι Persei	3 2	+49	4.1	0.11	2.69	Y
β Persei	2	+41	2.2	0.09	74.1	M
W.B. III, 113	9	+ 9	7.8	0.08	0.17	Y
e Eridani	16	−43	4.4	0.16	0.98	K
α Persei	17	+50	1.9	0.10	24.5	M
ε Eridani	28	−10	3.7	0.37	0.34	F
10 *Tauri*	32	0	4.4	0.07	5.01	Y
δ Eridani	38	−10	3.7	0.09	5.75	F
Ll. 6888–9 pr.	40	+41	8.2	0.05	0.30	Y
Gr. 745	48	+76	8.2	0.05	0.30	Y
Ll. 7443	3 56	+35	8.5	0.05	0.22	Y
o^2 Eridani	4 11	− 8	4.7	0.17	0.66	M
α Tauri	30	+16	1.2	0.10	46.8	M
Gr. 864	34	+42	7.1	0.03	2.29	Y
Gr. 884	44	+46	6.5	0.13	0.21	Y
W.B. IV, 1189	56	− 6	6.3	0.30	0.048	M
Gould's Z.C., V.243	5 8	−45	8.5	0.32	0.005	K
α Aurigae	9	+46	0.3	0.08	158.	M
λ Aurigae	12	+40	5.0	0.09	1.74	M
β Tauri	20	+29	1.8	0.07	55.0	P
W.B., V. 592	26	− 4	8.7	0.07	0.095	F
δ Orionis	27	0	2.4	0.08	24.	F
Gr. 990	30	+51	7.9	0.03	1.00	Y
ε Orionis	31	− 1	1.8	0.06	74.	F
P., V. 146	33	+53	6.3	0.14	0.22	Y
Ll. 10797–8	39	+37	7.1	0.08	0.32	Y
Ll. 11196 pr.	50	+14	7.0	0.06	0.62	Y
α Orionis	50	+ 7	1.2	0.03	524.	K
β Aurigae	5 52	+45	2.1	0.06	129.	M
κ Aurigae	6 9	+30	4.6	0.06	14.1	*
β Canis Majoris	18	−18	2.0	0.16	8.91	F
23 *H. Camelop.*	29	+80	5.5	0.04	5.62	Y

* 11.5 to 0.02.

* Jost, *Ast. Nach.* 3888.

Star	R.A.	Decl.	m	π	L	Auth'y
	h m	°		″		
γ *Geminorum*	6 32	+16	2.0	0.07	45.7	F
Sirius	41	−17	−1.3	0.38	32.	M
97 *Monocerotis*	45	0	6.7	0.26	0.043	F
P. VI, 305	6 57	+30	6.0	0.07	0.56	M
Ll. 13849	7 4	+21	7.0	0.11	0.19	Y
η *Canis Majoris*	20	−29	2.4	0.05	62.	F
α *Geminorum*	28	+32	1.8	0.06	74.	F
Procyon	34	+ 5	0.7	0.30	8.3	M
β *Geminorum*	39	+28	1.5	0.07	72.	K
Lac. 2957	42	−34	6.0	0.07	1.15	M
Lal. 15290	47	+31	8.2	0.05	0.30	M
9 *Puppis*	48	−14	5.3	0.04	3.55	F
Ll. 15547	54	+21	8.5	0.09	0.069	Y
Ll. 15565	7 54	+30	7.5	0.06	0.39	M
P. VII, 321	8 5	+33	6.7	0.05	1.18	Y
W.B.VIII,181–2	12	+31	8.3	0.09	0.083	F
Ll. 16304	13	−12	6.0	0.13	0.33	F
o *Ursae Majoris*	21	+61	3.4	0.09	7.59	F
A. Oe. 9342	46	+71	8.5	0.10	0.056	F
55 ρ *Cancri*	47	+29	6.2	0.07	0.95	Y
6 *Ursae Majoris*	52	+48	3.2	0.08	11.5	M
10 *Ursae Majoris*	8 54	+42	4.2	0.09	3.63	Y
Ll. 18115	9 8	+53	7.5	0.14	0.072	M
Σ3121	12	+29	7.0	0.13	0.13	F
Ll. 18397	16	+41	7.5	0.06	0.39	Y
d *Ursae Majoris*	26	+70	4.6	0.05	8.13	Y
θ *Ursae Majoris*	26	+52	3.4	0.08	9.55	M
Ll. 19022	37	+43	8.1	0.07	0.17	K
Gr. 1596	55	+56	8.2	0.07	0.15	Y
20 *Leonis Minoris*	9 55	+32	5.6	0.07	1.66	M
α *Leonis*	10 3	+12	1.8	0.03	302.	K
A. Oe. 10603	5	+50	7.0	0.16	0.089	M
Ll. 19821	6	+24	8.3	0.05	0.27	Y
39 *Leonis*	12	+24	6.5	0.09	0.44	Y
μ *Ursae Maj.*	16	+42	3.1	0.05	32.4	Y
Gr. 1646	22	+49	6.3	0.11	0.35	K
P., X 96	28	+50	7.4	0.05	0.62	K
Ll. 21008–10	51	+28	8.3	0.05	0.27	Y
β *Ursae Majoris*	56	+57	2.4	0.09	19.	P
Ll. 21185	58	+37	7.5	0.37	0.010	M
α *Ursae Maj.*	10 58	+62	2.0	0.05	89.	P
51 *Leonis Min.*	11 0	+26	7.8	0.04	0.27	Y
Ll. 21258	1	+44	8.5	0.24	0.0098	M
Ll. 21368	6	+31	8.5	0.05	0.22	Y
Σ1516	8	+71	6.5	0.13	0.21	M
ξ *Ursae Majoris*	13	+32	3.8	0.18	1.35	Y
A. Oe. 11677	15	+66	9.0	0.20	0.0089	M
Br. 1584	29	−32	6.2	0.21	0.11	F
Gr. 1812	33	+46	6.7	0.04	1.86	K
Gr. 1822	40	+48	8.0	0.03	1.00	K
β *Leonis*	11 44	+15	2.2	0.09	22.9	M
β *Virginis*	46	+ 2	3.7	0.12	3.24	Y
Gr. 1830	47	+38	6.6	0.15	0.14	K
γ *Ursae Maj.*	49	+54	2.5	0.10	14.1	P
Lac. 4955	11 53	−27	7.7	0.04	0.74	F
Gr. 1855	12 5	+41	7.3	0.07	0.35	K
Ll. 22908	8	+11	8.0	0.09	0.11	Y
α *Crucis*	21	−63	1.0	0.06	155.	K
8 *Canum Venat.*	29	+42	4.3	0.10	2.69	Y
γ *Virginis*	36	− 1	3.5	0.08	8.7	R
ε *Ursae Majoris*	50	+56	1.8	0.07	55.	M
ε *Virginis*	12 57	+11	3.0	0.03	100.	F
42 *Comae*	13 5	+18	4.5	0.12	1.55	F
Ll. 24504	6	+10	8.5	0.03	0.63	Y
β *Comae*	7	+28	4.5	0.12	1.55	M
61 *Virginis*	13	−18	4.8	0.15	0.76	F
W.B. XIII, 241	15	+36	9.0	0.08	0.055	Y
Ll. 24774	16	+44	8.0	0.18	0.028	Y
ζ *Ursae Majoris*	20	+55	2.4	0.08	24.	F
70 *Virginis*	24	+14	5.2	0.15	0.52	Y
Ll. 25224	34	+11	5.6	0.26	0.12	F
Ll. 25372	41	+15	8.5	0.27	0.0078	M
τ *Bootis*	42	+18	4.5	0.04	14.1	Y
β *Centauri*	13 57	−60	1.2	0.04	295.	K
α *Bootis*	14 11	+20	0.3	0.04	676.	M
Berlin B. 5072	21	+24	9.0	0.04	0.22	R
Ll. 26481	26	−15	8.0	0.31	0.0093	F
α^2 *Centauri*	33	−60	0.9	0.76	1.07	K
ϵ^2 *Bootis*	41	+27	2.6	0.05	52.	F
Ll. 27026	46	−24	8.2	0.04	0.47	F
Ll. 27298	52	+54	7.5	0.09	0.17	M
P., XIV, 212	14 52	−21	6.3	0.15	0.19	M
Ll. 27742	15 8	+20	6.7	0.05	1.18	Y
Ll. 27744	9	− 1	7.0	0.13	0.13	F
6 *Serpentis*	13	+ 1	5.5	0.13	0.52	F
5 *Serpentis*	14	+ 2	5.1	0.21	0.30	Y
Ll. 27958	15	+26	8.0	0.03	1.00	Y
W.B. XV, 268	18	+ 2	8.7	0.12	0.032	Y
μ^2 *Bootis*	21	+38	4.3	0.03	30.	F
Ll. 28358	26	+58	6.9	0.03	2.75	F
α *Coronae Bor.*	30	+27	2.3	0.05	68.	F
Ll. 28607	37	−11	7.0	0.03	2.51	F
39 *Serpentis*	48	+14	6.3	0.04	2.69	Y
χ *Herculis*	49	+43	4.5	0.10	2.24	Y
γ *Serpentis*	52	+16	4.0	0.08	5.50	M
ρ *Coronae Bor.*	15 57	+34	5.2	0.03	13.2	Y
Gr. 2305	16 2	+39	6.5	0.07	7.24	Y
Ll. 29439	3	+39	8.4	0.05	0.25	Y
τ *Coronae Bor.*	5	+37	5.2	0.12	0.81	F
α *Scorpii*	23	−26	1.2	0.03	525.	K

Star	R.A. h m	Decl. °	m	π ″	L	Auth'y
Ll. 30024–6	16 24	+19	7.0	0.09	0.28	Y
ζ *Herculis*	38	+32	3.1	0.18	2.57	Y
η *Herculis*	39	+39	3.7	0.16	1.86	F
Ll. 30699	43	+68	7.6	0.05	0.51	Y
Ll. 30694	16 48	0	7.0	0.14	0.11	Y
Ll. 31055	17 0	− 5	7.5	0.12	0.10	F
Ll. 31132	0	+47	6.6	0.14	0.17	Y
36 *Ophiuchi*	9	−26	4.7	0.37	0.15	F
Br. 2179	10	−26	6.8	0.24	0.039	F
δ *Herculis*	11	+25	3.2	0.09	9.12	*
72 *Herculis*	17	+33	5.4	0.12	0.68	M
W.B. XVII, 322	21	+ 2	8.0	0.18	0.028	M
Fed. 2895	25	+67	6.5	0.05	1.41	Y
26 *Draconis*	34	+62	5.3	0.09	1.32	Y
A. Oe. 17415–16	37	+68	9.0	0.28	0.0046	M
μ *Herculis*	42	+28	3.5	0.13	3.31	Y
γ *Draconis*	17 54	+51	2.4	0.06	42.7	P
70 *Ophiuchi*	18 0	+ 3	4.2	0.18	0.93	M
δ *Ursae Min.*	4	+87	4.4	0.04	15.5	†
χ *Draconis*	23	+73	3.7	0.12	3.24	Y
α *Lyrae*	34	+39	0.4	0.15	43.6	M
110 *Herculis*	41	+20	4.1	0.05	12.9	‡
Σ2398	18 42	+59	8.2	0.33	0.0069	F
31 *Aquilae*	19 20	+12	5.3	0.06	3.39	M
Br. 2459	21	+25	6.0	0.06	1.55	Y
Ll. 37120–1	30	+33	7.0	0.06	0.62	Y
σ *Draconis*	33	+69	4.7	0.20	0.47	F
θ *Cygni*	34	+50	4.6	0.07	4.17	Y
α *Aquilae*	46	+ 9	1.1	0.24	8.71	M
Ll. 38287	19 58	+15	7.0	0.09	0.28	Y
Ll. 38383	20 0	+23	7.0	0.11	0.19	Y
15 *Sagittae*	0	+17	6.0	0.17	0.18	Y
Ll. 38380	0	+30	5.8	0.04	4.27	Y
Gr. 3042	4	+53	5.7	0.06	2.04	Y
P., XX, 23	7	+16	7.0	0.08	0.35	Y

* Leavenworth. † de Ball., *Ast. Nach.* 2667.
‡ Jost, *Ast. Nach.* 3888

Star	R.A. h m	Decl. °	m	π ″	L	Auth'y
Gr. 3150	20 16	+67	6.2	0.12	0.32	Y
γ *Cygni*	19	+40	2.3	0.11	14.1	M
Ll. 39866	35	+ 5	8.4	0.06	0.17	Y
Fed. 3562–3	39	+75	7.2	0.06	0.51	Y
ε *Cygni*	42	+34	2.6	0.13	7.59	P
η *Cephei*	43	+61	3.6	0.10	5.13	Y
B.D. 37, 4131	55	+37	7.8	0.05	0.43	*
Gr. 3357	20 56	+40	6.7	0.08	0.46	†
Ll. 40844	21 0	+ 7	8.8	0.17	0.015	Y
61 *Cygni*	2	+38	6.1	0.30	0.057	M
δ *Equulei*	9	+10	4.6	0.03	22.9	F
ρ *Cygni*	30	+45	4.2	0.05	18.6	Y
ε *Pegasi*	39	+ 9	2.5	0.09	17.4	P
κ *Pegasi*	40	+25	4.2	0.03	33.	F
Ll. 42883–5	54	+29	7.4	0.03	1.74	Y
ε *Indi*	21 56	−57	4.8	0.28	0.22	K
α *Gruis*	22 2	−47	1.9	0.03	275.	K
Gr. 3689	3	+53	7.9	0.04	1.55	Y
Ll. 43492	12	+12	7.0	0.15	0.10	Y
34 *Pegasi*	21	+ 4	5.7	0.07	1.51	F
ξ *Pegasi*	42	+12	4.1	0.09	3.63	Y
α *Piscium Aust.*	52	−30	1.4	0.14	20.0	K
Lac. 9352	22 59	−36	7.1	0.29	0.024	K
α *Pegasi*	23 0	+15	2.6	0.09	15.8	P
Fed. 4371	1	+68	7.5	0.06	0.39	Y
Br. 3077	8	+57	6.0	0.14	0.29	K
W.B.XXIII,175	12	−14	8.2	0.05	0.30	F
Ll. 45755	17	+44	7.3	0.04	1.23	Y
ι *Piscium*	35	+ 5	4.3	0.15	1.20	Y
Ll. 46650	44	+ 2	8.7	0.22	0.010	M
85 *Pegasi*	57	+27	5.7	0.07	1.55	M

* Bergstrand, *Ast. Nach.* 3734. † Jost, *Ast. Nach.* 3888.

SECULAR PERTURBATIONS OF *MARS* FROM THE ACTION OF *VENUS*.

By ERIC DOOLITTLE.

The elements employed in the following computation are from Dr. G. W. Hill's "*New Theory of Jupiter and Saturn*," pages 192 and 554 :

Mars.

$$\begin{aligned} \pi &= 333^\circ\ 17'\ 51''.74 \\ i &= 1\ \ 51\ \ \ 2.24 \\ \Omega &= 48\ \ 23\ \ 54.59 \\ e &= 0.093\,26803 \\ n &= 689\,050''.784 \\ \log a &= 0.182\,8971 \\ m &= 1 \div 3\,093\,500 \end{aligned}$$

Epoch 1850.0 G.M.T.

Venus.

$$\begin{aligned} \pi' &= 129^\circ\ 27'\ 42''.83 \\ i' &= 3\ \ 23\ \ 35.01 \\ \Omega' &= 75\ \ 19\ \ 53.08 \\ e' &= 0.006\,84311 \\ n' &= 2\,106\,641''.357 \\ \log a' &= 9.859\,3378 \\ m' &= 1 \div 408\,134 \end{aligned}$$

Epoch 1850.0 G.M.T.

The values obtained for the preliminary constants are as follows :

$$
\begin{array}{rcl|rcl}
I &=& 1^\circ\,56'\,2''.460 & \log k &=& p9.9999\,439 \\
\Pi &=& 232\;18\;22.07 & \log k' &=& p9.9998\,087 \\
\Pi' &=& 28\;26\;43.81 & \log e &=& p5.3891\,826 \\
K &=& 203\;52\;27.49 & e &=& +0.0000\,24500\,932 \\
K' &=& 203\;50\;49.03 & & &
\end{array}
$$

The orbit of *Mars* was divided into twelve parts in regard to the eccentric anomaly; the sums of the functions corresponding respectively to the odd and even points of division were in as close agreement as could have been anticipated when the high eccentricity of the orbit of *Mars* is considered. After the computation was finished it was duplicated from the beginning, the form of the equations being changed as much as possible in the duplication. All known tests were also applied; the equation arising from the constancy of the major-axis,

$$\sin q \,.\, \tfrac{1}{2} A_1^{(s)} + \cos q \,.\, B_0^{(c)} = 0$$

was found to give the residual +0.000,000,007.

If the mass of *Venus* is left indefinite the values of the differential coefficients are as follows:

			log coeff
$\left[\dfrac{de}{dt}\right]_{00} =$	$+\quad 324''.6318$	m'	$p2.511\,39105$
$\left[\dfrac{d\chi}{dt}\right]_{00} =$	$+201915.56$	m'	$p5.305\,16975$
$\left[\dfrac{di}{dt}\right]_{00} =$	$-\quad 5236.2608$	m'	$n3.719\,02125$
$\left[\dfrac{d\Omega}{dt}\right]_{00} =$	$+126021.28$	m'	$p5.100\,4439$
$\left[\dfrac{d\pi}{dt}\right]_{00} =$	$+201981.28$	m'	$p5.305\,31115$
$\left[\dfrac{dL}{dt}\right]_{00} =$	$+1681713.6$	m'	$p6.225\,7520$

The Flower Observatory, 1907 *June* 20.

If we adopt the above value of m', ($m' = 1 \div 408\,134$), the following results are obtained:

$$\left[\frac{de}{dt}\right]_{00} = +0''.00079\,54049$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +0.49472\,856$$

$$\left[\frac{di}{dt}\right]_{00} = -0.01282\,9757$$

$$\left[\frac{d\Omega}{dt}\right]_{00} = +0.30877\,426$$

$$\left[\frac{d\pi}{dt}\right]_{00} = +0.49488\,961$$

$$\left[\frac{dL}{dt}\right]_{00} = +4.12049\,33$$

The results obtained by LEVERRIER were published in the *Annales de l'Observatoire de Paris*, Tome II, page 59 and Tome VI, page 189; those obtained by NEWCOMB are in "*The Secular Variations of the Orbits of the Four Inner Planets*," pages 336 and 378. If these results are all reduced to the above value of m', they will compare with those here given as follows:

	LEVERRIER	NEWCOMB	Method of GAUSS
$\left[\dfrac{de}{dt}\right]_{00} =$	$+0''.000\,80$	$+0''.000\,79$	$+0''.000\,795$
$e\left[\dfrac{d\pi}{dt}\right]_{00} =$	$+0.046\,18$	$+0.046\,14$	$+0.046\,1574$
$\sin i\left[\dfrac{d\Omega}{dt}\right]_{00} =$	$+0.009\,93$	$+0.009\,98$	$+0.009\,972$
$\left[\dfrac{di}{dt}\right]_{00} =$	$-0.012\,80$	$-0.012\,84$	$-0.012\,830$
$\left[\dfrac{dL}{dt}\right]_{00} =$	$+4.117$	....	$+4.120\,493$

CONTENTS.

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OBSERVATIONS OF THE FIFTH SATELLITE OF *JUPITER* AT THE OPPOSITION OF 1906–7,

By E. E. BARNARD.

During the opposition of 1906–7, the atmospheric conditions were very poor for observations of such a faint object as the fifth satellite of *Jupiter*. The great brilliancy of the planet itself made the sky near the disc very white when the seeing was poor, and the satellite became lost in this, not only from its blurred image, but also for the want of contrast by a dark background. The present nearness of the planet, therefore, does not seem to have been so favorable for observations of this satellite as others apparently less favorable in point of distance.

At the observations of Dec. 8, the measures had to be hurriedly closed, as the telescope was wanted for other work. In the haste I failed to record the reading of the position-circle for the setting of the wires. They had been placed closely parallel with the belts, so that they will not differ very much from the position-angle determined for the belts that night.

In the observations of this object the proximity of one or the other of the bright satellites is often a source of annoyance, and in some cases the faint satellite is entirely lost because of the brightness of the larger one. This was especially the case on Dec. 18. At the first measures on that date when satellite V was west of the planet, satellite I was near and made it exceedingly difficult to see the faint satellite, and finally stopped the observation. Then later in the night, when the satellite was on the east side, satellite I had passed to that side of the planet also and again interfered and for awhile entirely blotted out V.

The observations are in Central Standard Time ($6^h\ 0^m\ 0^s$ slow of G.M.T.).

Following are determinations of the position-angles of the belts of *Jupiter*. The numbers in parenthesis are the independent settings of the wires.

		h	m	°	
1906 Sept.	15	15	5	96.29	(6)
	29	15	40	96.38	(5)
Oct.	2	14	20	96.30	(5)
1906 Oct.	6	15	15	96.66	(5)
	16	13	40	97.16	(5)
	30	13	0	97.08	(5)
Nov.	10	12	15	96.96	(5)
	13	12	50	96.53	(5)
1907 Feb.	3	13	0	91.83	(5)*
	5	6	0	92.64	(5)
	19	7	45	92.56	(6)

* Seeing so excessively bad that belts only seen once in a while.

Following are the computed values of the apparent semi-diameters of *Jupiter* used in the reductions. They are from my values of the diameters of the planet printed in *A.J.* 325.

		Apparent Polar Semidiameter	Apparent Equatorial Semidiameter
		″	″
1906 Dec.	8	22.187	....
	15	22.368	....
	18	22.413	23.908
1907 Feb.	5	....	22.615

The measures of the satellite follow:

SATELLITE PRECEDING.

1906 Dec. 8 h	m	s	From limb ″	Appt. latitude ″	Obs.	
12	9	30	19.96	+2.23	5	From N. limb
12	13	12	25.88	+3.70	6	S. "
12	17	29	19.57	+2.62	6	N. "
12	21	45	25.43	+3.24	5	S. "

The wires were set parallel to the belts of *Jupiter*, but the observations were interrupted, and I failed to get the readings of the position-circle.

The satellite was well seen, but observations could not be carried further, as the telescope was required for other work.

SATELLITE FOLLOWING.

1906 Dec. 15 h m s	From limb ″	Appt. latitude ″	Obs.	
16 38 20	24.35	+1.98	6	From S. limb
16 41 58	21.46	+0.91	6	N. "
16 46 10	24.35	+1.98	6	S. "
16 50 19	21.48	+0.89	6	N. "
16 54 17	23.71	+1.34	6	S. "
16 58 5	21.62	+0.75	6	N. "
17 2 11	23.59	+1.23	6	S. "
17 6 10	21.86	+0.51	6	N. "
17 10 36	23.45	+1.08	6	S. "
17 23 44	21.95	+0.42	4	v.faint N. "

Position-angle of wires 95°.87.

Satellite fairly well seen in most of the measures. Sky very transparent. In the above measures there seems to be a constant difference of about 1″ in the apparent latitudes as determined from the N. and S. limbs, which I am unable to account for.

SATELLITE PRECEDING.

1906 Dec. 18 h m s	From limb ″	Appt. latitude ″	Obs.	
9 18 27	23.77	− 1.35	5	From N. limb
9 23 12	21.34	− 1.07	5	S. "
	From limb	From center	Obs.	
9 32 21	32.65	56.55	3	From P. limb
9 43 53	34.31	58.22	3	P. "
9 54 11	35.10	59.01	2	P. "

Position-angle of wires at latitude measures 94°.85.

Very poor seeing and a bright satellite near makes the measures very difficult.

(*Continued.*)

SATELLITE FOLLOWING.

h m s	From limb ″	Appt. latitude ″	Obs.	
15 18 47	23.26	+0.84	3	From S. limb
15 20 21	23.35	+0.94	3	S. "
15 21 51	21.42	+0.99	3	N. "
15 23 17	22.26	+0.15	3	N. "

Position-angle of wires 93°.75.

h m s	From limb ″	From center ″	Obs.	
15 26 36	32.20	56.11	3	From F. limb
15 28 38	32.58	56.49	4	F. "
15 30 47	80.73	56.83	3	P. "
15 32 48	81.26	57.36	4	P. "
15 35 11	33.70	57.61	3	F. "
15 37 25	33.73	57.64	4	F. "
15 40 22	81.96	58.05	3	P. "
15 42 19	82.40	58.49	4	P. "
15 44 22	34.60	58.51	3	F. "
15 46 33	34.86	58.77	4	F. "
15 49 16	83.33	59.42	3	P. "
15 51 40	83.57	59.64	5	P. "
15 53 59	25.15	59.06	3	F. "
15 55 44	35.63	59.53	3	F. "
15 58 4	35.48	59.39	3	From F. limb
15 59 42	35.44	59.35	3	F. "
16 1 21	35.23	59.14	2	F. "
16 3 12	83.68	59.77	3	P. "
16 6 3	84.56	60.65	4	P. "
16 8 31	35.22	59.13	3	F. "
16 10 44	35.31	59.22	3	F. "
16 13 17	34.81	58.71	3	F. "
16 16 9	34.87	58.78	3	F. "
16 18 41	35.42	59.33	3	F. "
16 20 46	35.09	58.99	3	F. "
16 22 50	35.10	59.00	3	F. "

The proximity of satellite I completely blotted out V for half an hour at this time.

(*Continued.*)

h m s	From limb ″	Appt. latitude ″	Obs.	
16 53 39	23.51	−1.10	4	From N. limb
16 57 42	23.80	−1.38	4	N. "
17 1 54	21.08	−1.33	4	S. "
17 4 53	21.53	−0.88	4	S. "
17 7 49	24.16	−1.75	3	N. "
17 10 7	24.49	−2.08	3	N. "

Position-angle of wires 93°.75.

The phase correction on preceding limb was 0″.02, but has not been applied in the observations.

1907 Feb. 5. h m s	From limb ″	From center ″	Obs.	
10 58 2	30.30	52.71	3	From F. limb
11 1 2	30.41	52.81	3	F. "
11 3 7	31.34	53.74	2	F. "
11 5 27	76.34	53.72	3	P. "
11 8 3	76.77	54.16	3	P. "
11 12 27	31.81	54.22	3	F. "
11 15 40	32.59	55.00	3	F. "
11 18 53	32.81	55.22	3	F. "
11 21 11	32.92	55.33	3	F. "
11 25 58	33.89	55.29	3	F. "
11 28 50	34.21	56.61	3	F. "
11 32 22	33.60	56.00	3	F. "
11 36 7	33.87	56.28	3	F. "
11 38 45	33.78	56.19	3	F. "
11 41 22	34.05	56.46	3	F. "
11 44 15	33.28	55.69	3	F. "
11 47 52	34.04	56.45	3	F. "
11 51 23	34.33	56.74	3	F. "
11 58 7	33.91	56.31	3	F. "
12 4 25	33.27	55.67	3	F. "
12 8 55	32.23	54.64	2	F. "
12 10 40	32.33	54.74	2	F. "

The satellite was seen with the utmost difficulty throughout. Sky whitish and bad seeing. The phase, for which the measures have been corrected, was 0″.208 on the following limb.

Yerkes Observatory, 1907 *June* 3.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENN., WITH A $4\frac{1}{2}$-INCH REFRACTOR,

BY A. W. QUIMBY.

1907		Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Jan.	1	8	5	6	44	2	fair
	2	8	–	5	24	2	poor
	4	2	1	6	38	1	fair
	5	9	–	6	59	2	fair
	6	8	–	6	54	3	fair
	7	8	–	2	19	2	fair
	9	8	2	3	11	2	poor
	10	8	1	5	24	2	fair
	11	9	1	4	22	3	fair
	20	8	3	4	18	2	poor
	21	8	1	4	11	2	poor
	22	4	–	2	5	–	poor
	23	8	1	5	18	2	poor
	24	8	1	6	14	2	poor
	25	9	–	6	12	2	poor
	26	4	–	5	12	3	good
	27	8	–	4	7	2	poor
	28	2	2	5	14	2	fair
	29	8	1	6	22	1	fair
	30	8	–	6	18	1	fair
	31	8	1	7	48	2	fair
Feb.	1	10	–	6	25	1	poor
	2	12	–	4	60	3	good
	3	8	–	4	40	2	poor
	5	4	3	7	31	3	poor
	6	2	1	7	45	2	fair
	7	9	–	7	44	3	poor
	8	2	–	6	50	2	fair
	9	3	–	5	55	2	fair
	10	8	–	5	38	2	poor
	11	8	1	6	53	2	poor
	12	8	1	7	84	2	poor
	13	8	–	6	80	1	fair
	14	8	1	6	54	3	poor
	15	8	–	5	64	3	poor
	16	8	1	5	46	3	fair
	17	8	–	4	56	2	fair
	18	8	–	4	23	2	fair
	19	9	1	4	20	1	fair
	21	8	1	5	28	1	fair
	22	8	–	5	30	1	fair
	23	4	1	6	46	3	fair
	24	8	1	7	28	2	fair
	25	8	–	6	22	1	fair
	26	8	–	5	11	2	poor
	27	8	2	7	12	2	poor
	28	8	1	7	10	3	poor
Mar.	1	8	–	5	20	3	fair
	2	3	1	7	20	3	fair
	3	8	–	7	23	3	fair
	4	8	1	8	27	3	fair
	5	8	–	7	28	3	fair
	6	11	–	6	20	4	fair
	7	8	–	5	24	3	fair
Mar.	8	2	–	5	55	3	fair
	9	4	–	4	38	2	fair
	11	8	–	4	17	2	poor
	12	7	–	3	12	2	fair
	13	6	–	3	7	3	poor
	14	12	–	4	11	2	fair
	15	5	4	7	11	4	fair
	16	9	1	8	15	4	fair
	17	7	–	6	14	4	fair
	18	7	–	4	8	3	fair
	20	7	–	3	12	2	fair
	21	7	–	3	8	2	fair
	22	7	–	2	7	1	poor
	23	7	–	2	8	2	poor
	24	7	–	2	13	2	fair
	25	7	–	2	5	2	fair
	26	10	1	2	3	2	poor
	27	8	–	2	3	2	poor
	28	8	–	1	2	2	fair
	29	8	2	3	5	2	fair
	30	7	–	2	2	1	fair
Apr.	1	4	3	5	20	2	fair
	2	8	–	4	15	2	fair
	3	7	–	5	17	3	fair
	4	7	–	4	34	3	fair
	5	7	1	4	28	1	fair
	6	8	–	4	18	1	fair
	7	9	–	4	14	1	poor
	8	4	–	4	17	1	poor
	9	4	–	1	2	–	poor
	10	7	–	4	16	3	fair
	11	7	–	3	12	5	fair
	12	8	–	3	3	2	poor
	13	5	2	4	15	1	fair
	14	7	–	4	12	3	poor
	15	7	–	2	12	3	fair
	16	4	–	1	12	3	fair
	17	7	–	1	10	1	poor
	18	7	–	2	13	2	fair
	19	5	2	3	7	3	poor
	20	7	–	3	7	2	fair
	21	7	1	4	27	3	fair
	22	7	–	3	12	2	fair
	23	4	–	3	7	1	poor
	24	7	–	3	7	1	fair
	25	7	1	3	14	2	fair
	26	7	–	2	12	2	fair
	27	4	–	2	10	2	fair
	28	4	–	2	12	3	fair
	29	4	–	2	12	3	fair
	30	8	–	2	6	2	fair
May	1	8	1	1	1	1	poor
	2	7	–	1	1	–	poor
	3	4	2	2	2	8	fair
May	4	4	–	2	28	2	fair
	5	6	–	2	32	2	fair
	6	9	–	2	40	2	fair
	8	7	2	4	36	1	fair
	9	5	–	3	32	1	fair
	10	7	–	2	34	1	fair
	11	7	–	2	30	1	fair
	12	7	–	2	36	2	fair
	13	7	–	2	21	2	fair
	14	7	2	4	24	2	fair
	15	6	–	4	14	3	fair
	17	6	1	2	4	2	fair
	18	7	–	2	5	2	fair
	19	7	1	2	2	2	fair
	20	5	1	3	7	2	fair
	21	6	–	3	5	2	fair
	22	6	–	3	4	1	fair
	23	9	1	4	5	2	fair
	24	7	–	1	3	–	poor
	25	6	1	2	6	3	fair
	27	4	–	1	1	2	fair
	28	6	–	2	6	2	fair
	29	6	1	3	4	3	fair
	30	6	–	2	2	2	fair
	31	6	–	2	2	2	fair
June	3	7	1	2	5	1	fair
	4	7	1	3	7	3	fair
	5	9	–	2	3	3	fair
	6	7	1	3	5	2	fair
	7	7	–	2	3	2	fair
	8	4	–	2	2	2	fair
	9	6	1	3	3	3	fair
	10	6	2	3	4	3	fair
	11	11	–	2	4	1	poor
	12	7	–	2	14	1	poor
	13	6	2	5	19	2	fair
	14	5	–	5	33	1	fair
	15	6	–	5	33	1	fair
	16	6	–	4	37	2	fair
	17	6	–	4	64	2	good
	18	6	–	3	78	3	good
	19	6	–	3	94	3	good
	20	6	–	2	65	3	good
	21	6	–	1	50	4	fair
	22	6	–	1	94	3	good
	23	6	–	1	65	2	fair
	24	7	–	1	34	4	fair
	25	6	2	3	30	2	fair
	26	6	–	2	2	3	fair
	27	6	–	1	2	3	fair
	28	6	–	1	5	2	fair
	30	7	–	1	6	1	poor

ELEMENTS AND EPHEMERIDES OF PLANET 1907 *XP*,

By ASAPH HALL, Jr.

[Communicated by Rear-Admiral ASA WALKER, U.S.N., Superintendent U.S. Naval Observatory.]

From three of the observations made by Messrs. HAMMOND and FREDERICKSON with the 26-inch equatorial on February 14, March 2, and March 20, 1907, I have computed the following elements and ephemerides of Asteroid 1907, *XP*. A comparison, made by Mr. HAMMOND, of the Washington observations with the ephemeris is also given. This asteroid is one of those found by the Reverend J. H. METCALF.

To find the elements, the method of GAUSS was used, as given in the *Theoria Motus*, translation into English of DAVIS, pp. 185 and following. The dates of the ephemeris for 1907 include all the Washington measures. It is sent for possible use in the reduction of observations which have been made.

I have been interested to read in the journals criticisms of the various methods of computing orbits. Apparently some of the writers fail to notice the statement made by GAUSS, *Theoria Motus*, as referred to above, p. 205. "It is incorrect to call one method more or less exact than another. That method alone can be considered to have solved the problem by which any degree of precision whatever is, at least, attainable. Wherefore one method excels another in this respect only, that the same degree of precision may be reached by one more quickly, and with less labor, than by the other."

In the case of this orbit it was necessary to recompute but once the P and Q of GAUSS. Probably the original assumptions would have been sufficient. The checks given for ☊ and ω were not exactly satisfied, values being found which differed by about 3″. But as nothing more was desired than very approximate elements, the computations were not examined.

1907 March 2.5, Berlin Mean Time.

$$M = 171^\circ\ 51'\ 57''.75$$

$$\left.\begin{aligned}\omega &= 294\ \ 7\ 53.9\\ \Omega &= 35\ 24\ 23.5\\ i &= 7\ 56\ 27.7\end{aligned}\right\}1907.0 \qquad \left.\begin{aligned}\omega_1 &= 294^\circ\ \ 7'\ 56''.2\\ \Omega_1 &= 35\ 25\ 11.45\\ i_1 &= 7\ 56\ 27.8\end{aligned}\right\}1908.0$$

$$\phi = 9\ 57\ 10.5$$

$$\mu = 714''.6833$$

$$\log a = 0.463929$$

For the middle observation in the sense Computed — Observed

$$d\lambda \cos\beta = -2''.1 \quad , \quad d\beta = +0''.6$$

For 1907.0 and 1908.0 the heliocentric equatorial co-ordinates are as follows, the ephemerides being computed from these co-ordinates:

1907.0

$$\begin{aligned}x &= r[9.998605] \sin(\ 59^\circ\ 16'\ 40''.35 + v)\\ y &= r[9.938265] \sin(331\ 55\ \ 1.4\ + v)\\ z &= r[9.702307] \sin(321\ 21\ 54.65 + v)\end{aligned}$$

1908.0

$$\begin{aligned}x &= r[9.998603] \sin(\ 59^\circ\ 17'\ 30''.5 + v)\\ y &= r[9.938270] \sin(331\ 55\ 54.3 + v)\\ z &= r[9.702294] \sin(321\ 22\ 34.2 + v)\end{aligned}$$

MEAN EQUINOX OF 1907.0

1907 Berlin M.T.	α h	m	s	δ °	′	″	log Δ	Ab. time m	s
Feb. 6.5	9	51	57.29	+24	23	56.8	0.386049	20	12.3
10.5	9	48	27.03	24	40	16.5	0.385680	20	12.1
14.5	9	44	54.12	24	55	6.2	0.386191	20	12.7
18.5	9	41	21.94	25	8	10.9	0.387568	20	16.6
22.5	9	37	53.95	25	19	20.2	0.389788	20	22.8
26.5	9	34	33.30	25	28	23.9	0.392815	20	31.4
Mar. 2.5	9	31	22.86	25	35	18.5	0.396597	20	42.2
6.5	9	28	25.13	25	40	0.6	0.401088	20	55.1
10.5	9	25	42.47	25	42	30.7	0.406222	21	10.0
14.5	9	23	16.96	25	42	49.8	0.411940	21	26.8
18.5	9	21	10.31	25	41	2.2	0.418169	21	45.4
22.5	9	19	23.71	25	37	13.3	0.424838	22	5.6
26.5	9	17	57.92	+25	31	29.5	0.431873	22	27.3

Opposition in α February 13.

1907	O−C α	O−C δ	1907	O−C α	O−C δ
	s	″		s	″
Feb. 14	+0.09	−1.0	Mar. 2	+0.01	−0.6
15	+0.14	−2.4	6	+0.02	−1.3
18	+0.17	−1.9	11	+0.09	+0.2
22	−0.08	+0.6	15	+0.15	+2.5
			20	+0.11	−0.6

MEAN EQUINOX OF 1908.0.

1908 Berlin M.T.	α			δ			log Δ	Ab. time	
	h	m	s	°	′	″		m	s
Mar. 20.5	13	56	5.44	−8	55	28.6	0.354995	18	48.7
24.5	13	53	39.87	8	47	39.2	0.348524	18	32.0
28.5	13	50	57.91	8	38	50.1	0.342761	18	17.3
Apr. 1.5	13	48	1.61	8	29	5.4	0.337784	18	4.8
5.5	13	44	53.50	8	18	35.5	0.333654	17	54.6
9.5	13	41	36.33	8	7	34.5	0.330418	17	46.6
13.5	13	38	14.31	7	56	17.5	0.328115	17	40.9
17.5	13	34	47.83	7	44	58.9	0.326743	17	37.6
21.5	13	31	22.25	7	33	52.6	0.326331	17	36.6
25.5	13	28	0.07	7	23	14.2	0.326864	17	37.9
29.5	13	24	44.25	7	13	17.9	0.328330	17	41.5
May 3.5	13	21	37.93	7	4	19.6	0.330685	17	47.2
7.5	13	18	43.83	6	56	32.7	0.333880	17	55.1
11.5	13	16	4.19	6	50	8.4	0.337848	18	5.0
15.5	13	13	40.93	−6	45	16.4	0.342517	18	16.7

$m_0 = 12.35 \qquad g = 8.65$

Opposition in α April 15. Mag. = 12.75

THE ECCENTRICITY OF THE ORBIT OF COMET 1894 II,

By HENRY A. PECK.

Several years ago I published in *A.J.* 496-7 a definitive orbit of Comet 1894 II. While the orbit is distinctly elliptic in its character, yet it is indeterminate within certain limits. Owing to lack of experience this indetermination was stated in terms of the SCHÖNFELD function ∂r. More mature reflection has convinced me that the elements should be transformed so that they would be functions of the correction to the eccentricity. In the present state of comet theory, this element is the most important. It is of especial interest to find within what limits it can vary without bringing the orbit into conflict with the observations upon which it rests.

A reference to the paper in question, shows that the observations extended over four and one-half months, during which time the comet covered eleven hours in right ascension and ninety-eight degrees in declination. This long series of observations was united into eleven normal places and the twenty-two equations of condition were formed according to the method of SCHÖNFELD. On account of their lack of homogeneity, new variables were substituted and the revised equations were united into the following normals, the coefficients being given by their logarithms:

$$
\begin{array}{lllllll}
+0.6147\,u & -0.5974\,v & -0.5104\,w & -0.4745\,x & +0.0047\,y & +9.8511\,z & +0.3446 = 0 \\
-0.5974 & +0.6407 & +0.6311 & +0.2490 & -9.8430 & +8.0682 & -9.9619 \\
-0.5104 & +0.6311 & +0.7734 & +9.6764 & +9.6895 & +0.4479 & +9.3304 \\
-0.4745 & +0.2490 & +9.6764 & +0.7088 & -0.1941 & -0.2638 & -0.6394 \\
+0.0097 & -9.8430 & +9.6895 & -0.1941 & +0.4111 & +0.4148 & +8.7076 \\
+9.8511 & +8.0682 & +0.4479 & -0.2638 & +0.4148 & +0.7317 & +0.0149
\end{array}
$$

The quantity w is a function of the form

$$w = 0.6783\,\frac{q}{1+e}\,\partial\frac{1}{a}$$

where q a and e denote the perihelion distance, the semi-major axis and the eccentricity. It is therefore the quantity in terms of which the remaining variables are to be expressed. Using the GAUSS method of substitution

$$\begin{aligned}
u - 9.9827\,v - 9.8598\,x + 9.3900\,y + 9.2364\,z &= -9.7299 + 9.8957\,w \\
v - 0.2824\,x + 9.6836\,y + 0.0857\,z &= -0.3268 - 0.3106\,w \\
x - 9.5476\,y + 7.9293\,z &= +9.7090 - 9.6201\,w \\
y + 0.0008\,z &= +9.7706 - 9.6096\,w \\
z &= -9.2548 - 9.6703\,w
\end{aligned}$$

and therefore

$$\begin{aligned}
\log u &= -9.9386 \quad -0.2033w \\
\log v &= -9.8874 \quad -0.3530w \\
\log x &= +9.8946 \quad -9.5924w \\
\log y &= +9.8863 \quad +8.7919w \\
\log z &= -9.2548 \quad -9.6703w
\end{aligned}$$

Using the relations stated in the article cited above

$$\begin{aligned}
\log \partial k &= -0.1234 \ +9.2982\,\partial e \\
\log \frac{k\partial T}{\sqrt{p}} &= -9.8535 \ +9.2292\,\partial e \\
\log \partial q &= +0.0608 \ +8.6687\,\partial e \\
\log \partial\lambda &= +0.2187 \ -8.0344\,\partial e \\
\log \partial\nu &= -9.9060 \ +9.2306\,\partial e
\end{aligned}$$

From these are found as corrections to the elements

$$\begin{aligned}
\partial T &= -0.000281 \ +0.0000066 8\,\partial e \\
\partial\omega &= -1''.28 \ +0.204\,\partial e \\
\partial\Omega &= -0''.87 \ -0.090\,\partial e \\
\partial i &= -1''.62 \ +0.144\,\partial e \\
\partial q &= -0.0000056 \ +0.000000226\,\partial e
\end{aligned}$$

When these corrections are substituted in the equations of condition the most probable value of ∂e is 243″ and if this value of ∂e is varied the following table may be constructed:

∂e	pvv		
0	1711		
		−199	
−50	1512		+45
		154	
100	1358		47
		107	
150	1251		45
		62	
200	1189		46
		−16	
250	1173		44
		+28	
300	1201		46
		74	
350	1275		45
		119	
400	1394		47
		166	
450	1560		46
		+212	
−500	1772		

Syracuse University, 1907 *Aug.* 20.

It is here to be noted that for fifty seconds on either side of the most probable value of ∂e, pvv shows only a slight proportionate change. In the following elements, the correction to the eccentricity is assumed as −250″ and there is given the variation of the remaining elements corresponding to a variation of ±100″ in the eccentricity.

T =	April 13.038995	±0.00668 Gr.M.T.	
w =	324° 11′ 30″.7	±20″.4	1894.0
Ω =	206 24 15.9	∓ 9.0	1894.0
i =	86 58 41.5	±14.4	1894.0
q =	0.983032	±0.000024	
e =	0.989889	±0.000485	

according to the above equations

$$\partial\nu = -0''.805 \quad +0.170\,\partial e$$

therefore when $\partial e = -250''$, $\partial\nu = -43''.3$ and these elements are found to be identical with those previously published when the value of e is then corrected by removing the decimal point in the coefficient of $\partial\nu$ one place to the right.

No great reliance can be placed upon the value of the eccentricity beyond the third place of decimals, and the remaining places must be considered as belonging to the accidents of computation rather than representing any fact. Translated into time it represents a period of about nine hundred and fifty-nine years, and a variation of one hundred seconds in the eccentricity is equivalent in round numbers to a variation of half a century in the resulting period.

REQUEST FOR UNPUBLISHED OBSERVATIONS OF *U GEMINORUM*,

By J. A. PARKHURST.

Mr. J. van der Bilt, Astronomer at the observatory, Utrecht, Holland, has undertaken the definitive reduction of all available observations of this remarkable variable, and would be very glad to have copies of any unpublished observations, in such detail that they can be reduced by a normal photometric light-scale. They may be sent to him direct, address *Maliesingel* 58, *Utrecht*, or if sent to the undersigned they will be transmitted to him.

Yerkes Observatory, Williams Bay, Wis.

MAXIMA OF LONG-PERIOD VARIABLES.

By IDA WHITESIDE

The maxima of the following long-period variables were determined by the single-light curves, deduced from observations made with a four-inch DOLLAND telescope. The predicted times of maximum are those given in CHANDLER'S "Ephemerides of Long-Period Variables." The last column gives the authority for the magnitudes of the comparison stars used.

Star	Date of Maximum	Predicted Date	Mag.	No. Obs.	Time Covered by Observations	Comp. Star
434 *S Piscium*	Jan. 18, 1907	Dec. 22, 1906	9.70	10	Oct. 12, 1906–Mar. 6, 1907	Harvard
782 *R Arietis*	Jan. 18, 1907	Jan. 3, 1907	8.20	11	Nov. 14, 1906–Mar. 18, 1907	Harvard
1113 *U Arietis*	Jan. 7, 1907	Jan. 10, 1907	8.55	10	Nov. 14, 1906–Mar. 20, 1907	Harvard
1222 *R Persei*	Mar. 13, 1907	Mar. 25, 1907	9.00	7	Jan. 21, 1907–Apr. 1, 1907	Harvard
1623 *T Camelopardalis*	Feb. 21, 1907	Mar. 6, 1907	8.30	10	Jan. 21, 1907–Apr. 18, 1907	Hagen
1717 *V Tauri*	Mar. 12, 1907	Mar. 12, 1907	8.75	9	Jan. 21, 1907–Apr. 18, 1907	Hagen
1761 *R Orionis*	Dec. 11, 1906	Jan. 25, 1907	9.30	10	Nov. 14, 1906–Mar. 25, 1907	Harvard
1805 *V Orionis*	Feb. 17, 1907	Mar. 17, 1907	9.05	9	Jan. 21, 1907–Apr. 18, 1907	Hagen
2478 *R Lyncis*	Nov. 3, 1906	Dec. 9, 1906	7.25	12	Oct. 12, 1906–Mar. 6, 1907	Harvard
2625 *V Geminorum*	Apr. 11, 1907	Apr. 24, 1907	8.25	9	Mar. 6, 1907–May 13, 1907	Harvard
2780 *T Geminorum*	Apr. 8, 1907	Apr. 22, 1907	8.60	12	Feb. 15, 1907–May 23, 1907	Hagen
2942 *RT Cygni*	June 13, 1907	June 26, 1907	6.25	11	May 13, 1907–Aug. 22, 1907	B.D.
2976 *V Cancri*	Mar. 13, 1907	Feb. 22, 1907	7.70	14	Jan. 11, 1907–May 23, 1907	Harvard
3493 *R Leonis*	Feb. 23, 1907	Mar. 12, 1907	5.50	11	Jan. 21, 1907–May 3, 1907	Harvard
3825 *R Ursae Majoris*	Dec. 28, 1906	Jan. 16, 1907	7.05	12	Nov. 14, 1906–Mar. 25, 1907	Harvard
4315 *R Comae*	June 29, 1907	June 29, 1907	7.30	10	Apr. 11, 1907–July 18, 1907	Harvard
4511 *T Ursae Majoris*	Apr. 11, 1907	Apr. 3, 1907	7.90	15	Feb. 21, 1907–July 3, 1907	Harvard
4521 *R Virginis*	Apr. 8, 1907	May 2, 1907	6.40	11	Mar. 6, 1907–June 8, 1907	Harvard
4557 *S Ursae Majoris*	July 4, 1907	June 26, 1907	7.80	15	Apr. 11, 1907–Aug. 14, 1907	Harvard
5237 *R Bootis*	May 18, 1907	May 18, 1907	7.75	13	Apr. 1, 1907–July 30, 1907	Harvard
5770 *R Herculis*	June 5, 1907	May 10, 1907	9.50	11	Apr. 11, 1907–July 30, 1907	Harvard
6512 *T Herculis*	June 30, 1907	June 26, 1907	7.80	11	May 13, 1907–Aug. 14, 1907	Harvard

South Cambridge, N.Y.

OBSERVATIONS OF COMETS,

MADE WITH THE 11-INCH EQUATORIAL AT SMITH COLLEGE OBSERVATORY, NORTHAMPTON, MASS.

By HARRIET W. BIGELOW.

1906 Gr. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$		Red. to App. Pl.	
			COMET *d* 1906 (FINLAY).							
	h m s		m s	′ ″	h m s	° ′ ″			s	″
Aug. 14 19 3 28	1	7 , 8	+0 4.04	+1 2.0	3 42 17.83	+ 7 47 12.8	9.578	0.732	+1.10	+3.0
15 19 35 6	3	12 , 9*t*	−1 5.50	−3 57.6	3 50 36.61	+ 8 31 32.5	9.544	0.720	+1.09	+2.5
16 19 14 17	5	7 , 8	−0 11.04	−1 48.0	3 58 28.86	+ 9 12 45.3	9.576	0.722	+1.13	+2.4
17 19 16 7	7	9 , 7	−0 17.57	−0 7.1	4 6 17.27	+ 9 53 1.0	9.579	0.719	+1.08	+1.7
18 19 26 56	9	7 , 7	−0 15.22	−3 19.1	4 13 58.09	+10 31 44.8	9.572	0.713	+1.07	+1.3
Sept. 14 20 18 15	11	7 , 7	+0 24.68	−0 4.0	6 37 47.66	+19 14 28.3	9.572	0.634	+1.12	−5.0
15 20 24 0	12	7 , 7	−0 6.20	−0 57.8	6 41 27.16	+19 21 49.5	9.563	0.628	+1.12	−5.1
17 20 25 50	14	6 , 8	−0 16.90	−6 13.9	6 48 30.31	+19 34 41.9	9.560	0.624	+1.13	−5.4
18 19 28 35	15	8 , 8	+0 1.40	+3 12.4	6 51 47.84	+19 40 26.2	9.625	0.669	+1.13	−5.4
23 19 13 40	16	10 , 10	−0 12.48	+0 56.7	7 7 44.92	+20 4 14.8	9.635	0.675	+1.19	−6.2
27 19 14 2	18	8 , 8	+0 2.13	+7 0.2	7 19 21.24	+20 17 57.6	9.632	0.670	+1.26	−6.8
			COMET *g* 1906 (THIELE).							
Nov. 14 18 29 15	19	6 , 6	+0 12.46	+5 10.4	9 34 22.90	+17 28 18.5	9.617	0.682	+1.77	−13.5
16 19 23 21	21	7 , 7	−0 32.66	+5 9.8	9 44 14.95	+20 7 31.6	9.564	0.620	+1.78	−14.9
Dec. 11 18 53 44	22	5 , 6	−0 12.12	+3 30.1	12 36 5.87	+51 5 44.0	9.831	0.425	+0.64	−25.9
18 19 37 1	24	12 , 7*t*	+0 43.94	−1 17.8	13 35 27.89	+55 39 9.6	9.872	0.292	−0.07	−25.0

Measures for $\Delta\alpha$ marked *t* were made by transits, all others directly with the micrometer.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	3 42 12.69	+ 7 46 7.8	Micr. comp. with 2	14	6 48 46.08	+19 41 1.2	A.G. 2467, Berlin A
2	3 42 49.96	+ 7 39 2.5	A.G. 1391, Leipzig II	15	6 51 45.41	+19 37 19.2	A.G. 2489, Berlin A
3	3 51 41.02	+ 8 35 27.6	Micr. comp. with 4	16	7 7 56.21	+20 3 24.3	Micr. comp. with 17
4	3 52 9.95	+ 8 39 1.1	A.G. 1452, Leipzig II	17	7 6 16.58	+20 2 29.1	A.G. 2824, Berlin B
5	3 58 38.77	+ 9 14 30.9	Micr. comp. with 6	18	7 19 17.85	+20 11 4.2	A.G. 2933, Berlin B
6	3 52 49.72	+ 9 21 16.6	A.G. 1457, Leipzig II	19	9 34 8.67	+17 23 21.6	Micr. comp. with 20
7	4 6 33.76	+ 9 53 6.4	Micr. comp. with 8	20	9 35 13.57	+17 23 50.1	A.G. 3888, Berlin A
8	4 8 25.64	+ 9 58 24.2	A.G. 1548, Leipzig II	21	9 44 45.83	+20 2 36.7	A.G. 3854, Berlin B
9	4 14 12.24	+10 35 2.6	Micr. comp. with 10	22	12 36 17.35	+51 2 39.8	B.D. 51°.1775 Micr. comp. with 23
10	4 16 31.63	+10 32 56.1	A.G. 1273, Leipzig I	23	12 36 58.90	+50 55 53.3	A.G. 4121, Camb. U.S.
11	6 37 21.86	+19 14 37.3	A.G. 2337, Berlin A	24	13 34 44.02	+55 40 52.4	Micr. comp. with 25
12	6 41 32.24	+11 22 52.4	Micr. comp. with 13	25	13 34 46.55	+55 47 51.9	A.G.7644, Hels.–Gotha
13	6 42 23.68	+19 16 55.0	A.G. 2390, Berlin A				

THE TRANSIT OF COMET 1819 II ACROSS THE SOLAR DISC,

By HENRY A. PECK.

On the morning of the 26th of June, 1819, several days before its discovery, the second comet of 1819 made a transit across the solar disc. As often happens, several amateur astronomers produced evidence to substantiate the claim that they had been witnesses of the interesting event and at various times this evidence has been debated. These claims have been perpetuated in several of the text books and popular works of the past generation, notably in Webb's "Celestial Objects for Common Telescopes," where a diagram is given reproducing a drawing which Pastorff, of Bucholtz, is said to have made. Schumacher discusses this observation adversely in the *Beilage* to the *Astronomische Nachrichten*, No. 87. Olbers agreed with Schumacher in declaring there was no evidence that Pastorff had seen the comet. Hind undertook his orbit for the purpose of examining the question. While perhaps the discussion may now be considered as no longer of particular interest, yet it may be well to place on record the apparent path described by the comet during the few hours in question on that June morning.

In the *A.J.* No. 593, I have given definitive parabolic elements as follows.

$$T = 1819 \text{ June } 27.71814 \text{ Gr.M.T.}$$

$$\left.\begin{array}{rl} \Omega = & 273^\circ\ 42'.23 \\ i = & 80\ 44.83 \\ w = & 13\ 26.36 \end{array}\right\} 1819.0$$

$$\log q = 9.53341$$

Syracuse University, 1907 Aug. 29.

Using five-place logarithms, as being within the limits of accuracy desired, we have the following positions of the comet and sun corrected for aberration but not for parallax:

	λ	β	⊙	λ−⊙
	° ′	′	° ′	′
June 25.5	93 33.1	−58.3	93 30.0	+3.1
.6	38.4	37.4	35.7	2.7
.7	43.7	−16.6	41.4	2.3
.8	49.0	+ 4.2	47.1	1.9
25.9	54.3	25.0	52.8	1.5
26.0	59.6	+45.7	58.6	+1.0

The apparent path is here seen to be almost perpendicular to the ecliptic and the comet at its nearest approach to the center of the sun was only about two minutes distant in the direction of increasing longitudes. Except in the neighborhood of the point of nearest approach the latitudes and distances from the center of the sun do not differ from one another by more than one or two-tenths of a minute of arc. If the radius of the sun be taken at 15′.8 a simple interpolation in the table of distances shows the first contact of the nucleus to have occurred June 25.705 and that the time of final passage from the disc was June 25.855. An examination of both Pastorff's diagram, as well as the observation of Canon Stark, serves to confirm Hind's conclusion. Whatever may have been observed by them, it certainly is scarcely within the range of possibilities that they saw the comet.

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OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 26-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By J. C. HAMMOND and M. FREDERICKSON.

[Communicated by Rear-Admiral Asa Walker, U.S.N., Superintendent.]

1907 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(568) *Cheruskia*.					
h m s			m s	′ ″	h m s	° ′ ″			s	″
Jan. 5 10 30 1	1	25 , 5	−1 48.15	−0 45.9	6 57 17.97	+5 15 19.2	n9.240	0.700	+0.51	−6.0†
5 10 50 26	2	25 , 5	−1 57.25	−5 41.5	6 57 17.28	+5 15 14.8	n9.131	0.688	+0.51	−6.0†
10 11 24 0	3	25 , 5	+2 11.41	+9 12.1	6 52 23.53	+4 53 37.3	n8.311	0.690	+0.56	−6.9*
10 11 35 45	4	25 , 5	−0 1.49	−2 36.9	6 52 23.06	+4 53 35.2	7.492	0.690	+0.56	−6.8*
					(351) *Yrsa*.					
Jan. 5 12 13 24	5	25 , 5	−2 17.95	−3 26.5	8 7 19.66	+23 58 16.2	n9.078	0.365	+0.38	−6.4†
10 12 34 37	6	15 , 5	+0 17.32	−6 20.0	8 3 0.10	+24 36 59.1	n8.342	0.332	+0.48	−6.5†
20 10 39 15	7	25 , 5	−1 26.17	−2 6.8	7 53 41.88	+25 50 38.8	n9.224	0.328	+0.62	−6.2*
20 10 56 18	8	25 , 5	−1 33.82	−1 31.3	7 53 41.17	+25 50 43.8	n9.118	0.314	+0.62	−6.2*
					(245) *Vera*.					
Jan. 26 11 16 38	9	15 , 3	−1 45.96	−5 14.5	8 38 53.13	+25 34 4.4	n9.125	0.323	+0.65	−6.7†
27 9 56 11	10	25 , 5	−2 32.55	−3 49.3	8 38 1.35	+25 37 35.4	n9.455	0.399	+0.66	−6.6*
Feb. 6 8 33 43	11	25 , 5	−0 42.90	−0 34.7	8 29 10.99	+26 9 36.6	n9.539	0.434	+0.74	−6.0*
8 9 55 29	12	25 , 5	+0 30.52	−0 51.6	8 27 27.34	+26 14 53.2	n9.240	0.318	+0.75	−5.9*
11 9 10 32	13	25 , 5	−0 18.69	+1 37.3	8 25 3.60	+26 21 36.0	n9.374	0.348	+0.76	−5.7*
12 10 43 45	14	25 , 5	−1 22.80	−4 40.8	8 24 13.95	+26 23 47.1	n8.438	0.274	+0.76	−5.6†
15 8 39 58	15	25 , 5	−2 11.38	−1 7.6	8 22 4.02	+26 28 58.5	n9.415	0.360	+0.76	−5.3*
18 7 50 38	16	25 , 5	−0 1.31	−3 7.8	8 20 0.94	+26 33 13.0	n9.512	0.407	+0.74	−5.0†
20 12 20 20	17	20 , 4	+2 59.55	−1 26.4	8 18 36.80	+26 35 41.8	9.413	0.356	+0.72	−4.9*
25 8 23 10	18	25 , 5	+1 25.08	−0 31.0	8 15 53.40	+26 39 10.6	n9.308	0.321	+0.69	−4.6*
27 8 28 57	19	25 , 5	+1 36.47	+2 24.6	8 14 55.03	+26 39 53.2	n9.238	0.305	+0.67	−4.3*
Estimated magnitude: Feb. 6, 11.5; Feb. 15, 11.5.										
					(509) *Jolanda*.					
Feb. 10 9 44 29	20	25 , 5	+2 2.99	−3 36.2	8 10 25.26	−2 14 55.4	n9.113	0.760	+0.77	−8.9†
10 10 5 34	21	25 , 5	+2 17.98	+6 34.8	8 10 24.68	−2 14 52.5	n8.948	0.760	+0.77	−9.0†
13 9 54 4	22	25 , 5	+0 20.81	+0 15.0	8 8 24.90	−1 59 45.0	n8.925	0.758	+0.76	−9.1*
13 10 11 7	23	25 , 5	+1 58.17	+1 10.5	8 8 24.47	−1 59 40.8	n8.702	0.758	+0.76	−9.2*
					(588) *Achilles*.					
Feb. 6 13 23 1	24	6 , 6	+0 12.72	+3 1.2	11 49 39.93	−6 55 29.9	n9.206	0.795	+0.50	−1.2*
11 12 38 7	25	12 , 4	−1 29.68	−2 56.1	11 48 0.02	−6 54 56.7	n9.310	0.792	+0.60	−2.2*
15 12 44 44	26	18 , 5	+0 54.43	−1 24.2	11 46 29.03	−6 52 58.8	n9.205	0.794	+0.68	−2.9*
Mar. 6 11 54 24	27	9 , 2	+0 33.05	−1 39.1	11 37 42.06	−6 27 18.9	n8.983	0.794	+0.95	−5.7*
15 12 35 51	28	25 , 5	−1 55.48	−1 57.9	11 33 2.26	−6 7 28.3	8.826	0.792	+1.01	−6.7†
20 12 49 11	29	30 , 6	−0 3.85	+3 42.9	11 30 27.04	−5 55 5.7	9.135	0.788	+1.02	−7.2†
Estimated magnitude; Feb. 6, 14–15; Mar. 15, 14.0.										

1907 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(105) *Artemis*.					
h m s			m s	′ ″	h m s	° ′ ″			s	″
Feb. 12 9 53 55	30	25 . 5	−1 0.77	+ 1 36.6	9 28 38.90	−11 14 16.5	n9.389	0.815	+0.92	−6.8†
15 10 37 25	31	25 . 5	−2 2.72	+ 4 39.6	9 25 54.49	−10 41 54.9	n9.137	0.821	+0.93	−7.2*
15 10 54 48	32	25 . 5	−2 15.36	+ 2 53.6	9 25 53.52	−10 41 47.8	n9.012	0.822	+0.93	−7.2*
Estimated magnitude; Feb. 15, 11.5.										
					[1907 *XP*.]					
Feb. 14 10 1 48	33	25 . 5	+1 19.24	− 3 1.2	9 44 45.42	+24 55 36.7	n9.424	0.403	+0.80	−6.6†
15 9 27 31	34	25 . 5	−1 0.89	− 0 37.7	9 43 53.52	+24 58 55.5	n9.503	0.438	+0.80	−6.5*
18 9 7 39	35	25 . 5	−3 4.74	+ 0 30.3	9 41 15.44	+25 8 29.2	n9.516	0.441	+0.82	−6.3†
22 9 47 38	36	18 . 4	+2 40.96	− 6 47.7	9 37 45.96	+25 19 41.2	n9.341	0.367	+0.83	−6.0*
Mar. 2 8 12 9	37	25 . 5	−1 33.72	+ 0 6.7	9 31 18.86	+25 35 23.1	n9.514	0.430	+0.84	−5.2†
6 9 41 15	38	25 . 5	+1 18.99	− 0 45.7	9 28 18.75	+25 40 5.1	n9.060	0.314	+0.81	−4.8*
11 8 22 54	39	25 . 5	+2 34.68	+ 3 45.8	9 25 0.91	+25 42 47.0	n9.362	0.362	+0.78	−4.4*
15 8 24 52	39	18 . 5	+0 14.22	+ 3 29.9	9 22 40.41	+25 42 31.6	n9.278	0.342	+0.74	−3.9*
20 7 54 53	40	30 . 5	+0 35.36	+ 1 47.6	9 20 12.52	+25 39 15.0	n9.313	0.351	+0.68	−3.5*
Estimated magnitude; Feb. 14, 12.8; Feb. 15, 13.0; Mar. 2, 13.1; Mar. 6, 13.5; Mar. 11, 13.7; Mar. 15, 13.7.										
					(375) *Ursula*.					
Feb. 14 10 57 0	41	25 . 5	+1 28.94	− 4 39.8	10 21 31.21	+13 7 20.3	n9.330	0.599	+0.76	−6.7†
					(52) *Europa*.					
Feb. 25 9 17 52	42	25 . 5	−1 24.77	− 5 57.2	10 22 2.58	+15 7 40.4	n9.494	0.598	+0.84	−6.9*
27 9 15 16	43	25 . 5	−0 40.62	− 1 39.9	10 20 32.02	+15 20 36.5	n9.480	0.591	+0.85	−6.8*
Mar. 4 9 41 29	44	25 . 5	−0 55.79	+ 2 37.0	10 16 50.02	+15 51 30.7	n9.336	0.559	+0.87	−6.5*
Estimated magnitude; Feb. 25, 11.0.										
					[1907 *XO*.]					
Mar. 11 10 40 1	45	20 . 4	+3 28.80	− 5 51.2	9 45 59.94	+20 56 10.9	8.264	0.429	+0.81	−5.5*
16 8 55 35	46	25 . 5	+3 16.94	− 3 51.7	9 41 27.90	+20 33 9.8	n9.178	0.456	+0.77	−5.0†
Estimated magnitude: Mar. 11, 12.5; Mar. 16, 12.5.										
					(581) *Tauntonia*.					
Mar. 11 11 41 6	47	25 . 5	−1 30.04	− 1 7.4	12 19 57.66	+29 43 12.5	n9.276	0.205	+0.94	−7.3*
Mar. 20 10 23 46	48	20 . 4	+1 9.22	+ 4 37.0	12 13 15.41	+30 39 31.1	n9.423	0.230	+1.04	−5.7*
20 10 34 49	49	20 . 4	−1 37.05	− 1 26.7	12 13 15.37	+30 39 30.1	n9.384	0.209	+1.04	−5.8*
Estimated magnitude: Mar. 11, 13.5.										
					[1907 *YC*.]					
Mar. 15 10 24 43	50	25 . 5	−1 46.77	+ 4 21.3	10 56 40.23	+13 44 13.2	n9.097	0.576	+0.92	−6.4*
16 10 32 17	51	25 . 5	+2 20.99	+ 2 6.9	10 55 46.82	+13 48 34.6	n9.001	0.573	+0.92	−6.4†
20 8 54 49	52	30 . 6	+0 32.50	+ 0 54.4	10 52 24.11	+14 4 5.5	n9.397	0.593	+0.91	−6.2*
Apr. 3 10 44 47	53	30 . 6	+0 47.40	− 1 29.6	10 42 18.42	+14 39 36.8	8.990	0.559	+0.82	−5.2*
14 10 25 53	54	20 . 4	+4 2.52	− 1 20.5	10 37 26.71	+14 43 36.8	9.193	0.565	+0.70	−4.3*
Estimated magnitude: Mar 15, 12.5; Mar. 16, 13.2; Apr. 14, 14.2.										
					[1907 *YD*.]					
Mar. 16 12 21 28	55	30 . 6	−0 14.14	− 2 49.0	11 25 20.65	− 2 32 20.1	8.786	0.763	+0.99	−6.9†
20 12 3 26	56	20 . 1	+3 3.11	−10 53.6	11 22 7.75	− 2 16 40.5	8.798	0.761	+0.99	−7.3*
Apr. 3 11 32 32	57	25 . 5	−0 30.39	+ 7 28.9	11 12 2.66	− 1 21 2.2	9.113	0.752	+0.94	−7.8*
11 10 6 44	58	25 . 5	+1 10.33	+ 0 19.2	11 7 15.63	− 0 53 11.5	8.495	0.748	+0.89	−7.8†
12 9 51 27	59	25 . 5	−1 4.97	+ 4 1.8	11 7 19.18	− 0 50 4.6	7.976	0.748	+0.89	−7.8*
17 11 53 48	60	25 . 5	−2 1.80	+ 5 3.7	11 5 26.61	− 0 35 42.5	9.442	0.745	+0.85	−7.7*
20 11 35 58	61	25 . 5	+1 31.23	− 0 52.0	11 4 38.28	− 0 28 37.4	9.429	0.744	+0.81	−7.7†
20 11 35 58	62	25 . 5	+0 59.18	− 8 16.6	11 4 38.33	− 0 28 35.2	9.429	0.744	+0.81	−7.7†
Estimated magnitude: Mar. 16, 12.5; Apr. 3, 12.5; Apr. 11, 12.6; Apr. 17, 12.5.										
					[1907 *YE*.]					
Mar. 20 13 52 21	63	30 . 6	−0 3.14	+ 3 6.1	11 40 17.31	+ 2 8 40.5	9.368	0.722	+0.98	−6.8†
Apr. 9 10 4 22	64	20 . 4	+0 11.37	− 1 58.9	11 27 21.78	+ 3 29 26.0	n8.152	0.705	+0.95	−6.8†
16 11 7 13	65	12 . 5	0 58.79	+ 5 31.0	11 21 14.13	+ 3 48 35.9	9.193	0.704	+0.90	−6.5†
Estimated magnitude. Mar. 20, 13.0; Apr. 9, 13.7.										

[1907 XZ.]

1907 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
			m s	′ ″	h m s	° ′ ″			s	″
Mar. 21 10 0 18	66	28 , 6	+2 39.46	− 8 59.1	11 38 4.09	− 5 44 41.7	n9.304	0.784	+1.02	−7.1†
22 10 30 57	67	20 , 4	+2 54.98	+ 2 26.2	11 37 10.18	− 5 43 28.0	n9.130	0.787	+1.03	−7.2*
25 10 58 28	68	24 , 5	+1 54.45	+ 5 28.5	11 34 33.51	− 5 39 30.4	n8.721	0.788	+1.03	−7.5*
28 12 10 40	69	18 , 6	+0 42.38	− 0 39.0	11 32 0.22	− 5 35 9.7	9.079	0.786	+1.03	−7.6†
Apr. 1 9 55 41	70	25 , 5	+1 3.88	−11 41.4	11 28 53.37	− 5 29 14.3	n9.045	0.786	+1.02	−7.8*
12 7 58 17	71	25 , 5	−1 43.80	− 5 20.2	11 21 30.97	− 5 13 16.5	n9.370	0.779	+0.96	−8.2*
16 8 23 4	72	25 , 5	+2 20.88	+ 4 13.9	11 19 25.29	− 5 8 24.5	n9.197	0.782	+0.92	−8.4†
20 10 4 24	72	25 , 5	+0 36.88	+ 8 7.5	11 17 41.26	− 5 4 31.0	8.889	0.783	+0.89	−8.5†
24 10 24 50	73	25 , 5	+3 28.22	− 9 26.8	11 16 22.58	− 5 1 54.0	9.177	0.781	+0.84	−8.5*
May 18 9 13 3	72	8 , 8	−0 11.76	− 5 29.8	11 16 52.35	− 5 18 7.7	9.282	0.782	+0.62	−7.9*

Estimated magnitude: Apr. 16, 11.8.

(302) *Clarissa.*

1907 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Mar. 23 13 52 42	74	30 , 6	+0 15.35	+ 7 54.0	11 37 1.01	+ 3 39 8.6	9.414	0.710	+0.98	−6.8†
Apr. 9 11 15 0	75	25 , 5	+2 4.10	+11 29.9	11 23 18.51	+ 4 39 3.2	9.079	0.694	+0.93	−6.7†
17 10 36 30	76	25 , 5	+2 8.69	+11 10.7	11 18 56.57	+ 4 54 10.1	9.060	0.691	+0.86	−6.4*

Estimated magnitude: Apr. 9, 12.7.

(31) *Euphrosyne* = [1907 *ZB*.].

1907 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
Apr. 24 11 23 9	77	30 , 6	−1 17.17	− 4 19.1	13 41 11.67	− 8 6 37.6	n8.299	0.807	+1.26	−5.2*
25 10 34 0	78	30 , 6	+0 55.05	− 6 9.7	13 40 15.65	− 8 7 28.9	n9.035	0.805	+1.26	−5.3*
30 10 45 43	79	15 , 3	−1 12.90	+ 1 48.5	13 35 32.13	− 8 12 18.9	n8.561	0.807	+1.28	−5.3†
May 4 8 45 33	80	30 , 6	+0 52.62	−10 18.1	13 32 0.63	− 8 16 42.8	n9.361	0.799	+1.27	−5.6†
9 9 38 50	81	25 , 5	−2 38.03	+ 2 17.5	13 27 46.13	− 8 23 20.2	n8.924	0.808	+1.27	−5.5*
12 10 28 33	82	30 , 6	+0 54.29	− 9 33.4	13 25 23.92	− 8 27 55.7	8.649	0.809	+1.25	−5.6*
17 8 54 4	83	25 , 5	0 45.03	+ 6 8.9	13 21 53.25	− 8 36 32.4	n8.995	0.809	+1.24	−5.7*
21 9 39 32	84	25 , 5	+1 26.92	+ 5 14.2	13 19 21.36	− 8 44 41.5	8.466	0.811	+1.21	−5.8†
27 9 12 2	85	25 , 5	−2 22.12	− 3 25.3	13 16 11.71	− 8 58 44.5	8.443	0.813	+1.19	−5.5*
June 5 10 24 19	86	5 , 1	−1 56.66	− 6 24.0	13 12 48.46	− 9 24 40.9	9.381	0.805	+1.12	−5.5*
6 10 18 4	87	30 , 6	−1 45.83	+ 4 26.4	13 12 32.36	− 9 27 52.5	9.375	0.806	+1.11	−5.5†
6 10 18 13	88	30 , 6	−1 48.76	+ 1 15.3	13 12 32.42	− 9 27 52.1	9.375	0.806	+1.11	−5.5†

Estimated magnitude: Apr. 24, 11.0; May 9, 11.5; May 12, 11.5; May 17, 11.5; June 6, 11.0

[1907 *ZD*.]

1907 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
May 4 12 52 13	89	30 , 6	+1 56.83	− 0 28.7	13 34 40.85	− 7 24 26.9	9.380	0.793	+1.27	−5.5†
9 10 47 27	90	15 , 5	−2 12.58	− 0 39.9	13 30 31.50	− 7 24 36.9	8.685	0.802	+1.27	−5.3*
11 11 6 11	91	23 , 5	+2 52.36	− 2 6.0	13 28 55.57	− 7 25 12.7	9.019	0.800	+1.25	−5.4*
14 9 10 58	92	25 , 5	−0 43.66	+ 7 34.6	13 26 43.68	− 7 26 40.5	n8.991	0.801	+1.26	−5.3†

Estimated magnitude: May 4, 13.0; May 9, 13.2; May 11, 13.5; May 14, 13.5.

[1907 *ZP*.]

1907 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
May 11 12 9 4	93	30 , 6	−1 3.03	− 1 56.9	15 16 51.37	− 8 49 35.5	8.182	0.812	+1.45	−2.4*
12 11 11 24	94	25 , 5	+1 41.51	+ 3 15.3	15 16 3.87	− 8 45 11.7	n8.960	0.810	+1.45	−2.5*
14 10 17 56	95	25 , 5	−1 50.13	− 8 2.8	15 14 26.86	− 8 36 29.7	n9.242	0.805	+1.47	−2.4*
17 9 37 14	96	21 , 7	−0 17.93	− 6 43.7	15 12 0.56	− 8 23 47.2	n9.351	0.800	+1.49	−2.4*
21 11 4 13	97	8 , 8	−0 7.21	+ 4 6.9	15 8 43.79	− 8 7 36.7	n8.302	0.807	+1.51	−2.4†

Estimated magnitude: May 11, 12.8; May 12, 12.5.

[1907 *ZQ*.]

1907 Wash. M.T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
May 17 10 46 37	98	25 , 5	+1 29.56	+ 4 4.7	15 28 27.37	− 8 3 9.0	n9.098	0.804	+1.50	−2.0*
20 11 3 4	99	25 , 5	+1 28.28	+ 9 13.2	15 25 49.06	− 8 3 26.6	n8.809	0.806	+1.52	−2.0*
June 6 11 52 28	100	25 , 5	−2 11.50	−10 16.5	15 12 49.49	− 8 29 26.2	9.280	0.803	+1.58	−1.5†
8 9 21 52	101	25 , 5	+2 19.64	− 5 7.8	15 11 42.74	− 8 34 59.3	n8.951	0.809	+1.57	−1.6†
15 10 26 48	102	25 , 5	−3 40.70	+ 2 21.8	15 8 20.92	− 9 0 4.1	9.013	0.811	+1.57	−1.4†

Estimated magnitude: May 17, 12.5; May 20, 12.5; June 8, 12.5.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	
	h m s	° ′ ″		
1	6 59 5.61	+ 5 16 11.1	Leipzig II, A.G.	3475
2	6 59 14.02	+ 5 21 2.3	" "	3479
3	6 50 11.56	+ 4 44 32.1	Albany, A.G.	2498
4	6 52 23.99	+ 4 56 18.9	½ {Albany 2521+ Leipzig II, 3379}	
5	8 9 37.23	+24 1 49.1	Berlin (B.), A.G.	3306
6	8 2 42.30	+24 43 25.6	" "	3273
7	7 55 7.43	+25 52 51.8	Camb. Eng., A.G.	4288
8	7 55 14.37	+25 52 21.3	" "	4289
9	8 40 38.44	+25 39 25.6	" "	4685
10	8 40 33.24	+25 41 31.3	" "	4683
11	8 29 53.15	+26 10 17.3	" "	4595
12	8 26 56.07	+26 15 50.7	" "	4570
13	8 25 21.53	+26 20 4.4	" "	4559
14	8 25 35.99	+26 28 33.5	" "	4560
15	8 24 14.64	+26 30 11.4	" "	4555
16	8 20 1.51	+26 36 25.8	" "	4523
17	8 15 36.53	+26 37 13.1	" "	4491
18	8 14 27.63	+26 39 46.2	" "	4483
19	8 13 17.89	+26 37 32.9	" "	4471
20	8 8 21.50	− 2 11 10.3	Strassburg, A.G.	3104
21	8 8 5.93	− 2 21 18.3	" "	3103
22	8 8 3.33	− 1 59 50.9	" "	3102
23	8 6 25.54	− 2 0 42.1	" "	3089
24	11 49 26.71	− 6 58 29.9	Vienna, "	4410
25	11 49 29.10	− 6 51 58.4	" "	4411
26	11 45 33.92	− 6 51 31.7	" "	4396
27	11 37 8.06	− 6 25 34.1	" "	4350
28	11 34 56.73	− 6 5 23.7	Strassburg, A.G.,	4375
29	11 30 29.87	− 5 58 41.4	" "	4355
30	9 29 38.75	−11 15 46.3	Camb. U.S., A.G.	Zones
31	9 27 56.28	−10 46 27.3	" " "	"
32	9 28 7.95	−10 44 34.2	" " "	"
33	9 43 25.38	+24 58 44.5	Camb. Eng., A.G.	5095
34	9 44 53.61	+24 59 39.7	" "	5109
35	9 44 19.36	+25 8 5.2	" "	5103
36	9 35 4.17	+25 26 34.9	" "	5054
37	9 32 51.74	+25 35 21.6	" "	5040
38	9 26 58.95	+25 40 55.6	" "	4998
39	9 22 25.45	+25 39 5.6	" "	4962
40	9 19 36.18	+25 37 30.9	" "	4941
41	10 20 1.51	+13 12 6.8	Leipzig I, A.G.	4013
42	10 23 26.51	+15 13 41.5	Berlin (A), A.G.	4162
43	10 21 11.79	+15 22 23.2	" "	4142
44	10 17 41.94	+15 49 0.2	" "	4129
45	9 42 30.33	+21 2 7.6	Berlin (B), A.G.	3845
46	9 38 10.19	+20 37 6.5	" "	3830
47	12 21 26.76	+29 44 27.2	½ {Leiden 4673+ Cambridge (Eng.) 6112}	
48	12 12 5.15	+30 34 59.8	Leiden, A.G.	4638
49	12 11 51.38	+30 41 2.6	" "	4646
50	10 58 26.08	+13 39 58.3	Leipzig I, A.G.	4179
51	10 53 21.91	+13 46 31.1	" "	4163
52	10 51 50.70	+14 3 17.3	Leipzig I, A.G.	4152
53	10 41 30.20	+14 41 11.6	" "	4106
54	10 33 23.49	+14 45 1.6	½ {Leipzig I, 4073+ Ber. (A) 4212}	
55	11 25 33.80	− 2 29 24.5	Newc's Fund. Cat.	729
56	11 19 3.65	− 2 5 39.6	Nicolajew, A.G.	3245
*57	11 12 32.11	− 1 28 23.3	{Micrometer Comparison with Nicolajew 3219 and 3228}	
58	11 6 34.41	− 0 53 22.9	Nicolajew, A.G.	3197
59	11 8 23.26	− 0 53 58.6	" "	3209
60	11 7 30.56	− 0 40 38.5	" "	3204
61	11 3 6.24	− 0 27 37.7	" "	3185
62	11 3 38.34	− 0 20 10.9	" "	3187
63	11 40 19.47	+ 2 5 41.2	Albany, A.G.	4345
64	11 26 36.46	+ 3 34 31.7	" "	4297
65	11 25 12.32	+ 3 43 8.4	" "	4296
66	11 35 23.61	− 5 35 35.5	Strassburg A.G.	4380
67	11 34 14.17	− 5 45 47.0	" "	4370
68	11 32 38.03	− 5 44 51.4	" "	4364
69	11 31 16.81	− 5 34 23.1	" "	4358
70	11 27 48.47	− 5 17 25.1	" "	4340
71	11 23 13.81	− 5 7 48.1	" "	4315
72	11 17 3.49	− 5 12 30.0	" "	4287
73	11 12 53.52	− 4 52 18.7	" "	4268
74	11 36 44.68	+ 3 31 21.4	Albany, A.G.	4331
75	11 21 13.48	+ 4 27 40.0	" "	4273
76	11 16 47.02	+ 4 43 5.8	½ {Albany 4254+ Leipzig II, 5781}	
77	13 42 27.58	− 8 2 13.3	Vienna, A.G.	4913
78	13 39 19.34	− 8 1 13.9	" "	4900
79	13 36 13.75	− 8 14 2.1	Newc's Fund. Cat.	859
80	13 31 6.74	− 8 6 19.1	Vienna, A.G.	4849
81	13 30 22.89	− 8 25 32.2	" "	4844
82	13 24 28.38	− 8 18 16.7	" "	4812
83	13 22 37.04	− 8 42 35.6	" "	4806
84	13 17 53.23	− 8 49 49.9	" "	4787
85	13 18 32.64	− 8 55 13.7	" "	4790
86	13 14 44.00	− 9 18 11.4	" "	4773
87	13 14 17.08	− 9 32 13.4	" "	4769
88	13 14 20.07	− 9 29 1.9	" "	4771
†89	13 32 42.75	− 7 23 52.7	Vienna A.G. 4858 (S. Pr.)	
90	13 32 42.81	− 7 23 51.7	" "	4858
91	13 26 1.96	− 7 23 1.3	" "	4820
92	13 27 26.08	− 7 34 9.8	" "	4826
93	15 17 52.95	− 8 47 36.2	" "	5371
94	15 14 20.91	− 8 48 24.5	" "	5354
95	15 16 15.52	− 8 28 24.5	" "	5362
96	15 12 17.00	− 8 17 1.1	" "	5342
97	15 8 49.49	− 8 11 41.2	" "	5317
98	15 26 56.31	− 8 7 11.7	" "	5421
99	15 24 19.26	− 8 12 37.8	" "	5408
100	15 14 59.41	− 8 19 8.2	" "	5356
101	15 9 21.53	− 8 29 49.9	" "	5322
102	15 12 0.05	9 2 24.5	Newc's Fund. Cat.	967

* Star, No. 57, which is Nicolajew 3220, has considerable proper motion in α.

† Star, No. 89, is the S. pr. component of a double. The position of the mean of the two components is given in the Vienna A.G. Catalogue. A correction of $0^s.06$ in α, and $1''.0$ in δ, has been applied to that position.

The star places from the Cambridge (U.S.) A.G. Zones were furnished through the courtesy of the Director of the Observatory at that place.

OBSERVATIONS OF COMETS,

MADE WITH THE 12-INCH AND 26-INCH EQUATORIALS AT THE U.S. NAVAL OBSERVATORY,

By H. L. RICE, J. C. HAMMOND AND M. FREDERICKSON.

[Communicated by Rear-Admiral ASA WALKER, U.S.N., Superintendent.]

COMET 1906 VII

1906-7 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$ (α)	log $p\Delta$ (δ)	Red. to App. Pl. (α)	Red. to App. Pl. (δ)
1906 h m s			m s	′ ″	h m s	° ′ ″			s	″
Nov. 12 15 15 0	1	15 , 3	+3 52.28	+10 7.4	9 25 33.08	+15 4 4.0	n9.493	0.598	+1.82	−12.1†
13 14 11 10	2	25 , 5	+1 55.83	+5 47.5	9 29 53.68	+16 15 5.4	n9.602	0.627	+1.81	−12.8†
15 16 27 46	3	22 , 5	−0 53.07	−10 56.2	9 39 47.36	+18 55 50.5	n9.284	0.500	+1.79	−14.4*
22 14 10 20	4	17 , 6	+0 53.84	+9 40.4	10 16 41.72	+28 21 38.3	n9.654	0.508	+1.76	−19.0*
23 15 6 53	5	12 , 4	−1 37.55	−10 6.7	10 22 52.45	+29 49 7.9	n9.583	0.385	+1.72	−19.8*
24 14 0 13	6	21 , 5	+1 58.95	−5 22.2	10 28 42.46	+31 9 18.8	n9.680	0.497	+1.74	−20.3*
24 14 26 42	7	25 , 5	+0 55.33	−3 16.1	10 28 49.09	+31 10 56.1	n9.652	0.444	+1.73	−20.3*
25 14 59 30	8	22 , 5	−2 16.83	+3 24.2	10 35 14.25	+32 36 36.0	n9.616	0.350	+1.67	−20.9*
28 15 57 24	9	20 , 5	−2 5.92	−3 28.2	10 55 21.29	+36 46 41.2	n9.537	0.054	+1.57	−22.6*
29 15 57 18	10	21 , 5	−3 57.86	−5 36.1	11 2 18.44	+38 6 31.3	n9.553	9.995	+1.50	−23.1*
Dec. 11 14 43 43	11	23 , .	−0 32.23	. .	12 36 27.32	+51 8 .	n9.830		+0.66	−25.9*
18 15 30 56	12	15 , 6	+1 6.27	+8 25.7	13 35 49.53	+55 40 30.1	n9.863	9.803	−0.06	−24.9*

Nov. 12, observed by HAMMOND. Nov. 13, observed by FREDERICKSON. Other observations by RICE.

COMET 1907 *a* (*Giacobini*).

1906-7 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$ (α)	log $p\Delta$ (δ)	Red. to App. Pl. (α)	Red. to App. Pl. (δ)
1907										
Mar. 11 7 47 56	13	25 , 5	+1 34.38	−0 48.7	6 58 22.98	−16 19 50.0	7.882	0.858	+0.30	−16.0*
13 7 40 28	14	31 , 6	+0 11.77	+1 7.8	6 53 8.72	−14 30 1.1	8.282	0.847	+0.23	−15.6*
15 7 37 53	15	21 , 7	+0 42.67	+2 11.9	6 48 20.96	−12 43 53.7	8.598	0.837	+0.16	−15.3*
16 9 12 47	16	18 , 6	+1 45.21	−3 46.8	6 45 57.91	−11 48 55.6	9.371	0.819	+0.11	−15.2*
17 8 41 51	17	12 , 4	+0 40.39	−3 9.3	6 43 52.56	−10 59 33.9	9.277	0.819	+0.08	−15.0*
Apr. 15 7 58 28	18	10 , 2	+1 49.68	−8 9.7	6 12 50.05	+6 25 41.1	9.543	0.696	−0.64	−9.3†
May 11 8 37 10	19	3 , 3	−0 47.08	+8 33.6	6 11 25.63	+14 43 9.9	9.674	0.713	−0.96	−6.0†

April 15, observed by HAMMOND. Other observations, by RICE.

COMET 1907 *b* (*Mellish*).

1906-7 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$ (α)	log $p\Delta$ (δ)	Red. to App. Pl. (α)	Red. to App. Pl. (δ)
Apr. 15 8 42 37	20	12 , 4	−0 31.61	+11 20.6	6 52 32.45	+13 33 45.8	9.560	0.636	−0.44	−6.6*
16 8 41 2	21	20 , 7	−0 35.07	+5 11.2	7 2 53.52	+18 32 43.9	9.558	0.576	−0.42	−4.7*
17 9 0 29	22	15 , 5	+2 9.45	+5 5.7	7 12 13.77	+22 47 6.7	9.592	0.536	−0.42	−3.1*
May 4 9 41 29	23	5 , 4	+0 6.22	−1 11.8	8 28 15.29	+46 35 17.2	9.776	0.114	−0.42	+6.5†
11 9 56 1	24	6 , 3	−1 40.51	+8 0.0	8 43 3.72	+49 4 0.2	9.808	0.202	−0.51	+7.7†

Observed by RICE.

COMET 1907 *c* (*Giacobini*).

1906-7 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$ (α)	log $p\Delta$ (δ)	Red. to App. Pl. (α)	Red. to App. Pl. (δ)
June 3 10 1 15	25	6 , 6	+0 22.73	−0 55.6	10 23 45.73	+23 49 44.3	9.659	0.590	0.00	+2.3†
8 9 49 41	26	5 , 6	+3 12.17	+1 34.2	10 45 26.80	+23 5 33.6	9.644	0.581	+0.05	+2.7†

Observed by HAMMOND.

COMET 1907 *d* (*Daniel*).

1906-7 Wash. M.T.	*	Comp.	$\Delta\alpha$	$\Delta\delta$	App. α	App. δ	log $p\Delta$ (α)	log $p\Delta$ (δ)	Red. to App. Pl. (α)	Red. to App. Pl. (δ)
June 14 14 33 23	27	25 , 5	−1 56.63	−8 46.7	0 2 9.96	+0 1 43.4	n9.596	0.740	+0.23	+1.6†
15 14 37 43	28	20 , 4	+1 54.30	+0 5.1	0 5 4.28	+0 16 27.2	n9.590	0.739	+0.27	+1.8†
16 15 51 23	29	12 , 4	−0 18.39	−4 45.4	0 8 11.12	+0 32 13.3	n9.466	0.736	+0.27	+1.8†
19 14 46 27	30	25 , 5	−0 35.47	−0 24.4	0 17 19.37	+1 18 5.7	n9.575	0.733	+0.31	+1.8†
21 14 28 6	31	20 , 4	−3 15.28	−6 27.4	0 23 49.15	+1 50 19.6	n9.596	0.731	+0.32	+1.8†
26 15 8 10	32	30 , 6	+0 56.40	+3 49.7	0 41 42.53	+3 17 38.8	n9.539	0.718	+0.41	+2.2*
30 15 39 14	33	25 , 5	+1 3.71	−4 17.4	0 57 47.58	+4 33 49.5	n9.482	0.705	+0.45	+2.2†
July 5 15 32 32	34	25 , 5	−0 52.38	+6 5.8	1 20 27.40	+6 17 1.5	n9.503	0.692	+0.46	+2.0†
8 14 51 14	35	25 , 5	−1 8.27	−3 17.4	1 35 35.75	+7 22 57.2	n9.577	0.694	+0.48	+1.8†
13 15 19 36	36	25 , 5	−2 23.16	+12 43.9	2 4 27.69	+9 21 7.4	n9.551	0.673	+0.47	+1.6†
16 15 5 2	37	25 , 5	+0 47.67	−0 19.0	2 23 50.62	+10 34 24.0	n9.584	0.672	+0.48	+1.2†
20 15 20 45	38	25 , 5	+2 46.32	−9 4.8	2 52 36.95	+12 13 16.6	n9.583	0.657	+0.46	+0.6†
30 15 14 43	39	19 , 4	−2 54.04	+4 35.1	4 17 24.92	+15 48 18.0	n9.644	0.662	+0.29	−0.8*
Aug. 12 15 29 28	40	15 , 3	+1 14.92	−5 56.3	6 19 10.28	+17 20 26.8	n9.676	0.695	+0.09	−1.7†
18 15 57 22	41	15 , 3	−1 46.25	+0 9.4	7 10 58.73	+16 42 38.6	n9.674	0.698	+0.01	−1.9†
25 16 23 38	42	20 , 4	+2 14.93	+2 16.1	8 5 20.98	+15 18 1.1	n9.671	0.703	−0.02	−1.5*
Sept. 13 16 44 8	43	11 , 4	−0 41.62	−4 5.0	10 7 53.15	+9 34 41.6	n9.666	0.732	−0.03	−1.0*

June 14, 15, 19, Aug. 12, observed by HAMMOND. July 8, 13, 16, 20, Aug. 18, observed by FREDERICKSON. Other observations by RICE.

* Observed with the 12-inch equatorial. † Observed with the 26-inch equatorial.

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	9 21 38.98	+14 54 8.7	½ {Berlin (A) 3809 + Leipzig I. 3748}	23	8 28 9.49	+46 36 22.5	{Microm. obsns with Bonn A.G. 6676; $\Delta\alpha = -2^m 10^s.57$; $\Delta\delta = +7' 31''.6$}
2	9 27 56.04	−16 9 30.7	Berlin (A), A.G. 3845	24	8 44 44.74	+48 55 52.5	Bonn. A.G. 6806
3	9 40 38.64	−19 7 1.1	" " " 3918	25	10 23 23.00	+23 50 37.6	Berlin (B), A.G. 4024
4	10 15 46.12	+28 12 16.9	Camb. Eng., A.G. 5311	26	10 42 14.58	+23 3 56.7	" " " 4098
5	10 24 28.28	−29 59 34.4	Leiden, A.G. 4192	27	0 4 6.36	+ 0 10 28.5	Nicolajew, A.G. 11
6	10 26 41.77	+31 15 1.3	" " 4206	28	0 3 9.71	+ 0 16 20.3	" " 9
7	10 27 52.03	+31 14 32.5	" " 4217	29	0 8 29.24	+ 0 36 56.9	" " 19
8	10 37 29.41	+32 33 32.7	" " 4258	30	0 17 54.53	+ 1 18 28.3	½(Nicol. 51 + Albany 65)
9	10 57 25.64	−36 50 32.0	Lund, A.G. 5100	31	0 27 4.11	+ 1 56 45.2	Albany, A.G. 99
10	11 6 14.80	+38 12 30.5	" " 5142	32	0 40 45.72	+ 3 13 46.9	" " 183
11	12 36 58.89	+50 55 53.5	Camb. U.S., A.G. 4121	33	0 56 43.42	+ 4 38 4.7	" " 261
12	13 34 43.32	+55 32 29.3	Hels.-Gotha, A.G. 7643	34	1 21 19.32	+ 6 10 53.7	Leipzig, II A.G. 526
13	6 56 48.30	−16 18 45.3	Wash'gton, A.G. Zones	35	1 36 43.54	+ 7 26 12.8	" " " 637
14	6 52 56.72	−14 30 53.3	" " "	36	2 6 50.38	+ 9 8 21.9	" " " 829
15	6 47 38.13	−12 45 50.3	Camb. U.S., A.G. Zones	37	2 23 2.47	+10 34 41.8	" I. " 718
16	6 44 12.59	−11 44 53.6	" " " "	38	2 49 50.17	+12 22 20.8	" " " 866
17	6 43 12.09	−10 56 9.6	" " " "	39	4 20 18.67	+15 43 43.7	Berlin (A), A.G. 1171
18	6 11 1.01	+ 6 34 0.1	Leipzig II, A.G. 2752	40	6 17 55.27	+17 26 24.8	" " " 2111
19	6 12 13.67	+14 34 42.3	" I. " 2145	41	7 12 44.97	+16 42 31.1	Newc. Fund. Cat. 469
20	6 53 4.50	+13 22 31.8	" I. " 2623	42	8 3 6.07	+15 15 46.5	Berlin (A), A.G. 3207
21	7 3 29.01	+18 27 37.4	Berlin, (A), A.G. 2636	43	10 8 34.80	+ 9 38 47.6	Leipzig II, A.G. 5399
22	7 10 4.74	+22 42 4.1	Berlin, (B), A.G. 2856				

The star places from the Cambridge (U.S.) A.G. Zones were furnished through the courtesy of the Director of the Observatory at that place.

ELEMENTS AND EPHEMERIS OF COMET *d* 1907 (MELLISH).

By ELEANOR A. LAMSON.

[Communicated by Rear-Admiral ASA WALKER, U.S.N., Superintendent.]

The following elements were computed from observations made by Mr. FREDERICKSON, at Washington, on Oct. 15, 18 and 21:

ELEMENTS.

T = 1907 Sept. 14.872250 G.M.T.

$$\left.\begin{aligned} \pi &= 349^\circ\ 27'\ 51''.3 \\ \Omega &= \ \ 54\ \ 22\ \ \ 2.7 \\ i &= 119\ \ 46\ \ 53.9 \end{aligned}\right\}\ 1907.0$$

q = 0.985448

Residuals (O−C): $\cos\beta\Delta\lambda = -0.3$

$\Delta\beta = \ \ 0.0$

HELIOCENTRIC COORDINATES.

$$x = r\,[9.850515]\sin(350^\circ\ 22'\ 37''.5 + v)$$
$$y = r\,[9.984040]\sin(\ 64\ \ 25\ \ 25.1 + v)$$
$$z = r\,[9.877361]\sin(320\ \ 30\ \ \ 3.1 + v)$$

EPHEMERIS.

Gr. M.T.	α	δ	Light
1907	h m s	° ′ ″	
Nov. 6.5	5 53 20.8	+12 25 45	3.18
8.5	5 22 33.0	15 59 20	3.36
10.5	4 48 46.5	19 25 38	3.26
12.5	4 13 21.9	22 27 43	3.06
14.5	3 38 6.1	24 52 54	2.82
16.5	3 4 44.3	26 36 58	2.48
18.5	2 34 31.9	27 43 24	2.14
20.5	2 8 5.2	28 19 59	1.78
22.5	1 45 26.2	28 35 30	1.47
24.5	1 26 17.7	28 37 14	1.20
26.5	1 10 13.0	28 30 39	1.00
28.5	0 56 44.0	28 19 27	0.84
30.5	0 45 25.4	28 6 5	0.70
Dec. 2.5	0 35 54.9	+27 52 3	0.58

Brightness of Oct. 15 taken as unit.

OBSERVATIONS OF MINOR PLANETS,

MADE WITH THE 12-INCH EQUATORIAL AT THE U.S. NAVAL OBSERVATORY,

By H. L. RICE.

[Communicated by Rear-Admiral ASA WALKER, U.S.N. Superintendent.]

(28) *Bellona.*

1906 Wash. M.T. (h m s)	*	Comp.	Δα (m s)	Δδ (′ ″)	App. α (h m s)	App. δ (° ′ ″)	log pΔ		Red. to App. Pl. (s)	(″)
June 28 11 26 46	1	30 . 6	+3 3.33	+ 3 28.2	18 27 49.10	−12 57 2.8	n8.861	0.837	−2.21	− 8.2*
29 12 52 1	2	17 . 6	−0 40.04	+ 8 6.7	18 26 53.84	−12 59 27.7	9.045	0.836	+2.22	− 8.4
29 13 43 48	3	24 . 5	−2 8.21	+ 8 30.8	18 26 51.94	−12 59 33.2	9.323	0.828	−2.22	+ 8.4
July 12 11 37 30	4	25 . 5	−1 58.10	+ 3 29.4	18 16 2.63	−13 34 51.6	8.934	0.840	+2.32	+ 8.6
12 12 9 12	5	20 . 4	−2 31.26	− 0 48.3	18 16 1.56	−13 34 58.1	9.173	0.837	+2.32	− 8.6
12 12 49 23	6	20 . 4	−2 45.17	− 3 34.5	18 16 0.11	−13 35 2.8	9.352	0.830	−2.32	+ 8.6

* Observed on the 26-inch equatorial by M. FREDERICKSON.

(80) *Sappho.*

1906 Wash. M.T. (h m s)	*	Comp.	Δα (m s)	Δδ (′ ″)	App. α (h m s)	App. δ (° ′ ″)	log pΔ		Red. to App. Pl. (s)	(″)
July 5 11 31 4	7	20 . 4	−2 35.12	− 2 51.8	18 27 50.14	− 8 42 52.6	n7.878	0.811	+2.24	− 9.1
5 11 53 30	8	25 . 5	−0 26.09	− 4 47.7	18 27 49.04	− 8 42 51.4	8.578	0.811	+2.23	− 9.0
6 11 1 52	8	24 . 5	−1 24.83	− 3 4.7	18 26 50.31	− 8 41 8.3	n8.752	0.810	+2.24	− 9.1
13 10 40 36	9	20 . 4	−1 43.40	− 7 43.7	18 19 58.55	− 8 33 37.8	n8.479	0.810	+2.27	− 9.3
19 11 58 22	10	24 . 6	−0 25.19	+ 1 1.2	18 14 38.01	− 8 34 11.8	9.257	0.805	+2.28	+ 9.5
19 12 24 13	11	20 . 4	+1 34.31	+ 5 47.2	18 14 37.01	− 8 34 12.5	9.357	0.801	+2.27	− 9.4

(7) *Iris.*

1906 Wash. M.T. (h m s)	*	Comp.	Δα (m s)	Δδ (′ ″)	App. α (h m s)	App. δ (° ′ ″)	log pΔ		Red. to App. Pl. (s)	(″)
July 26 12 10 50	12	30 . 6	−1 30.19	− 7 58.6	20 35 9.85	−11 24 34.6	n8.232	0.829	+2.37	−14.0
26 12 10 50	13	30 . 6	−2 25.11	− 6 25.3	20 35 10.00	−11 24 33.6	n8.232	0.829	+2.37	+14.0
Aug. 15 11 32 38	14	21 . 7	−0 28.53	− 7 20.6	20 14 53.92	−12 2 16.2	9.025	0.830	+2.49	+14.4
15 11 52 6	15	20 . 4	−1 9.39	− 1 13.5	20 14 53.31	−12 2 17.6	9.159	0.828	+2.49	+14.4
15 12 12 52	16	25 . 5	−1 33.28	+ 8 4.8	20 14 52.48	−12 2 19.7	9.265	0.826	+2.49	+14.4

(126) *Velleda.*

1906 Wash. M.T. (h m s)	*	Comp.	Δα (m s)	Δδ (′ ″)	App. α (h m s)	App. δ (° ′ ″)	log pΔ		Red. to App. Pl. (s)	(″)
Oct. 6 10 20 4	17	25 . 5	−1 57.21	+11 6.4	22 59 46.22	− 8 57 39.0	8.601	0.813	+2.55	+16.9
8 10 34 54	17	25 . 6	−2 58.00	+ 9 38.3	22 58 45.42	− 8 59 7.1	8.942	0.812	+2.54	+16.9
15 11 20 49	18	19 . 4	+2 23.36	− 1 21.5	22 56 7.64	− 8 58 31.4	9.364	0.803	+2.47	+16.5
15 11 42 40	19	20 . 4	+1 39.99	+ 5 7.6	22 56 7.61	− 8 58 30.9	9.429	0.800	+2.47	+16.5

(2) *Pallas.*

1906 Wash. M.T. (h m s)	*	Comp.	Δα (m s)	Δδ (′ ″)	App. α (h m s)	App. δ (° ′ ″)	log pΔ		Red. to App. Pl. (s)	(″)
Oct. 13 10 18 35	20	25 . 5	+1 15.08	− 2 59.7	0 50 17.67	−14 28 35.1	n9.118	0.843	+2.67	+15.2
13 10 40 17	21	20 . 4	+2 39.46	− 4 2.1	0 50 16.98	−14 28 48.3	n8.943	0.846	+2.67	+15.2
13 11 0 12	22	20 . 4	−2 37.41	− 5 26.5	0 50 16.38	−14 28 59.2	n8.673	0.847	+2.67	+15.2
13 11 38 41	23	20 . 4	−3 15.40	+ 5 23.5	0 50 15.00	−14 29 20.6	8.510	0.847	+2.67	+15.1
29 8 40 38	24	24 . 5	−1 20.28	+ 8 52.3	0 38 57.79	−17 35 49.6	n9.256	0.855	+2.65	+12.9
29 9 1 51	25	19 . 4	−2 46.32	+ 7 12.9	0 38 57.18	−17 35 56.7	n9.141	0.859	+2.65	+12.9

(532) *Herculina.*

1906 Wash. M.T. (h m s)	*	Comp.	Δα (m s)	Δδ (′ ″)	App. α (h m s)	App. δ (° ′ ″)	log pΔ		Red. to App. Pl. (s)	(″)
Oct. 6 12 40 52	26	25 . 5	+1 41.58	+ 5 26.8	2 7 44.08	−11 56 27.0	n8.740	0.812	+2.56	+14.3*
15 12 48 52	27	19 . 4	+1 8.32	+ 0 38.2	2 0 43.29	−12 44 33.0	8.682	0.837	+2.67	+13.8
26 10 31 53	28	15 . 3	−3 46.45	+ 1 0.8	1 51 37.30	−13 24 48.8	n9.094	0.838	+2.74	+12.8
29 9 57 45	29	30 . 6	+0 1.03	+10 39.2	1 49 9.15	−13 31 38.2	n9.213	0.836	+2.75	+12.4
29 10 37 38	30	25 . 5	+1 59.93	+ 9 58.3	1 49 7.75	−13 31 45.3	n8.925	0.840	−2.76	−12.5
29 11 9 44	31	20 . 4	+3 23.70	−10 47.9	1 49 6.68	−13 31 47.1	n8.270	0.842	−2.76	+12.6

* Observed on the 26-inch equatorial by M. FREDERICKSON.

(1) *Ceres.*

1906 Wash. M.T. (h m s)	*	Comp.	Δα (m s)	Δδ (′ ″)	App. α (h m s)	App. δ (° ′ ″)	log pΔ		Red. to App. Pl. (s)	(″)
Dec. 3 10 5 17	32	17 . 4	−2 17.08	+ 9 27.8	4 51 38.89	+20 12 32.8	n9.381	0.491	+3.68	− 0.4
3 10 25 36	33	25 . 5	−0 55.83	+ 2 29.6	4 51 38.04	+20 12 35.5	n9.305	0.478	+3.69	− 0.5
3 10 51 45	34	20 . 4	+1 55.15	+ 2 52.3	4 51 36.94	+20 12 37.8	n9.175	0.463	+3.69	− 0.7
3 11 22 20	35	20 . 4	+3 4.87	+ 3 27.1	4 51 35.47	+20 12 39.7	n8.937	0.452	+3.69	+ 0.8

1907 Wash. M. T.	*	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
					(8) *Flora.*					
	h m s			m s	′ ″	h m s	° ′ ″			s	″
Jan. 4 10 18 33	36	20 , 4	−1 13.75	− 6 13.9	7 46 42.00	+20 58 19.3	n9.484	0.507	+0.40	− 6.1
4 10 37 41	37	20 , 4	−1 15.67	− 6 8.1	7 46 40.99	+20 58 24.6	n9.434	0.490	+0.40	− 6.1
10 10 47 32	38	25 , 5	−2 30.54	− 3 55.5	7 39 50.13	+21 39 11.8	n9.294	0.446	+0.50	− 6.3
					(16) *Psyche.*					
Feb. 25 10 55 30	39	20 , 4	+0 51.31	+ 7 29.7	10 28 6.99	+10 1 26.5	n9.164	0.632	+0.85	− 7.0
25 11 15 57	40	20 , 4	−3 12.28	− 3 44.7	10 28 6.32	+10 1 32.5	n9.024	0.629	+0.85	− 6.9
					(511) *Davida.*					
Feb. 25 12 57 27	41	20 , 5	+2 3.36	+11 45.0	9 46 7.37	+28 42 35.6	9.305	0.252	+0.87	− 5.6
					(356) *Liguria.*					
Mar. 15 10 37 18	42	24 , 5	+1 20.78	− 9 13.8	11 11 42.92	+ 6 54 56.7	n9.104	0.669	+0.94	− 6.8
22 10 31 59	43	25 , 5	+2 1.41	+ 6 7.6	11 5 26.74	+ 7 10 40.5	n8.852	0.664	+0.93	− 6.9

Mean Places of Comparison-Stars for the beginning of the year.

*	α	δ	Authority	*	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	18 24 43.56	−13 0 39.2	Camb. U.S., A.G. Zones	23	0 53 27.73	−14 34 59.2	Wash'gton, A.G. Zones
2	18 27 31.66	−13 7 42.8	" " " "	24	0 40 15.42	−17 44 54.8	" " "
3	18 28 57.93	−13 8 12.4	" " " "	25	0 41 40.85	−17 43 22.5	" " "
4	18 17 58.41	−13 38 29.6	" " " "	26	2 5 59.94	−12 2 8.1	Camb. U.S. A.G. Zones
5	18 18 30.50	−13 34 18.4	" " " "	27	1 59 32.30	−12 45 25.0	" " " "
6	18 18 42.96	13 31 35.9	" " " "	28	1 55 21.01	−13 26 2.4	" " " "
7	18 30 23.02	− 8 45 53.5	Vienna, A.G. 6231	29	1 49 5.37	−13 42 29.8	1/2 Washington+Cambridge (U.S.) 1/2 A.G. Zones
8	18 28 12.90	− 8 38 12.7	" " 6216	30	1 47 5.06	−13 41 56.1	1/2 Washington+Cambridge (U.S.) 1/2 A.G. Zones.
9	18 21 39.68	− 8 26 3.4	" " 6191	31	1 45 40.22	−13 21 11.8	Camb. U.S., A.G. Zones
10	18 15 0.92	− 8 35 22.5	" " 6155	32	4 53 52.29	+20 3 4.6	Berlin A, A.G. 1365
11	18 13 0.43	− 8 40 9.1	" " 6143	33	4 52 30.18	+20 10 5.4	1/2 Berlin B 1582+ 1/2 Berlin A, A.G. 1359
12	20 36 37.67	−11 16 50.0	Camb. U.S., A.G. Zones	34	4 49 38.10	+20 9 44.8	1/2 Berlin B 1568+ 1/2 Berlin A, A.G. 1346
13	20 37 32.74	−11 18 22.3	" " " "	35	4 48 26.91	+20 9 11.8	1/2 Berlin B 1553+ 1/2 Berlin A A.G. 1337
14	20 15 19.96	−11 55 10.0	" " " "	36	7 47 55.35	+21 4 39.3	Berlin (B), A.G. 3162
15	20 16 0.21	−12 3 45.5	" " " "	37	7 47 56.26	+21 4 38.8	" " " 3163
16	20 16 23.27	−12 10 38.9	" " " "	38	7 42 20.17	+21 43 13.6	" " " 3121
17	23 1 40.88	− 9 9 2.3	Vienna, A.G. 8216	39	10 27 14.83	+ 9 54 3.8	Leipzig I, A.G. 4048
18	22 53 41.81	− 8 57 26.4	" " 8173	40	10 31 17.75	+10 5 24.1	" " " 4065
19	22 54 25.15	− 9 3 55.0	" " 8177	41	9 44 3.14	+28 30 56.2	Camb. Eng., A.G. 5099
20	0 48 59.92	−14 25 50.6	Wash'gton, A.G. Zones	42	11 10 21.20	+ 7 4 17.3	Leipzig II, A.G. 5741
21	0 47 34.85	−14 25 1.4	" " "	43	11 3 24.40	+ 7 4 39.8	" " " 5697
22	0 52 51.12	−14 23 47.9	" " "				

126, 16 and 356 were found photographically by Mr. G. H. Peters.

The star places from the Cambridge (U.S.) Zones were furnished through the courtesy of the Director of the Observatory at that place.

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THE JOVIAN EVECTION IN THE LUNAR THEORY,

By G. W. HILL.

The researches of Professors S. NEWCOMB and E. W. BROWN having rendered it reasonably certain that some fault existed in the determination of this inequality in the *Astronomical Papers*, Vol. III, p. 373, I have reviewed my work with the result of finding a serious oversight. DELAUNAY's R contains the term $\frac{\mu}{2a}$, and his transformations lead to continual variation in the signification of this symbol. These circumstances make it impossible to derive from DELAUNAY's long expansion of terms (Tom. I, pp. 119–256) expressions for R and $\frac{\partial R}{\partial V}$ suitable for our case. An independent development must be made. A second imperfection in the work is insufficient approximation. Stopping with terms involving the factor $\frac{n'^2}{n^2}$ does not give the coefficients exact to 0″.01. The series are of unexpected slow convergence.

I have thought it worth while to remove these defects by repeating the calculations and carrying the developments to terms multiplied by $\frac{n'^5}{n^5}$. Pushing the approximation three orders farther, DELAUNAY operations have to be abandoned and expressions for the radius and longitude of the moon used. It is proposed to found the investigation upon DELAUNAY's determination of these quantities. But the degree of approximation set demands the knowledge of the two coordinates to quantities of the seventh order inclusive, while DELAUNAY has given the radius only to quantities of the fifth order. Some method must be devised for supplementing this defect. The differential equation for areas, when the longitude is known, constitutes a linear equation for the determination of the square of the radius; the integration of this will give what is wanted. However, it is necessary to know one coefficient, preferably the absolute one, from an independent source; but this can be obtained, with the requisite precision, from DELAUNAY's expression for $\frac{a}{r}$.

To the requisite degree of precision the equation of areas is

$$\frac{d}{ndt}\left[r^2\frac{dV}{ndt}\right]+\frac{3}{2}m^2r^2\sin 2(V-V') = 0$$

and the absolute term of r^2 is given by

$$\frac{r^2}{a^2} = 1-\frac{1}{3}m^2+\frac{407}{144}m^4+\frac{325}{24}m^5$$

Employing the value of V given by DELAUNAY and his notation for designating arguments, we arrive at the following expression:

$$\begin{aligned}\frac{r^2}{a^2} = 1 &- \frac{1}{3}m^2+\frac{407}{144}m^4+\frac{325}{24}m^5\\
&-\left[2m^2+\frac{19}{3}m^3+\frac{122}{9}m^4+\frac{1361}{54}m^5\right]\cos 2D\\
&-\left[\frac{1}{4}m^4+\frac{457}{240}m^5\right]\cos 4D\\
&+\left[-2+\frac{13}{6}m^2+\frac{465}{32}m^3+\frac{80671}{1152}m^4+\frac{517461}{2048}m^5\right]e\cos l\\
&-\left[\frac{15}{4}m+\frac{139}{16}m^2+\frac{20777}{768}m^3+\frac{941449}{9216}m^4+\frac{96342661}{221184}m^5\right]e\cos(2D-l)\\
&-\left[\frac{9}{8}m^2+\frac{25}{8}m^3+\frac{1435}{192}m^4+\frac{200363}{9216}m^5\right]e\cos(2D+l)\end{aligned}$$

$$
\begin{aligned}
&-\left[\frac{135}{64}m^3+\frac{4773}{256}m^4+\frac{2\,085\,887}{20480}m^5\right]e\cos(4D-l)\\
&-\left[\frac{61}{128}m^4+\frac{203}{60}m^5\right]e\cos(4D+l)\\
&-\frac{915}{1024}m^5 e\cos(6D-l)\\
&+\left[-\frac{1}{2}-\frac{1}{12}m^2+\frac{1275}{128}m^3+\frac{453998}{9216}m^4+\frac{2\,856\,023}{8192}m^5\right]e^2\cos 2l\\
&+\left[\frac{45}{8}m+\frac{801}{32}m^2+\frac{40313}{512}m^3+\frac{1\,673\,821}{6144}m^4+\frac{136\,064\,965}{147456}m^5\right]e^2\cos(2D-2l)\\
&-\left[\frac{13}{16}m^2+\frac{107}{48}m^3+\frac{32107}{5760}m^4+\frac{4\,843\,183}{345600}m^5\right]e^2\cos(2D+2l)\\
&-\left[\frac{225}{128}m^2+\frac{4275}{256}m^3+\frac{416263}{4096}m^4+\frac{22\,817\,063}{40960}m^5\right]e^2\cos(4D-2l)\\
&-\left[\frac{363}{512}m^4+\frac{1237}{256}m^5\right]e^2\cos(4D+2l)\\
&-\left[\frac{3645}{1024}m^4+\frac{89863}{2048}m^5\right]e^2\cos(6D-2l)
\end{aligned}
$$

Here a, m and e have the signification of DELAUNAY's last formulas.

From the foregoing expression must be derived the values of the two functions

$$r^2\cos 2(V-V') \text{ and } r^2\sin 2(V-V')$$

where, however, we may limit the expressions to the terms depending on the argument $2D-2l$. Use the subscript ($_0$) to distinguish elliptic values and put

$$2(V-V') = 2D+y$$

then the perturbations of

$$r^2\cos 2(V-V') \text{ and } r^2\sin 2(V-V') \text{ are}$$

$$\delta\left[r^2 \begin{matrix}\cos\\ \sin\end{matrix}(V-V')\right]=\left[\frac{r^2}{r_0^2}-1-\frac{1}{2}\frac{r^2}{r_0^2}y^2\right]r_0^2\begin{matrix}\cos\\ \sin\end{matrix}2(V_0-V')\mp\frac{r^2}{r_0^2}y r_0^2\begin{matrix}\sin\\ \cos\end{matrix}2(V-V')$$

In these formulas it suffices to put

$$\frac{a^2}{r_0^2} = 1+2e\cos l+\frac{5}{2}e^2\cos 2l$$

$$\frac{r_0^2}{a^2}\begin{matrix}\cos\\ \sin\end{matrix}2(V_0-V')=\begin{matrix}\cos\\ \sin\end{matrix}2D-3e\begin{matrix}\cos\\ \sin\end{matrix}(2D-l)+e\begin{matrix}\cos\\ \sin\end{matrix}(2D+l)+\frac{5}{2}e^2\begin{matrix}\cos\\ \sin\end{matrix}(2D-2l)+e^2\begin{matrix}\cos\\ \sin\end{matrix}(2D+2l)$$

Limiting the expressions to the terms necessary,

$$
\begin{aligned}
\frac{r^2}{r_0^2}-1-\frac{1}{2}\frac{r^2}{r_0^2}y^2 = &-\frac{1}{3}m^2+\frac{539}{576}m^4+\frac{1}{48}m^5\\
&+\left[\frac{105}{64}m^4+\frac{697}{60}m^5\right]\cos 4D\\
&+\left[\frac{3}{2}m^2+\frac{135}{32}m^3 \quad \frac{1567}{128}m^4-\frac{377613}{2048}m^5\right]e\cos l\\
&+\left[\frac{525}{64}m^3+\frac{16175}{256}m^4+\frac{6\,555\,989}{20480}m^5\right]e\cos(4D-l)\\
&+\left[\frac{5}{4}m^2+\frac{105}{128}m^3 \quad \frac{766699}{18432}m^4-\frac{291\,128\,215}{442368}m^5\right]e^2\cos 2l\\
&+\left[\frac{1575}{128}m^2+\frac{28725}{256}m^3+\frac{2\,784\,801}{4096}m^4+\frac{136\,507\,653}{40960}m^5\right]e^2\cos(4D-2l)
\end{aligned}
$$

$$\frac{r^2}{r_0^2}\,y = -\left[\frac{151}{128}m^4+\frac{309}{40}m^5\right]\sin 4D$$
$$+\left[\frac{75}{32}m^3+\frac{3013}{128}m^4+\frac{279279}{2048}m^5\right]e\sin l$$
$$-\left[\frac{75}{16}m^3+\frac{947}{32}m^4+\frac{385561}{3072}m^5\right]e\sin(4D-l)$$
$$-\left[\frac{7}{8}m^2+\frac{1065}{128}m^3-\frac{2383}{1536}m^4-\frac{484199}{8192}m^5\right]e^2\sin 2l$$
$$-\left[\frac{675}{128}m^2+\frac{8325}{256}m^3+\frac{572621}{4096}m^4+\frac{61\,543\,733}{122880}m^5\right]e^2\sin(4D-2l)$$

The substitution of these values in the formulas shows that $r^2\cos 2(V-V')$ and $r^2\sin 2(V-V')$ contain the terms

$$\frac{r^2}{a^2}\cos 2(V-V') = \left[\frac{5}{2}+\frac{2599}{384}m^2+\frac{20625}{256}m^3+\frac{2\,267\,495}{4608}m^4+\frac{2\,009\,963\,507}{884736}m^5\right]e^2\cos(2D-2l)$$
$$\frac{r^2}{a^2}\sin 2(V-V') = \left[\frac{5}{2}-\frac{4151}{384}m^2-\frac{19725}{256}m^3-\frac{7\,800\,475}{18432}m^4-\frac{1\,793\,089\,237}{884736}m^5\right]e^2\sin(2D-2l)$$

In addition it may be thought advisable to retain the terms involving the two arguments $2D-2l-l'$ and $2D-2l+l'$, in which, however, we limit ourselves to the first power of m:

$$\delta\left(\frac{r^2}{a^2}\right) = -2(1-3e\cos l)\,\delta\frac{a}{r}$$
$$= \frac{105}{8}me^2e'\cos(2D-2l-l') - \frac{45}{8}me^2e'\cos(2D-2l+l')$$
$$\delta\left[\frac{r^2}{a^2}\cos 2(V-V')\right] = \frac{135}{16}me^2e'\cos(2D-2l+l') - \frac{45}{16}me^2e'\cos(2D-2l-l')$$
$$\delta\left[\frac{r^2}{a^2}\sin 2(V-V')\right] = \frac{45}{8}me^2e'\sin(2D-2l+l') - \frac{45}{8}me^2e'\sin(2D-2l-l')$$

Putting θ for double the angular distance of the moon's perigee from the mean position of *Jupiter*, the first portion of R arising from the direct action is

$$R = m'\frac{a^2}{a'^3}\left\{0''.02585\left(\frac{45}{8}m+\frac{801}{32}m^2+\frac{40313}{512}m^3+\frac{1\,673\,821}{6144}m^4+\frac{136\,064\,965}{147456}m^5\right)+0''.00086\,m\right\}e^2\cos\theta$$

The second portion is

$$R = m'\frac{a^2}{a'^3}\left\{1''.0928\left(\frac{5}{2}-\frac{97}{64}m^2+\frac{225}{128}m^3+\frac{1\,269\,505}{36864}m^4+\frac{108\,437\,135}{884736}m^5\right)-0''.00068\,m\right\}e^2\cos\theta$$

The first portion of the indirect action is

$$-\frac{\partial R}{\partial V}\delta V' = m'\frac{a^2}{a'^3}\left\{1''.365\left(-\frac{15}{4}+\frac{15}{4}\gamma^2+\frac{4151}{256}m^2+\frac{59175}{512}m^3+\frac{7\,800\,475}{12288}m^4+\frac{1\,793\,089\,237}{589824}m^5\right)\right.$$
$$\left.+0''.020-0''.113\,m\right\}e^2\cos\theta$$

The second portion is

$$-3R\frac{\delta r'}{r'} = m'\frac{a^2}{a'^3}\left\{2''.868\left(\frac{15}{8}-\frac{15}{8}\gamma^2+\frac{45}{32}m+\frac{5803}{512}m^2+\frac{164063}{2048}m^3+\frac{3\,581\,267}{8192}m^4+\frac{2\,282\,093\,437}{1\,179\,648}m^5\right)\right.$$
$$\left.-0''.011+0''.080m\right\}e^2\cos\theta$$

The quantities a, m, e, however, have not the signification of a, $\frac{n'}{n}$, e before DELAUNAY's last transformation (See DELAUNAY, Tom. II, pp. 799, 800). We must replace

$$m \text{ by } \frac{n'}{n}+\frac{n'^3}{n^3}-\frac{387}{64}\frac{n'^5}{n^5}$$
$$a^2e^2 \text{ by } a^2e^2\left[1+\frac{13}{192}\frac{n'^2}{n^2}+\frac{2595}{128}\frac{n'^3}{n^3}+\frac{103835}{36864}\frac{n'^4}{n^4}-\frac{974989}{12288}\frac{n'^5}{n^5}\right]$$

After these substitutions our four series in powers of m are changed into the following:

$$\frac{45}{8}\frac{n'}{n}+\frac{801}{32}\frac{n'^2}{n^2}+\frac{10847}{128}\frac{n'^3}{n^3}+\frac{673117}{1536}\frac{n'^4}{n^4}+\frac{30\,485\,981}{18432}\frac{n'^5}{n^5}$$

$$\frac{5}{2}-\frac{517}{384}\frac{n'^2}{n^2}+\frac{13425}{256}\frac{n'^3}{n^2}+\frac{942377}{24576}\frac{n'^4}{n^4}-\frac{89\,475\,205}{884736}\frac{n'^5}{n^5}$$

$$-\frac{15}{4}+\frac{2043}{128}\frac{n'^2}{n^2}+\frac{10125}{256}\frac{n'^3}{n^3}+\frac{2\,020\,667}{3072}\frac{n'^4}{n^4}+\frac{2\,371\,604\,917}{589824}\frac{n'^5}{n^5}$$

$$\frac{15}{8}+\frac{45}{32}\frac{n'}{n}+\frac{1467}{128}\frac{n'^2}{n^2}+\frac{61247}{512}\frac{n'^3}{n^3}+\frac{24\,300\,385}{49152}\frac{n'^4}{n^4}+\frac{2\,662\,306\,723}{1\,179\,648}\frac{n'^5}{n^5}$$

For the direct action we have

$$R = m'\frac{a^2}{a'^3}\left\{2''.732-5''.46\,\gamma^2+0''.140\,\frac{n'}{n}-0''.824\,\frac{n'^2}{n^2}+59''.5\,\frac{n'^3}{n^3}+53''\,\frac{n'^4}{n^4}-68''\,\frac{n'^5}{n^5}\right\}e^2\cos\theta$$

and for the indirect action

$$R = m'\frac{a^2}{a'^3}\left\{0''.268-0''.26\,\gamma^2+4''.000\,\frac{n'}{n}+54''.66\frac{n'^2}{n^2}+462''.7\,\frac{n'^3}{n^3}+1912''\frac{n'^4}{n^4}+8730''\,\frac{n'^5}{n^5}\right\}e^2\cos\theta$$

and the sum

$$R = m'\,\frac{a^2}{a'^3}\left\{3''.000-5''.72\,\gamma^2+4''.140\,\frac{n'}{n}+53''.84\frac{n'^2}{n^2}+522''.2\,\frac{n'^3}{n^3}+1965''\,\frac{n'^4}{n^4}+8662''\frac{n'^5}{n^5}\right\}e^2\cos\theta$$

Employing the same numerical values as in the first investigation, and adding inductive estimates for the remainders of the series

$$R = 0''.00006495\,a^2n^2\cos\theta$$
$$e\,\frac{\partial R}{\partial e} = 0''.00012991\,a^2n^2\cos\theta$$
$$a\,\frac{\partial R}{\partial a} = 0''.00017886\,a^2n^2\cos\theta$$

$$\delta e = -0''.5822\cos\theta$$
$$e\delta l = -0''.5769\sin\theta$$
$$\delta(h+g+l) = +0''.246\sin\theta$$

The inequalities of the longitude are

$$\begin{aligned}\delta V = &-1''.159\sin(2h+2g+l-2h''-2g''-2l'')\\ &+0''.246\sin(2h+2g-2h''-2g''-2l'')\\ &-0''.235\sin(l-2h'-2g'-2l'+2h''+2g''+2l'')\end{aligned}$$

OBSERVATIONS OF THE TRANSIT OF *MERCURY*, 1907 NOVEMBER 13,

AT AMHERST COLLEGE OBSERVATORY,

By DAVID TODD.

Contact III was observed at

$20^h\,58^m\,12^s$	(Waltham Chronometer). Weight 2.
+4.4	Chronometer correction.
20 58 16.4	Amherst mean time of third (in time) contact.

Observation made with $3\frac{1}{2}$-inch equatorial, Power 120. Atmospheric disturbance very marked. Seeing weight 2 in scale of 5. The observation was made at the new Observatory, whose co-ordinates are given in the *American Ephemeris and Nautical Almanac* for 1910.

Observatory House, Amherst, Mass., 1907 *November* 14.

OBSERVATIONS OF THE TRANSIT OF *MERCURY* AT THE CINCINNATI OBSERVATORY,

By J. G. PORTER, DIRECTOR.

The transit was observed here by Dr. STEWART, with the 11-inch equatorial, and by myself, with the portable 4-inch equatorial.

The times of the third and fourth contacts were noted as follows:

		PORTER	STEWART	
		h m s	h m s	
Third contact	Nov. 13	20 10 27	20 10 38	Gr. M.T.
Fourth contact		20 12 55	20 12 56	"

The sky was clear, but the low altitude of the sun rendered the observations somewhat uncertain.

o CETI,

By CHARLES P. OLIVIER.

LEANDER McCORMICK OBSERVATORY, UNIVERSITY OF VIRGINIA.

Since the early part of 1904, *o Ceti* has been regularly observed here during the time of its maximum brightness. In general, the observations were made with the naked eye, but when the variable was below the fourth magnitude, or at other times when moonlight or slight haze interfered, a field glass magnifying four diameters was used. The observations were made usually by ARGELANDER's method, though when the variable was compared with only one star the differences were estimated in absolute magnitudes. In the tables the columns give the date to the tenth of an hour, the resulting magnitude and the number of comparison stars used. The letter *n* following shows that the observation was not a good one, from bad seeing, haze or some other such cause.

Date	M	d	h	Mag.	No. *
1901	7	19	15.	4.49	7
		21	15.	4.44	5
		31	15.	4.96	4
	8	8	14.	5.43	1
	7	14	15.5	4.52	4
1904	2	4	8.	5.17	2
		6		4.96	4
		11	7.2	4.03	7
		12	7.5	4.02	4
		13	7.7	4.07	2
		15	7.4	3.86	3
		16	7.5	3.71	3
		17	8.1	3.90	1
		22	7.7	3.90	1
		23	7.6	3.64	1
	3	4	7.4	3.26	2
		8	7.7	3.26	2*n*
		9	7.5	2.94	3
		12	7.6	3.17	3
		16	7.5	3.34	1*n*
		18	7.6	3.18	1*n*
1905	2	7	8.0	3.97	3
		10	7.7	3.94	3
		14	8.0	3.81	2
		15	8.5	3.81	2
		24	7.7	3.77	2
		28	8.0	3.64	1
	3	2	7.8	3.54	1*n*
		4	8.1	3.44	1*n*
	12	22	7.1	4.73	4
		29	7.6	4.80	2
		30	10.1	4.69	4
1906	1	6	10.	4.30	2
		12	7.4	4.38	3
		17	10.1	4.38	3
		24	7.7	4.42	2
	2	1	7.6	4.10	1*n*
		14	7.5	5.18	2
		15	7.7	5.18	1
		19	7.6	5.22	3
	11	21	10.8	2.60	4
		22	7.8	2.56	2
		23	12.3	2.54	2
		29	6.7	2.33	1*n*

Date	M	d	h	Mag.	No. *
1906	11	30	8.5	2.44	2
	12	4	7.8	2.23	1
		6	7.7	2.13	1
		7	8.1	2.23	1
		11	8.2	2.02	1
		14	8.6	2.08	1
		18	8.1	2.12	1
		25	8.4	2.12	1
		31	9.	2.50	2
1907	1	1	6.5	2.60	2
		4	8.0	2.60	2
		5	9.6	2.70	2
		7	9.6	2.70	2
		9	10.0	2.74	2
		26	9.0	3.26	2
		28	9.	3.38	2
	2	2	8.7	3.45	2
		6	7.5	3.74	2
		8	8.1	3.74	2
		11	8.2	3.80	2
		12	7.0	3.80	2
		14	8.2	3.83	2
		20	8.	3.90	1*n*
		25	8.	4.16	2*n*
	3	1	8.0	5.39	3
		5	8.	5.50	3
		6	7.8	5.56	2
	10	8	10.3	4.84	2
		9	10.1	4.77	2
		11	9.5	4.77	2
		15	10.0	4.03	2
		16	10.5	3.80	2
	11	4	9.5	3.38	2
		7	7.0	3.38	2
		13	.	3.51	2
		25	7.3	3.72	2
		26	7.5	3.77	2
	12	27	7.0	3.84	2
		29	10.3	3.84	2
		10	8.6	4.17	2
		11	7.3	4.17	2
		12	7.1	4.33	2
		21	7.3	4.91	2

The magnitudes of the comparison stars were taken from *Annals Harvard College Observatory*, Vol. 45. Not using the few scattered observations made in 1901 and 1902, five maxima are shown. The approximate dates are as follows: 1904 March 9, 1905 February 28±, 1906 January 6, 1906 December 11 and 1907 November 6. In

all cases there is some uncertainty as to the exact date, and this is especially true for the maximum of 1905, which seems very late, though the last two observations of that period were made under poor conditions and are of little weight. Taking the first and last date as being true times of maximum brightness, the period comes out 334 days, which accords well with the previous determinations. It will be noted how variable the maximum brightness is, the highest recorded being 2.02 in December 1906 and the lowest 4.30 in January 1906.

MAXIMA AND MINIMA OF LONG-PERIOD VARIABLES, DURING 1906–7,

BY MARY W. WHITNEY.

The predictions are based upon CHANDLER's revised elements *A.J.* 553.

107. *T Cassiopeae.*

The nine observations taken extend from 1906 March 5 to 1907 April 5. Those about maximum are too scattered for satisfactory determination. They indicate a maximum somewhat preceding predicted date 1906 July 9. Minimum occurred about 1906 Dec. 15, predicted date being 1907 Jan. 22.

243. *U Cassiopeae.*

Three observations in 1905 February and March, indicate that maximum followed predicted date (1905 Jan. 31), probably by fifty days or more.

Five observations, 1905 Oct. 4 to Dec. 26, also give a delay (55 days) beyond prediction, 1905 Nov. 3.

Four observations, 1906 Aug. 12 to 1907 April 5, indicate another maximum about 1906 Sept. 30.

432. *S Cassiopeae.*

Eight observations, 1906 Aug. 19 to 1907 April 5, give a maximum (7.6) on 1906 Oct. 12, 19 days before date of prediction.

678. *U Persei.*

Twelve observations, 1905 Oct. 24 to 1907 April 2, give a fairly well determined minimum on 1906 Dec. 15 (10.8), twenty days preceding prediction. Minimum also occurred, probably, about 1906 Jan. 3, and maximum about 1906 June 13, though the observations do not range well for decision of date.

906. *R Trianguli.*

Six observations, 1906 Oct. 25 to Dec. 11, indicate that maximum preceded predicted date Nov. 12.

Six observations, 1907 Jan. 26 to April 2, place minimum (11.5) on 1907 March 3; predicted date April 6.

1511. *T Ursae Majoris.*

Six observations about maximum of 1906 place the date Aug. 17, thirty days later than prediction.

Nine observations about maximum of 1907 give April 28, twenty-five days later than prediction.

4557. *S Ursae Majoris.*

Nine observations, 1906 Aug. 22 to 1907 June 7, place maximum (7.8) on 1906 Nov. 10, and minimum (11.0) on 1907 March 9, both close to predicted dates Nov. 13 and March 11.

4948. *R Canum Venaticorum.*

Six observations, 1906 Feb. 28 to Aug. 15, locate maximum on 1906 May 3, sixty-four days earlier than prediction.

The maximum of 1905 (*A.J.* 586) gave a like value for O – C.

5237. *R Bootis.*

Four observations, not well arranged, indicate a minimum near the predicted date 1906 June 8.

5677. *R Serpentis.*

Five observations, 1906 May 21 to Sept. 1, place maximum on 1906 June 22. The predicted date is Aug. 8.

5889. *U Herculis.*

Five observations, 1906 May 21 to Sept. 3, all lie on the downward slope. Maximum preceded prediction, May 28.7, by several days, perhaps twenty.

7085. *RT Cygni.*

A few observations, none well disposed, near the predicted maxima of 1905 Dec. 1, 1906 June 10 and 1906 Dec. 17, indicate little or no deviation from the computed dates.

7192. *Z Cygni.*

Six observations locate maximum on 1906 Sept. 28. The third observation fell on the predicted date, Oct. 13, but the star was then fainter than at times of preceding observations.

8324. *V Cassiopeae.*

Thirteen observations, 1905 March 11 to 1907 March 11, too scattered to give satisfactory values by curve, indicate that the dates of maximum and minimum do not vary much from those given by the elements. Possibly they occurred from ten to twenty days earlier than prediction.

Vassar College Observatory, 1907 July 9.

SUNSPOT OBSERVATIONS,

MADE AT BERWYN, PENNA., WITH A 4½-INCH REFRACTOR,

By A. W. QUIMBY.

1907	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
July 1	6	–	1	4	2	fair
*2	9	1	1	6	2	poor
*3	7	–	1	6	2	"
*4	6	–	1	10	2	fair
5	6	–	1	20	3	good
6	4	–	1	23	3	fair
7	6	–	1	30	2	good
8	6	–	1	18	2	fair
9	5	1	2	24	2	good
10	5	–	2	24	2	"
11	10	1	3	27	2	fair
12	6	3	5	20	2	"
13	7	–	4	27	2	"
14	5	–	3	38	1	"
15	6	2	5	70	2	good
16	6	–	4	70	2	"
17	7	–	3	62	2	fair
18	6	–	2	70	3	"
19	6	–	2	60	2	"
20	8	2	4	22	2	"
21	6	–	4	21	3	"
22	6	1	4	18	4	"
23	6	–	4	18	3	"
24	6	1	2	10	2	poor
25	6	2	4	18	3	fair
26	5	–	2	13	2	"
27	6	1	3	23	3	"
28	5	2	4	33	4	good
29	6	–	4	32	3	"
30	6	–	4	18	3	fair
31	6	–	4	10	2	"
Aug. 1	5	–	4	15	3	good
2	6	1	5	15	2	poor
3	6	–	5	13	1	fair
4	6	–	5	11	2	"
5	6	1	5	17	2	"
6	6	0	5	26	3	fair
7	6	1	5	18	3	poor
8	7	–	5	13	3	"
9	4	–	3	11	1	"
10	3	1	4	11	2	"
11	5	–	4	8	2	"
12	5	–	4	13	1	good
13	7	–	4	9	1	fair
14	7	1	4	23	2	"
15	6	–	4	25	3	"
16	6	1	4	25	4	"
17	6	–	4	31	2	"
18	7	–	4	21	1	poor
19	6	–	4	23	2	fair
20	7	–	4	21	3	"
21	7	–	4	21	3	"
22	6	–	4	25	2	"
*23	6	–	3	15	2	"
24	5	–	3	11	3	"
25	6	–	3	5	3	"
26	6	–	2	3	3	"
27	6	1	2	3	3	"

1907	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Aug. 28	9	–	2	42	2	good
29	6		2	25	3	fair
30	5	1	3	33	3	"
31	6	–	2	32	3	"
Sept. 1	6	2	4	33	3	"
2	6	–	3	26	3	"
3	7	–	3	36	3	"
4	6		3	38	3	"
5	11	–	3	30	2	"
6	5	3	6	58	4	good
7	6	–	6	38	4	fair
8	5	2	8	22	4	poor
9	10		3	11	–	v. poor
11	9	2	6	18	2	poor
12	8	–	6	38	4	fair
13	8	–	5	76	4	v.good
14	6	–	5	48	4	fair
15	6	–	4	37	4	"
16	6	–	4	27	5	"
*17	7	1	4	12	1	poor
*18	2	1	4	14	2	"
20	1	2	6	22	1	fair
21	7	–	6	22	2	"
22	7	1	6	18	2	"
24	7		4	18	2	"
25	7	1	4	26	3	"
26	7		4	26	2	"
27	7	1	5	35	3	"
28	7	2	7	34	3	"
29	3		5	32	3	"
30	1	1	7	22	2	"
Oct. 1	7	1	7	18	3	"
2	7	–	7	18	4	"
3	7	1	7	22	4	"
4	3	–	5	10	3	poor
5	7	–	5	7	3	fair
6	7	–	4	8	3	"
7	7	–	2	5	2	poor
8	3	–	3	15	2	"
9	7	2	4	20	3	fair
10	7	–	4	33	2	"
11	7	–	4	41	2	"
12	7	1	5	36	2	"
13	7	1	4	42	2	good
14	7	2	5	31	3	fair
15	7	–	5	52	4	good
16	7	–	5	52	3	"
17	7	–	5	72	3	"
18	7	–	5	44	3	fair
19	7	–	5	44	2	"
20	7	–	2	22	2	poor
21	7	1	3	36	1	fair
22	7	2	5	36	2	"
23	7	–	5	22	2	poor
24	7	–	4	19	2	"
25	7	–	4	11	2	fair
26	7	–	2	3	2	"

1907	Time	New Grs.	Total Grs.	Total Spots	Fac. Grs.	Def.
Oct. 27	7	1	1	6	2	poor
28	11	1	2	7	1	"
29	8	–	2	7	1	"
30	7		2	9	2	fair
31	7	1	3	18	2	"
Nov. 1	7	–	3	11	1	poor
3	7	–	3	15	2	fair
4	7	1	4	11	2	poor
5	7	–	3	11	2	fair
7	3	–	2	7	–	poor
8	7	3	5	22	2	fair
9	7	1	5	28	2	"
10	2	1	3	20	–	poor
11	8	1	5	60	1	fair
12	9	–	3	20		poor
13	8	–	4	50	3	fair
14	8	–	4	48	3	"
15	7	–	3	42	3	"
16	7	–	2	32	2	"
17	7	1	3	42	2	"
19	8	2	5	18	3	"
20	8	–	3	22	2	poor
22	9	2	4	12	1	"
23	8	–	3	16	1	"
25	9	–	3	12	1	fair
26	9	–	2	6	–	poor
27	8	–	2	5	–	fair
28	8	–	–	–	–	v. poor
29	8	–	–		2	fair
30	8	1	2	3	3	"
Dec. 1	8	–	1	2	2	poor
2	9	–	1	3	1	fair
3	8	–	1	3	1	"
4	11	1	2	5	1	"
5	8	1	3	5	1	"
6	9	1	4	15	1	poor
7	8	1	5	13	2	"
8	8	1	5	17	1	fair
9	8	–	5	13	1	poor
10	8	–	5	15	1	"
11	9	–	4	19	–	"
12	9	–	4	24	1	fair
13	9	–	3	29	3	"
16	3	2	3	17	2	"
17	8	–	3	21	2	"
19	9	1	4	15	2	"
20	8	–	4	15	2	"
21	8	–	4	13	2	"
22	8	–	3	7	2	"
23	3	–	2	3	1	poor
24	8	–	3	12	2	fair
25	8	–	3	7	1	fair
26	8	–	1	1	1	poor
27	12	1	1	1	1	"
28	8	–	2	3	1	"
29	9	1	3	7	2	fair
31	8	–	3	5	1	poor

* 2½-inch refractor.

OBSERVATIONS OF COMETS,

MADE WITH THE 16-INCH EQUATORIAL OF THE CINCINNATI OBSERVATORY,

By J. G. PORTER, Director.

1907 Cin. M.T	✱	Comp.	Δα	Δδ	App. α	App. δ	log pΔ		Red. to App. Pl.	
			Comet 1907 *a*.							
h m s			m s	′ ″	h m s	° ′ ″			s	″
Mar. 15 7 38 40	1	8 , 6	+0 39.79	+3 19.4	6 48 17.93	−12 42 45.3	8.616	0.838	+0.15	−15.3
16 8 27 19	3	6 , 6	+1 46.88	−4 19.1	6 45 59.75	−11 49 28.5	9.177	0.828	+0.10	−15.1
20 8 28 57	5	10 , 6	−0 6.71	0 38.3	6 38 1.99	− 8 33 54.5	9.287	0.805	−0.01	−14.5
21 8 56 48	7	8 , 6	+0 2.09	+0 5.9	6 36 13.95	− 7 46 46.7	9.406	0.795	−0.04	−14.3
Apr. 1 7 48 44	9	8 , 6	−2 12.42	+2 43.6	6 21 49.69	− 0 28 14.5	9.370	0.746	−0.34	−11.9
2 7 49 29	10	8 , 6	+0 53.04	−4 34.7	6 20 53.34	+ 0 6 59.6	9.388	0.741	−0.38	−11.7
			Comet 1907 *e*.							
Oct. 18 16 22 1	11	6 , 6	−2 15.72	2 40.3	8 16 6.96	− 7 17 21.2	n9.384	0.793	+1.15	+ 2.6
Nov. 4 14 0 10	12	10 , 10	−1 2.39	+3 56.4	6 17 55.55	+ 9 17 9.2	n9.220	0.645	+2.41	+ 0.2
6 12 39 21	13	10 , 12	−1 2.71	−5 6.6	5 51 39.19	+12 34 28.9	n9.403	0.617	+2.63	+ 0.5
8 9 31 30	14	10 , 10	+0 48.97	+2 43.7	5 23 16.77	+15 50 32.0	n9.651	0.672	+2.84	+ 1.6
9 11 40 50	15	8 , 8	+1 18.55	−4 31.8	5 5 22.39	+17 42 48.6	n9.415	0.546	+2.99	+ 2.3
13 9 53 10	16	8 , 8	−1 49.51	−1 54.0	3 57 55.91	+23 30 49.7	n9.496	0.470	+3.30	+ 6.6
25 6 2 40	17	8 , 6	+0 12.56	−4 26.6	1 23 21.81	+28 31 26.7	n9.575	0.416	+3.03	+19.2
26 10 46 12	18	10 , 6	+0 8.02	−2 29.7	1 14 3.66	+28 27 42.1	9.388	0.298	+2.96	+19.9

Mean Places of Comparison-Stars for the beginning of the year.

✱	α	δ	Authority	✱	α	δ	Authority
	h m s	° ′ ″			h m s	° ′ ″	
1	6 47 37.99	−12 45 49.1	Mun. 2153 & comp. with	10	6 20 0.68	+ 0 11 46.0	Nicolajew, A.G. 1666
2	6 44 9.25	−12 41 11.6	Rad.₂ 1694	11	8 18 21.53	− 7 14 43.5	Wien-Ott., A.G. 3206
3	6 44 12.77	−11 44 54.3	B.D.−11°1644 com. with	12	6 18 55.53	+ 9 13 12.6	Leipzig II, A.G. 2864
4	6 43 28.11	−11 37 19.8	Pos. Med. 775	13	5 52 39.30	+12 39 35.0	" I, " 1915
5	6 38 8.71	− 8 33 1.7	B.D.−8°1529 comp. with	14	5 22 24.96	+15 47 46.7	Gr.9-Yr.513, Ber.A, A.G.1529
6	6 39 35.61	− 8 19 43.1	Wien-Ott., A.G. 2127	15	5 4 0.85	+17 47 18.1	Berlin A, A.G. 1409
7	6 36 11.90	− 7 46 38.3	B.D.−7°1518 comp. with	16	3 59 42.12	+23 32 37.1	Berlin B, A.G. 1318
8	6 36 41.56	− 7 54 11.9	Wien-Ott., A.G. 2098	17	1 23 6.22	+28 35 34.1	Cambridge, A.G. 828
9	6 24 2.45	− 0 30 46.2	Nicolajew, A.G. 1699	18	1 13 52.68	+28 29 51.9	" " 755

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